1. A catheter of internal diameter \( d = 2 \text{ mm} \) and length \( l = 20 \text{ cm} \) is inserted in the blood vessel to measure intravascular blood pressure \( P \). The catheter is filled with fluid of viscosity \( \eta = 0.003 \text{ Pa s} \). A chamber filled with the same fluid at the other end of the catheter has a cylindrical geometry with \( D = 1 \text{ cm} \) internal diameter and \( L = 1 \text{ cm} \) length. A circular piston terminating the cylindrical chamber on the other side is suspended by a single strain gauge with gauge factor \( G = 100 \), Young’s modulus \( E = 20 \text{ kPa} \), and effective wire length \( L = 10 \text{ cm} \) and cross-section area \( A_G = 1 \text{ mm}^2 \). Hence the movement of the piston due to expansion of the volume under the pressure \( P_c \) of the fluid in the chamber results in strain proportional to this pressure. In turn, this strain is transduced to voltage \( V \) by a balanced Wheatstone bridge with identical nominal resistances \( R = 100 \text{ k}\Omega \) including that of the strain gauge, and with supply voltage \( V_s = 1 \text{ V} \). You may assume no air bubbles are present in the fluid-filled catheter and chamber.

(a) Find the sensitivity of the voltage output \( V \) with respect to the pressure \( P_c \) in the fluid chamber.

(b) Find the compliance \( C \) of the fluid chamber accounting for the piston and the strain gauge.

(c) Find the transfer function \( V(j\omega) / P(j\omega) \) of the intravascular blood pressure transducer. Identify the DC gain, resonance frequency, and damping factor of the second-order system.

2. Due to viscosity, blood flow in a vessel has a parabolic velocity profile that is zero in direct proximity to the vessel interior wall. In cylindrical coordinates: \( v(\rho) = v_{\text{peak}} \left(1 - \rho^2/R^2\right) \), where \( v_{\text{peak}} \) is the peak velocity reached at the midline of the cylindrical vessel, \( \rho \) is radial distance from the midline, and \( R \) is the radius of the vessel interior wall. It is important to account for this velocity profile to get an accurate estimate of volumetric blood flow (in units L/s) from the readings of Doppler, electromagnetic, and other flow transducers.

(a) Find the volumetric blood flow from a Doppler ultrasonic velocity measurement registering a relative frequency shift \( \Delta f/f = 0.0001 \), where the focused transmit and receive beams intersect the midline of a blood vessel at 45° angles with respect to the blood vessel. The diameter of the blood vessel is 3 mm, and the speed of ultrasonic wave propagation in body tissue and blood is \( c = 1,500 \text{ m/s} \).

(b) Find the volumetric blood flow now using an electromagnetic flow transducer spanning the 1 cm diameter of an artery, which registers a voltage of 1.5 mV for a transversal magnetic field of 100 mT. **Hint:** To find the relation between voltage and peak velocity, integrate the electric field between the two electrodes, along a straight line that crosses the vessel midline at the center (i.e., with \( \rho \) ranging from \(-R \) to \( R \)). You may assume that the magnetic field is constant throughout.

3. **Design Problem:** Design an automated sphygmomanometer that includes a digital readout of systolic and diastolic pressure and heart rate.

You have the following components available:

(a) Cuff with arbitrary digital control over applied pressure;

(b) Sonic transducer contacting skin over artery inside the cuff, with digital readout of the acquired sound waveform;
(c) Alphanumeric display;
(d) Programmable microcontroller;
(e) Start button and sleep button;
(f) Power supply with power on/off switch.

Sketch a diagram of the system, and outline the algorithm running on the microcontroller to control the cuff, acquire the sound signal, and compute and display the systolic pressure, diastolic pressure, and heart rate.

You do not need to write actual code, but be quantitative in your description with sufficient information for a programmer to be able to complete the design. You may use timing diagrams and equations to define the sequence of control variables, the corresponding measured quantities, and the computation of the output variables.

Define important parameters in the algorithm, and give numerical values based on the physiological range that you expect.