1. A variant on the Clark PO\textsubscript{2} electrode is used to measure the concentration of glucose in a blood sample. The Clark glucose electrode is based on the oxidase of glucose producing hydrogen peroxide:

\[
\alpha\text{-}D\text{-}glucose + O_2 \rightleftharpoons \text{D-glucono-1,5-lactone} + H_2O_2
\]

which in turn is reduced at the platinum electrode to form hydroxyl:

\[
H_2O_2 + 2 \, e^- \rightleftharpoons 2 \, (OH)^-
\]

Finally, (OH\textsuperscript{-}) exchanges with Cl\textsuperscript{-} in a saturated KCl salt buffer, and Cl\textsuperscript{-} combines with oxidation of Ag at the Ag/AgCl electrode to close the electrical circuit for amperometric measurement between the electrodes:

\[
2 \, KCl + 2 \, (OH)^- \rightleftharpoons 2 \, KOH + 2 \, Cl^- \\
2 \, Cl^- + 2 \, Ag \rightleftharpoons 2 \, AgCl + 2 \, e^-
\]

(a) What type of semi-permeable membrane is needed between the sample chamber and the reference chamber to measure glucose rather than oxygen in the blood? What other conditions need to be satisfied in order for the amperometric measurement with this Clark sensor to be linear in glucose concentration in the blood flowing through the sample chamber?

(b) Assuming these conditions are met, find the relationship between the current measured between the electrodes, the concentration of glucose in the blood sample, and the flow rate of the blood through the sample chamber.

(c) A new Ag/AgCl electrode is used, which has a net deposit of 100 mg AgCl over its 10 g Ag core. What is the total amount of glucose going through the sample chamber that can be detected before the electrode malfunctions? You may assume that the KCl solution is constantly being replenished. Explain.

2. Hemoglobin (Hb) in red blood cells serves important functions in the cardiocascular and respiratory systems by carrying oxygen from the lungs to organs across the body, and carrying carbon dioxide back to the lungs. Pulse oximetry is extensively used for non-invasive measurement of SO\textsubscript{2}, the saturation of oxygen in the hemoglobin, by measuring the absorbance \( A \) at two different wavelengths \( \lambda_1 \) and \( \lambda_2 \). The saturation SO\textsubscript{2} is then obtained as a function of the ratio of these two absorbances, \( SO_2 = f(A(\lambda_1)/A(\lambda_2)) \). Here we will consider an extension of pulse oximetry to also and simultaneously measure SCO\textsubscript{2}, the saturation of carbon dioxide in the hemoglobin, by conducting such measurements of absorbance at three different wavelengths. For simplicity we will assume that hemoglobin in individual red blood cells can be in one of three states: reduced (Hb) carrying nothing, oxygenated (oxyhemoglobin HbO\textsubscript{2}) carrying oxygen, or carbonated (carbaminohemoglobin HbCO\textsubscript{2}) carrying carbon dioxide. Hence:

\[
[HbO_2] = SO_2 \, W \tag{1}
\]

\[
[HbCO_2] = SCO_2 \, W \tag{2}
\]

\[
[Hb] = (1 - SO_2 - SCO_2) \, W \tag{3}
\]

where \( W \) is the volume concentration of hemoglobin in any of its states.
(a) Given absorptivities $a_r(\lambda)$ for Hb, $a_o(\lambda)$ for HbO$_2$, and $a_c(\lambda)$ for HbCO$_2$, use Beer’s law to write the absorbance $A(\lambda)$ in blood with oxygen saturation $SO_2$ and carbon dioxide saturation $SCO_2$, with hemoglobin volume concentration $W$, and with optical path length $L$, as a function of wavelength $\lambda$.

(b) Show that reliable non-invasive measurement of this absorbance $A(\lambda)$ in a pulsing blood vessel surrounded by stationary tissue illuminated at a wavelength $\lambda$ is obtained as the ratio of the AC pulsing to DC stationary components of measured transmitted or reflected intensity exiting the tissue. Explain how the effective path length $L$ relates to the pulsing variation in vessel diameter.

(c) From three such measurements $A(\lambda_1)$, $A(\lambda_2)$ and $A(\lambda_3)$ at three wavelengths $\lambda_1$, $\lambda_2$ and $\lambda_3$, estimate both the oxygen and carbon dioxide saturations $SO_2$ and $SCO_2$. Under what condition on the absorptivities at these three wavelengths does a unique solution exist?

(d) Show how these three measurements of absorbance can be made using three light emitting diodes (LEDs) each radiating at a different wavelength, and a single photodetector. What are some of the advantages of using a single photodetector rather than three, one each for the three wavelengths?

(e) Explain the relationships between $SCO_2$, $PCO_2$, and pH in blood, and the role of bicarbonate in the latter.

3. **Design Problem:** Repeat your design of the single-lead three-electrode ECG recording system that you did for HW #4, but now with a rechargeable battery and with additional safeguards against bodily harm and instrument damage. Include in your circuit a wireless charging station (“charger”) that connects to 110 V$_{rms}$ 60 Hz AC wall power using a standard three-prong receptacle, and that inductively couples to your wireless ECG recording system (“recorder”) to charge its battery. The inductive link, when activated by bringing the recorder in close proximity to the charger, serves as a transformer from the line power in the charger to a rectifier and regulator on the recorder to provide a 3V internal supply to the recorder, while also charging the 3V battery. The battery is connected directly to the internal supply, so that when the charger is not active, the recorder still has 3V power until the battery is drained. For simplicity you may assume that the battery charges as long as the regulator provides 3V to it.

Protect the subject and the recorder from accidental overvoltages on any of the three electrodes (signal electrode, reference electrode, and active ground electrode) interfacing with the body. As before, none of these electrodes should draw more than 10 $\mu$A from/into the body. Also protect the recorder from accidental shorting of the voltage output so that no more of 10 mA may be drawn from the output.

All other specifications are identical to the HW #4 assignment design problem. You may copy your solution, or any other solution, of that problem as a starting point for your expanded design. As before, show a complete circuit diagram and specify all types and values for active and passive components, including relative tolerances (in percentage error) for the passive components. Justify design choices to meet the specifications.