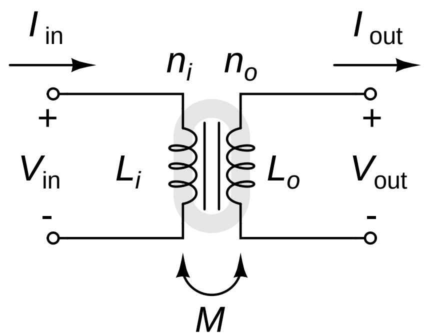
BENG 186B Winter 2025 HW #2

Due *Friday, January 31st* at 11:59pm on Canvas (Gradescope)

Parts of the homework done collectively in-class (Friday, January 24) are indicated below with **[IC]**.

1. [**20 pts**] A transformer is an electrical circuit that uses magnetic inductive coupling between coils at the input and output to transfer AC electrical energy from one side to the other, in either direction. An ideal transformer does this energy transfer without loss in net energy, by preserving the magnetic energy exchanged between the two coils. In particular, for the ideal transformer all magnetic field lines are contained in a ferromagnetic core loop so that all magnetic flux is shared between both coils.



The electrical characteristics of the transformer can be modeled according to the above equivalent circuit:

$$V_{in} = j\omega L_i \, I_{in} - j\omega M \, I_{out}$$
$$V_{out} = j\omega M \, I_{in} - j\omega L_o \, I_{out}.$$

For the ideal transformer, the inductance L_i of the input coil, inductance L_o of the output coil, and mutual inductance M between the coils are given by:

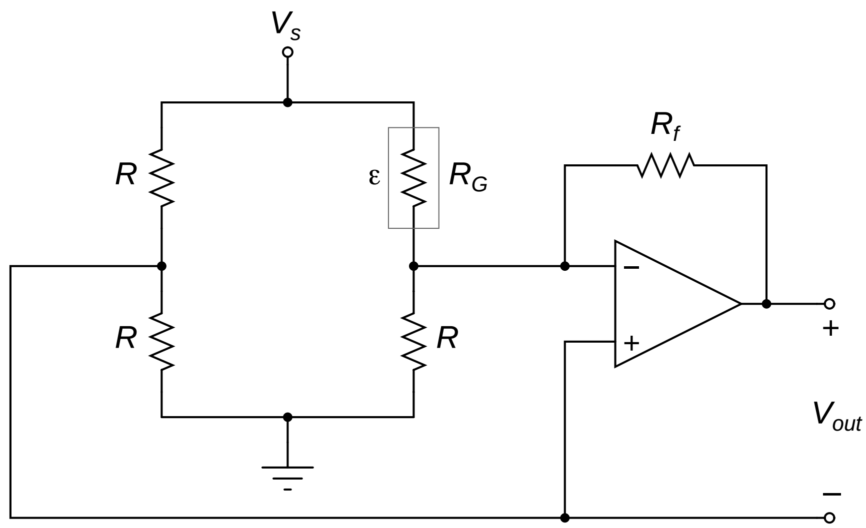
$$L_i = \mu \, {n_i}^2 \, G$$
$$L_o = \mu \, {n_o}^2 \, G$$
$$M = \mu \, n_i n_o \, G$$

where μ is the magnetic susceptibility of the core, n_i and n_o are the number of turns in the windings of the input and output coils, and G is a common geometric factor.

(a) **[IC]** [10 pts] Show that for an ideal voltage source at the input of the ideal transformer, the Thévenin equivalent at the output is an ideal voltage source proportional to the input voltage, with zero output impedance. Find the voltage gain $V_{out}(j\omega) \, / \, V_{in}(j\omega)$, and explain how it depends on the ratio of number of turns in the two coils.

(b) [10 pts] Now find the Norton equivalent at the output of the ideal transformer for an ideal current source at the input. Identify the output impedance, and find the current gain $I_{out}(j\omega) \, / \, I_{in}(j\omega)$ at zero output voltage. Compare this current gain for the current-driven transformer with the voltage gain that you found in (a) for the voltage-driven transformer, and explain.

2. [**20 pts**] Given below is an active sensor circuit for highly sensitive measurement of strain. The circuit combines a Wheatstone bridge with an active output stage implementing an inverting amplifier. R_G is a strain gauge with nominal resistance $R_{nom} = 100\,\text{k}\Omega$ and gauge factor $G = 100$. The other resistances in the bridge are identical, $R = R_{nom}$. The Wheatstone reference voltage is $V_s = 3$ V. The active feedback resistance is $R_f = 1\,\text{M}\Omega$, and you may assume that the operational amplifier (opamp) is ideal (open-loop gain $A_v = \infty$, and input resistance $R_{in} = \infty$).

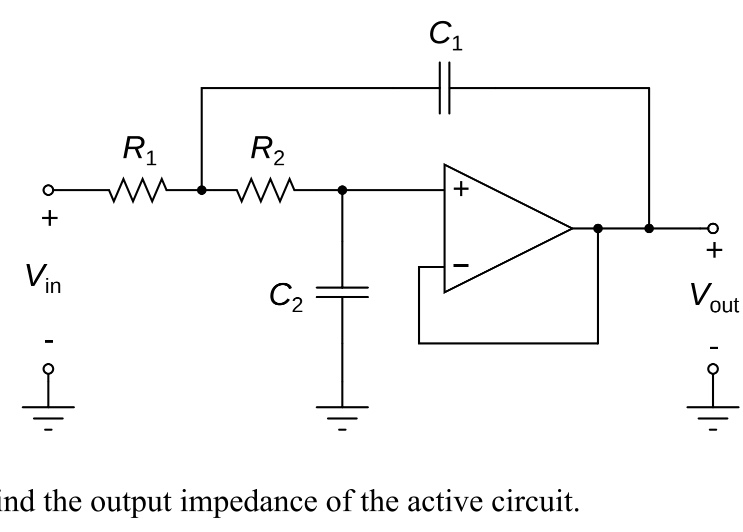


(a) **[IC]** [5 pts] Find the output impedance of the active sensor circuit.

(b) [10 pts] Find the voltage output V_{out} as a function of strain ϵ.

(c) [5 pts] Find the sensitivity and offset of the transducer, and compare with the standard Wheatstone bridge without the active output stage.

3. [**30 pts**] Consider the active circuit below depicting a second-order lowpass filter, known as a Sallen-Key filter. Assume the opamp in the active circuit is ideal.



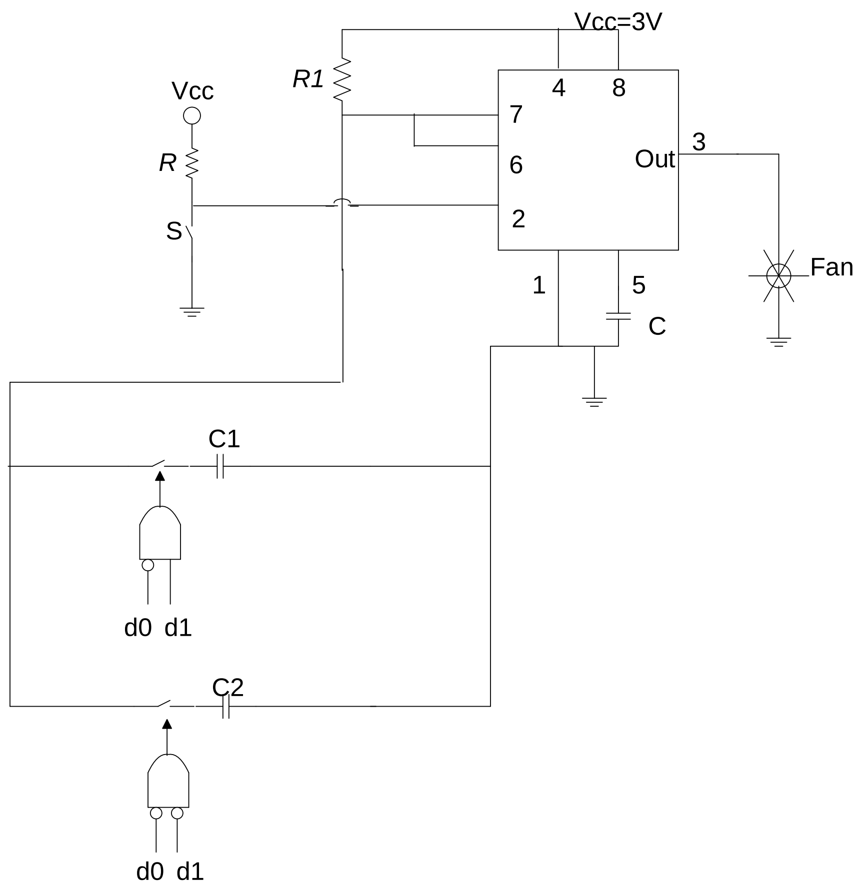
(a) **[IC]** [5 pts] Find the output impedance of the active circuit.

(b) [10 pts] Find the transfer function $H(j\omega) = V_{out}(j\omega)\,/\,V_{in}(j\omega)$.

(c) [5 pts] Find the range of natural frequencies ω_n, and the range of damping factors ζ, that can be realized with this circuit.

(d) [10 pts] Find suitable values for the components for a critically damped second-order low-pass response with cut-off frequency $f_c = 100$ Hz ($\omega_n = 2\pi f_c = 628.3$ rad/s).

4. **Design Problem [30 pts]:** As a BENG 199 intern in a biochemistry lab on campus you are preparing an experiment for which you need access to a fume hood. The fume hood available to you in the lab has a control panel for the exhaust fan with three mysterious switches. The control panel was designed by a former lab mate who since left for graduate school elsewhere. The control manual for the fume hood is missing, but luckily you find a copy of the circuit schematic below. You decide to carefully study the circuit schematic and the printed circuit board before testing its function by trying the different states of the switches S, d_0 and d_1 on the panel. In addition to the switches, the printed circuit circuit has a 3V supply, a 555 timer integrated circuit (IC), another IC with four NOR logic gates, and passive components $R = 100$ kΩ, $R_1 = 90$ kΩ, $C = 10$ nF, $C_1 = 50$ μF, and $C_2 = 250$ μF.



(a) [10 pts] You observe that no matter the state of the binary input control switches d_0 and d_1, the exhaust fan stays off when the pushbutton switch S is untouched (left in the OFF position). You also observe that when d_0 is in the binary 0 state, whenever the pushbutton S is pressed (goes briefly in the ON position) the exhaust fan goes on for some time, and then goes off until S is pressed again. The time duration of the fan blowing depends on the state of d_1. Explain why this happens, and find the time durations when the d_1 switch is in state 0, versus when it is in state 1.

(b) [10 pts] You find that when d_0 is in the binary 1 state, the exhaust fan goes on for a very brief time (about 50 ms) when the pushbutton S is pressed, regardless of the binary state of d_1. Why might this be happening?

(c) [10 pts] Can you figure a possible way how the logic for the d_0 and d_1 control in the circuit schematic was implemented with three NOR gates of the logic IC on the printed circuit board?