Parts of the homework done collectively in-class (Friday, February 14) are indicated below with **[IC]**.

1. **[30 pts]** An equivalent circuit model of the contact interface between skin and electrode, with electrolytic gel in between, is shown below (J.G. Webster, *Medical Instrumentation*, 4th Ed., Fig. 5.8).
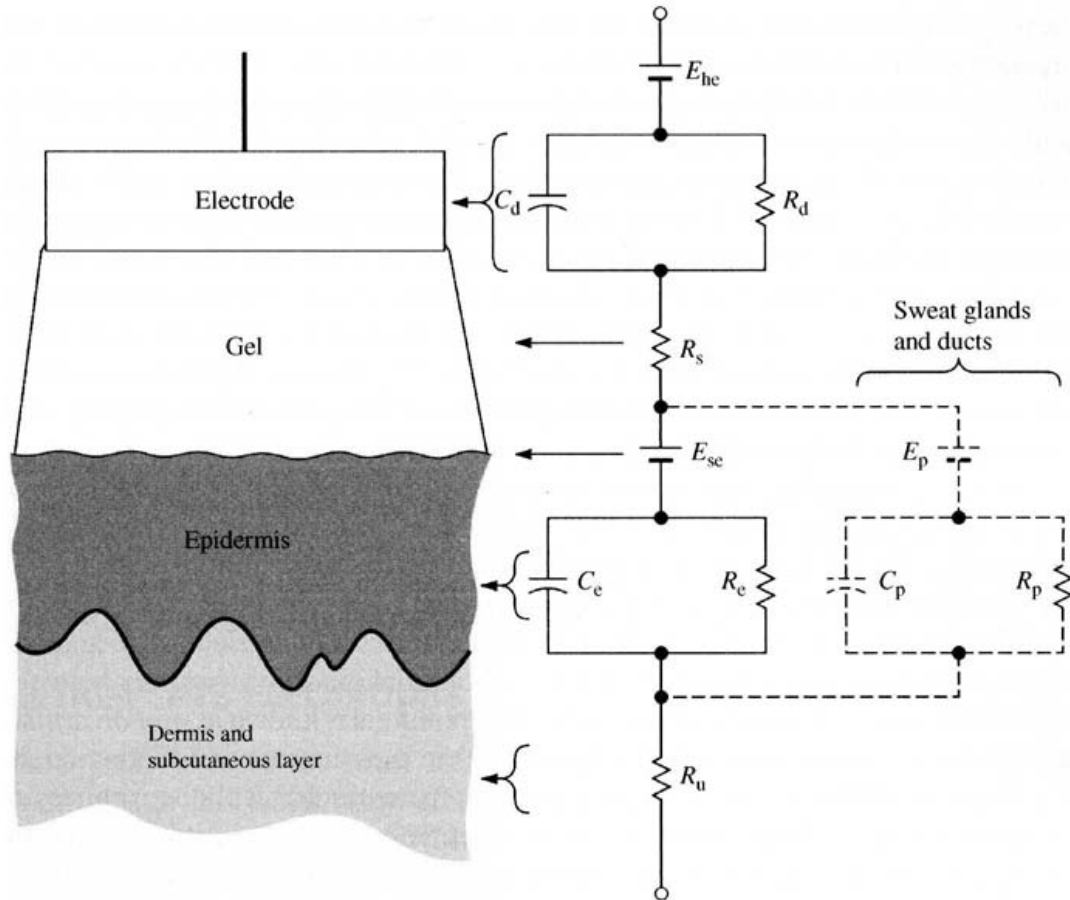
(a) **[IC]** [5 pts] What is E_{hc} when the electrode is made of silver coated with a thin layer of silver chloride?

(b) [10 pts] Ignoring the effect of sweat glands and ducts in this model, and considering that the time constant $R_e C_e$ of the epidermis-dermis interface internal to the skin is practically zero on any time scales of interest, reduce this model to an approximate standard electrolyte-electrode form with a single half-cell potential, a single parallel combination of double-layer resistance and capacitance, and a single series resistance.

(c) [10 pts] For $C_d = 10$ nF, $R_d = 100$ kΩ, $R_s = 10$ kΩ, $C_e = 10$ pF, and $R_e = R_u = 100\,\Omega$, show the Bode plot (amplitude and phase as a function of frequency) of the contact impedance for your reduced model, superimposed with that for the full model in the absence of sweat. How good is the reduced model at frequencies of physiological interest?

(d) [5 pts] What do you expect the effect of sweat to be? Does the contact impedance increase, or decrease, and why?

2. [**30 pts**] The simplified electrophysiological model of the electrocardiagram (ECG) postulated by the Dutch cardiologist pioneer Einthoven predicts that the electrode voltage V is given by the dot product of a cardiac vector **M**, and a lead vector **a** which points from one electrode to another located on the torso. Modeling the electrical dipole of depolarization and repolarization along the systolic wavefront in the beating heart, the cardiac vector moves around in space and is different during each wave of the cardiac cycle. During the QRS complex the cardiac vector is greatest in magnitude, and points down vertically from the heart to the left leg. This cardiac model is only valid for a fixed distance r from the heart to any of the electrodes of interest; indeed this distance r is approximately the same for electrodes placed on the left shoulder, right shoulder, and left hip.

   (a) [**IC**] [10 pts] Estimate the magnitude of the cardiac vector **M** during the QRS complex for the following physiological parameters for a typical adult: average torso volume conductivity $\sigma = 0.1\ \Omega^{-1}\mathrm{m}^{-1}$, peak dipole current $I = 100\ \mu\mathrm{A}$, dipole spacing $d = 1$ cm, and electrode distance $r \approx 25$ cm.

   (b) [10 pts] Estimate the corresponding electrode voltages measured on the RA, LA, and LL electrodes in the frontal plane, relative to the Wilson central terminal.

   (c) [10 pts] How do these results change for a 2-year old toddler, with an average electrode distance of 10 cm rather than 25 cm?

3. **Design Problem [40 pts]:** Design a complete 6-lead frontal electrocardiogram (ECG) recording system. The system takes as input the voltages on 4 electrodes RA, LA, LL and RL located on the right shoulder, left shoulder, left hip, and right hip respectively, and outputs the ECG amplified voltage any of the 6 frontal ECG leads (I, II, III, aVL, aVR, and aVF) on demand. Your design should include a single instrumentation amplifier with a low-pass corner of 1 kHz and a gain of 1,000, a driven right leg (DRL) auxiliary amplifier with gain -100 from the Wilson central terminal to the RL electrode for common-mode suppression, and analog multiplexers (digitally controlled switches) to select the desired lead from the input electrodes connected to the system. The lead should be selected by three binary control variables d_0, d_1, and d_2:

$d_2 d_1 d_0$	Output
0 0 0	I
0 0 1	II
0 1 0	III
1 0 0	aVL
1 0 1	aVR
1 1 0	aVF

The circuit is powered by a single 1.2V battery, and should not draw more than 12 μA from/into the body.

As before, show a complete circuit diagram and specify all types and values for active and passive components. Justify design choices to meet the specifications. You may find it useful to check the operation of your circuit and observe its behavior using an electronic simulator such as the Falstad Java applet with several circuit examples posted at https://www.falstad.com/circuit/.