

## Lecture 5

# Amplifiers and Active Linear Circuits for Signal Processing

### References

Webster, Ch. 3 (Sec. 3.1-3.4, 3.11, 3.12).

<http://en.wikipedia.org/wiki/Rectifier>

<http://en.wikipedia.org/wiki/Opamp>

[http://en.wikipedia.org/wiki/Active\\_filter](http://en.wikipedia.org/wiki/Active_filter)

# AMPLIFIERS AND SIGNAL PROCESSING

Webster, Chap. 3

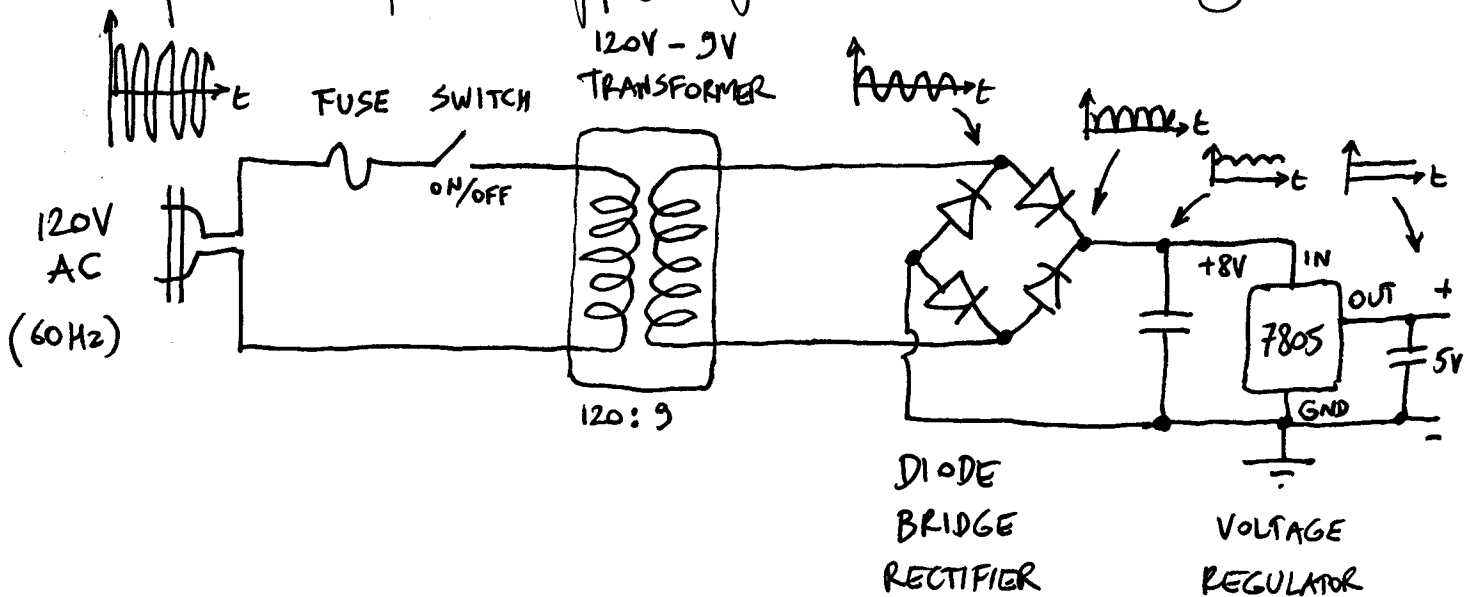
- Passive circuits, made of  $R$ ,  $L$  and  $C$ , cannot amplify signals: they can filter, but they do not provide gain (larger than one) because they do not provide power.
- Active circuits, such as amplifiers, are powered and can provide large amplification. Digital active circuits can further provide logic operations for digital signal processing. They require power supplies to function.

	PASSIVE CIRCUITS	ACTIVE CIRCUITS
COMPONENTS :	$R$ , $L$ , $C$ $M$ (mutual inductance)	Power supplies Op - amps Comparators Timers Digital gates etc...
USE :	Passive filtering	Amplification Buffering Active filtering Signal processing Decision making etc...

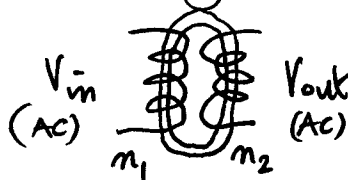
# Power supplies

- Most electronic devices, and all active devices, have a power supply
- Batteries are the easiest, but have limitations

Simple DC power supply from AC wall voltage:



- Transformer: transforms AC voltages through mutual inductance



$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{n_2}{n_1} \quad (\text{for } \omega > 0)$$

- Diode bridge: rectifies AC voltage to DC voltage



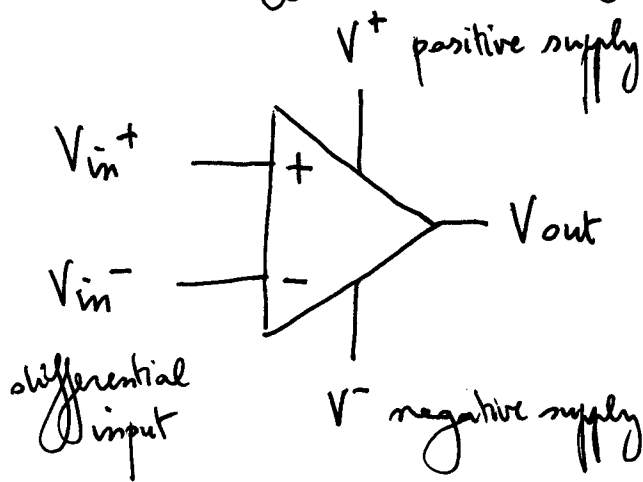
Diodes only conduct in one direction  
 $\Rightarrow$  only the positive part of the voltage waveform comes through

- Regulator: stabilizes DC voltage from IN to OUT  
 as long as  $IN > OUT$ .

LM 7805 : OUT = +5V  
 LM 7812 : OUT = +12V

# - Operational amplifiers (OP-AMPS)

→ differential amplifier with high DC gain



Typical op-amps:

$$V^+ = +12V \text{ max}$$

$$V^- = -12V \text{ max}$$

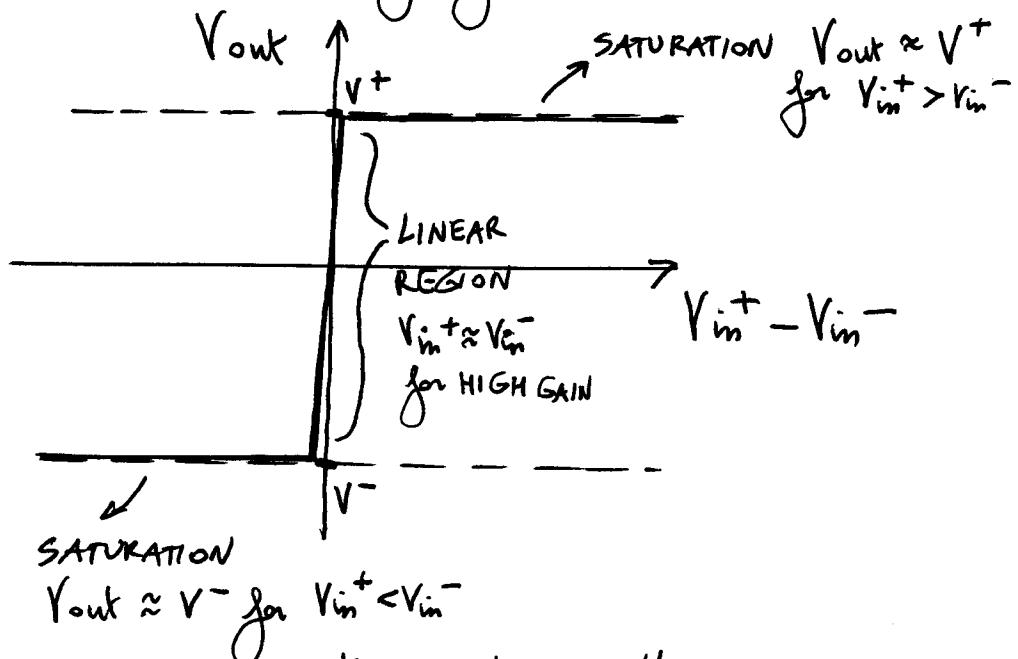
Modern (low voltage) op-amps:

$$V^+ = 5V, \text{ or } 3V$$

$$V^- = 0V \text{ (GND)}$$

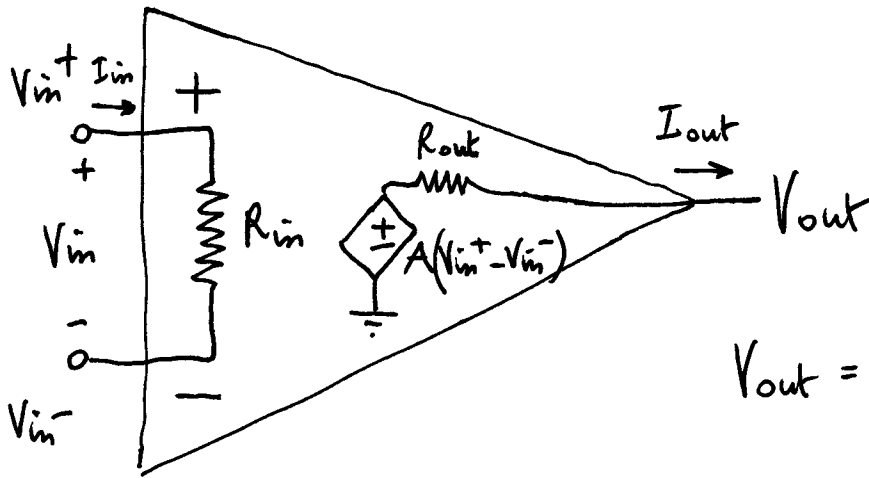
- The voltage supplies  $V^-$  and  $V^+$  are important!

The output  $V_{out}$  cannot go higher than  $V^+$  or lower than  $V^-$ , no matter the voltage gain



Op-amps are designed to operate in the LINEAR REGION, where the output is not saturated, and for which the two inputs are approximately equal  $V_{in}^+ \approx V_{in}^-$  in the limit of HIGH GAIN.

- Linear model of the op-amp :



$$V_{out} = A(V_{in}^+ - V_{in}^-) - R_{out} \cdot I_{out}$$

$$V_{in} = V_{in}^+ - V_{in}^- = R_{in} \cdot I_{in}$$

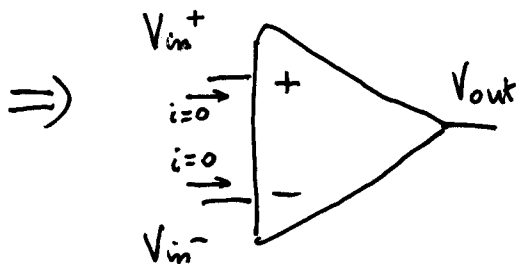
- $A$  : VOLTAGE GAIN (differential)
- $R_{in}$  : INPUT IMPEDANCE (resistance)
- $R_{out}$  : OUTPUT IMPEDANCE (resistance)

- Typical op-amp :  
e.g. "OP27"

- $A \sim 10,000$  (large)
- $R_{in} \sim 1T\Omega = 10^{12}\Omega$  (large)
- $R_{out} \sim 50\Omega$  (relatively small - depends on power consumption)

- "Ideal" op-amp :

- $A \rightarrow \infty$
- $R_{in} \rightarrow \infty$
- $R_{out} \rightarrow 0$



- no current into  $V_{in}^+$  node
- no current into  $V_{in}^-$  node
- $V_{in}^+ \approx V_{in}^-$

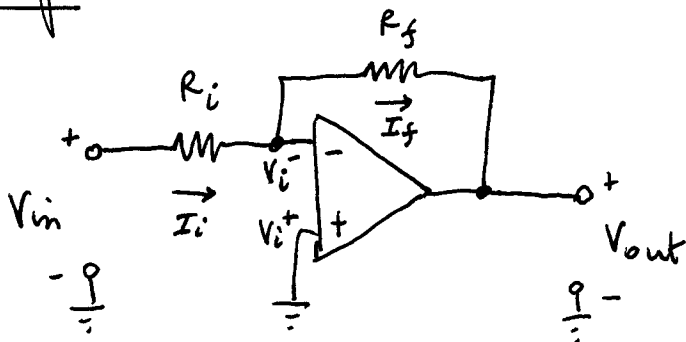
# Analysis of linear active circuits with IDEAL opamps:

PRINCIPLE: Nodal analysis (don't even attempt mesh analysis)

- Label all unknown node voltages
- Apply KCL on all nodes except op-amp OUTPUTS (as well as VOLTAGE SOURCES)
- For each op-amp, add an equation that  $V_{in+} = V_{in-}$

⇒ Should have same number of equations as unknowns  
 → Solve the linear set of equations for the node voltages

Example: INVERTING AMPLIFIER



$$\rightarrow V_i^- = V_i^+ = 0$$

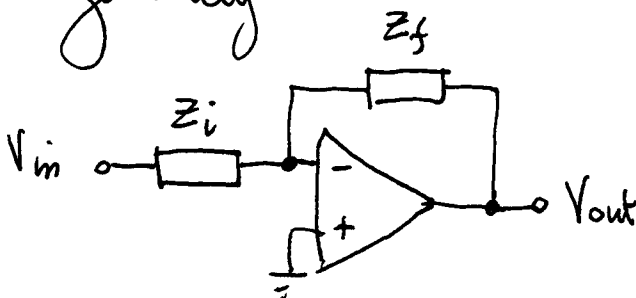
→ KCL @  $V_i^-$ : no current IN the op-amp ( $I_i = I_f$ )

$$\Rightarrow \frac{V_{in}}{R_i} = - \frac{V_{out}}{R_f}, \text{ or}$$

$$V_{out} = - \frac{R_f}{R_i} \cdot V_{in}$$

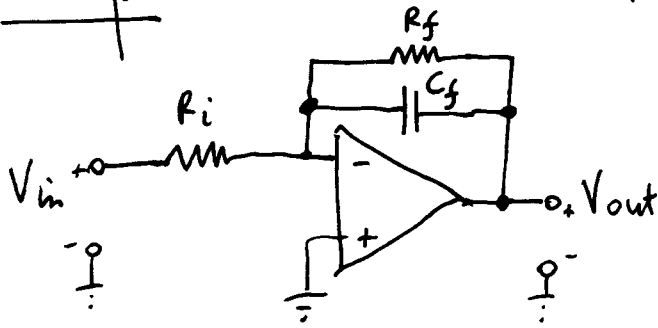
NEGATIVE GAIN (inverting)

More generally:



$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = - \frac{Z_f(j\omega)}{Z_i(j\omega)}$$

Example: ACTIVE FIRST-ORDER LOWPASS FILTER



: Inverting amplifier with

$$Z_i = R_i$$

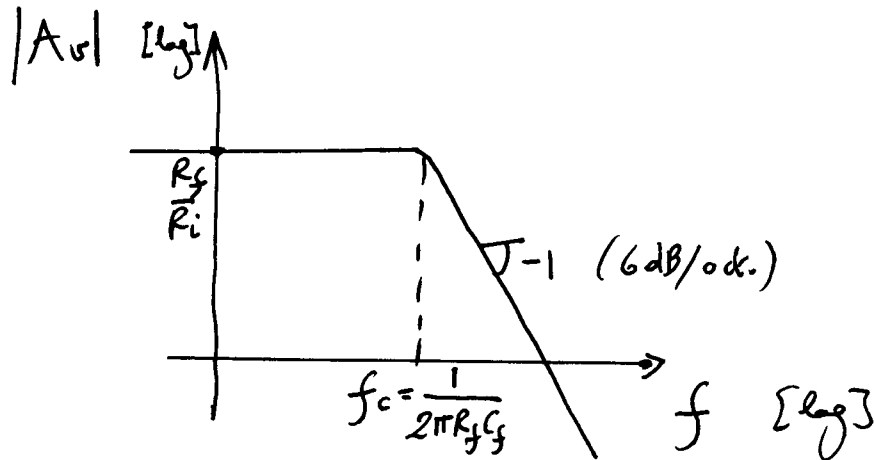
$$Z_f = R_f \parallel \frac{1}{j\omega C_f}$$

↓  
PARALLEL  
combination

$$\Rightarrow \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = - \frac{Z_f}{Z_i} = - \frac{R_f \cdot \frac{1}{j\omega C_f}}{R_i + \frac{1}{j\omega C_f}} = - \frac{R_f}{R_i} \cdot \frac{1}{1 + j\omega R_f C_f}$$

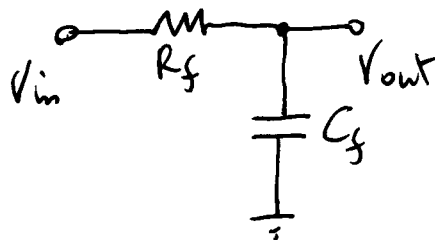
$$\Rightarrow \text{voltage gain } A_v(j\omega) = A_{v0} \cdot \frac{1}{1 + \frac{j\omega}{\omega_c}}$$

with  $\left\{ \begin{array}{l} A_{v0} = - \frac{R_f}{R_i} \\ \text{DC GAIN} \\ \omega_c = 2\pi f_c = \frac{1}{R_f C_f} \\ \text{CUT-OFF} \end{array} \right.$



In contrast to the PASSIVE R-C FILTER, the active filter has a gain that can be greater than unity (in absolute value)

PASSIVE R-C:

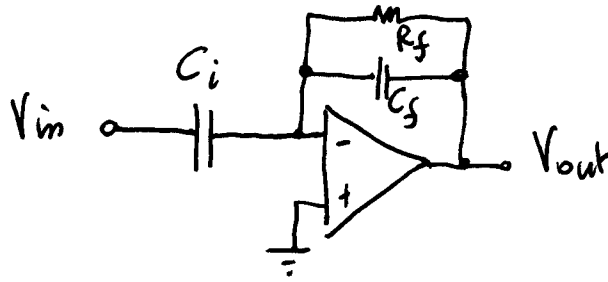


$$A_{v0} = 1$$

$$\omega_c = 2\pi f_c = \frac{1}{R_f C_f} \text{ (SAME)}$$

Variations:

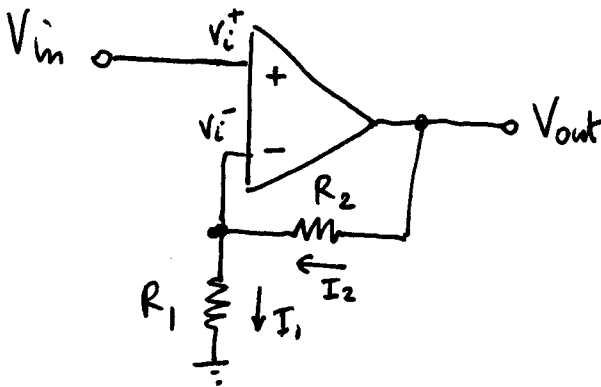
ACTIVE FIRST-ORDER HIGHPASS FILTER



$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{j\omega R_f C_i}{1 + j\omega R_f C_f}$$

etc...

Example: NON-INVERTING AMPLIFIER



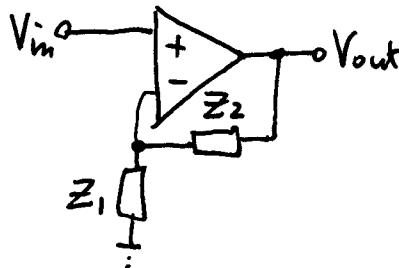
$$\rightarrow V_i^+ = V_i^- = V_{in}$$

$\rightarrow$  KCL @  $V_i^-$ : no current in the opamp  
 $I_1 = I_2$

$$\Rightarrow \frac{V_{in}}{R_1} = \frac{V_{out} - V_{in}}{R_2}, \text{ or } V_{out} = \left(1 + \frac{R_2}{R_1}\right) \cdot V_{in}$$

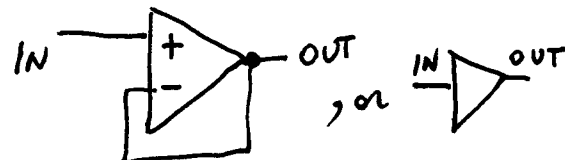
POSITIVE (non-inverting)  
 GAIN,  $A_v \geq 1$

• More generally:



$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = 1 + \frac{Z_2(j\omega)}{Z_1(j\omega)}$$

• Special case: BUFFER :  $Z_1 = \infty$   
 (UNITY GAIN AMP.) :  $Z_2 = 0$



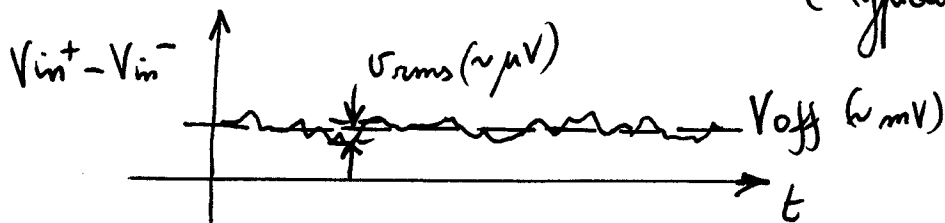
$\Rightarrow$  OUT = IN, but :  $Z_{in} = \infty$  : good to boost weak signals  
 $Z_{out} = 0$



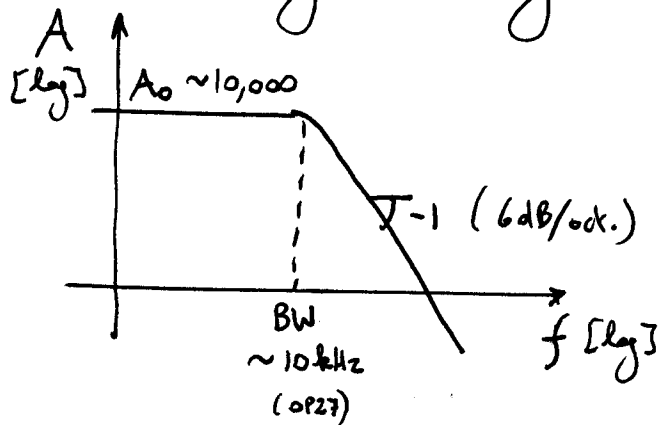
• Non-ideal characteristics of OP-AMPS :

current flowing in at zero input voltage

- finite  $A$ , finite  $R_{in}$ , non-zero  $R_{out}$ , non-zero bias current  
 $\sim 10,000$ ,  $\sim T\Omega$ ,  $\sim 50\Omega$ ,  $\sim \mu A$
- Input offset voltage:  $V_{in}^+ - V_{in}^- = V_{off}$ , typically mV  
 may drift with temperature
- Input referred voltage noise  $V_{in}^+ - V_{in}^- = \sigma_{rms}$   
 root-mean-square AC noise voltage (typically  $\mu V$ )



- Finite bandwidth: gain is a function of frequency:



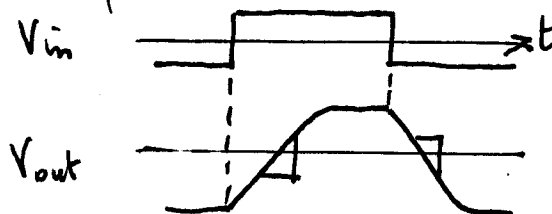
$A_0$ : DC gain  
 BW: bandwidth ( $f_c$ ) for maximum gain ( $A_0$ )

Gain-bandwidth product:  $A_0 \cdot BW = A(f) \cdot f$  at high frequency

- e.g:
- $A(10 kHz) = 10,000$
  - $A(1 MHz) = 100$
  - $A(100 MHz) = 1$

- Finite slew rate: slope  $dV_{out}/dt$  is limited, both upgoing &

downgoing:



typically  $\sim 100 V/\mu s$