• This final is closed book and closed notes. You may use a calculator for algebra and arithmetic.

• This final has 21 pages, including this cover sheet. Do not attach separate sheets. If you need more space, use the back of the pages.

• Circle or box your final answers and show your work on the pages provided.

• There are 10 problems. Points for each problem are given in [brackets]. There are 100 points total.

• You have 3 hours to complete this final.

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You may find the following equations useful:

\[ \omega_n = \frac{1}{\sqrt{LC}} \quad \zeta = \frac{1}{2} RC \omega_n \]

\[ R = R_G (1 + G \epsilon) \quad \sigma = E \epsilon \]

\[ V_o = A_d V_d + A_c V_cm \]

\[ V_d = V_b - V_a \]

\[ V_{cm} = (V_b + V_a)/2 \]

\[ V = \mathbf{M} \cdot \mathbf{r} = \cos \theta |\mathbf{M}| |\mathbf{r}| \]

\[ e = \int_0^\ell \mathbf{v} \times \mathbf{B} \, d\ell \]

\[ \Delta f = \frac{v}{c} (\cos \theta_r + \cos \theta_s) f_s \]

\[ V = E_{\text{glass}} - E_{\text{ref}} + E_{\text{Nernst}} \]

\[ E_{\text{Nernst}} = \frac{RT}{nF} \ln(10) \log_{10} \left( \frac{[A^n]_{\text{out}}}{[A^n]_{\text{in}}} \right) \text{ for some ion } A^n \text{ with valence } n \]

\[ V_m = \frac{RT}{F} \ln(10) \log_{10} \left( \frac{P_Na[Na^+]_o + P_K[K^+]_o + P_{Cl}[Cl^-]_i}{P_{Na}[Na^+]_i + P_K[K^+]_i + P_{Cl}[Cl^-]_o} \right) \]

\[ \frac{RT}{F} \ln(10) = 62 \text{ mV at room temperature} \]

\[ I = 4F [O_2] \phi \quad F = 96 485 \text{ C/mol} \]

\[ \log_{10} \text{PCO}_2 = -\text{pH} + \text{constant} \]

\[ O_2 + 2 H_2O + 4 e^- \rightleftharpoons 4 OH^- \]

\[ 4 \text{Ag} + 4 \text{Cl}^- \rightleftharpoons 4 \text{AgCl} + 4 e^- \]
\[ A(\lambda) = W L a(\lambda) \]

\[ I_d = \frac{1}{1 - \exp(-d/\tau)} I_r \]

The following diagrams may come handy as well:

\[ T_{hi} = 0.7(R_A + R_B)C \quad T_{lo} = 0.7R_B C \quad T = 1.1RC \]
1. [16 pts] Circle the best answer (only one answer per question):

(a) [1 pt] Noise in a measurement system affects its:
   i. sensitivity
   ii. accuracy
   iii. precision
   iv. resolution
   v. none of the above

(b) [1 pt] The circuit shown below outputs a high pulse at $V_{out}$ when:
   i. CTRL is activated
   ii. THR goes above $2/3 V_S$
   iii. $V_{trig}$ goes above $2/3 V_S$
   iv. $V_{trig}$ goes below $1/3 V_S$
   v. None of the above

(c) [1 pt] A cell membrane permeable to only one ion type has an equilibrium potential equal to its:
   i. half-cell potential
   ii. Nernst potential
   iii. action potential
   iv. concentration overpotential
   v. All of the above

(d) [1 pt] Which of the following biopotentials can be observed on electrodes placed over the skull:
   i. EEG
   ii. EMG
   iii. EOG
   iv. ECG
   v. All of the above
(e) [1 pt] The concentration of carbon dioxide in the blood can be derived from measurement of:
   i. pH and $[\text{HCO}_3^-]$
   ii. $\text{PO}_2$
   iii. $\text{SO}_2$, pH, and temperature
   iv. $\text{SCO}_2$
   v. All of the above.

(f) [1 pt] The Severinghaus electrode:
   i. Measures saturation of carbon dioxide in the blood.
   ii. Implements driven right leg active grounding.
   iii. Is an amperometric sensor requiring a transimpedance amplifier
   iv. Measures oxygen concentration in the blood at constant flow rate.
   v. None of the above.

(g) [1 pt] Which one of these devices provides non-invasive measurement of blood pressure?
   i. A micro–tipped manometer.
   ii. A fluid–filled catheter with pressure transducer.
   iii. A sphygmomanometer.
   iv. A tonometer.
   v. All of the above.

(h) [1 pt] Microshock:
   i. Is naturally less severe than macroshock.
   ii. Has low rates of mortality when properly treated immediately.
   iii. Is the state of disbelief when going through these kinds of questions.
   iv. Is a major cause of cardiac arrest.
   v. None of the above.
(i) [8 pts] Indicate for each statement below whether it is true or false:

i. **TRUE / FALSE**: A piezoelectric crystal transduces between stress, strain, charge, and voltage.

ii. **TRUE / FALSE**: An ion-sensitive field-effect transistor transduces concentration of an electrically charged chemical compound in solution to an electrical conductance.

iii. **TRUE / FALSE**: The current dipole generates an electric potential proportional to polarized current and inversely proportional to distance.

iv. **TRUE / FALSE**: The Ag/AgCl electrode is a non-polarizable electrode.

v. **TRUE / FALSE**: The P wave in the electrocardiogram manifests atrial depolarization.

vi. **TRUE / FALSE**: An underdamped second-order system responds with large transient oscillations to sharp changes in the input.

vii. **TRUE / FALSE**: An ideal transformer transfers electrical energy from input to output with minimal energy losses, and with galvanic isolation between input and output.

viii. **TRUE / FALSE**: Capacitive sensors are highly sensitive to displacement of the electrical plates.
2. **[14 pts]** Consider the active circuit below with a strain gauge in a modified Wheatstone bridge. The circuit is used as a transducer to measure stress $\sigma$ by an output voltage $V_{out}$. The strain gauge has Young’s modulus $E$, gauge factor $G$, and nominal resistance $R_G = R$. The other resistances $R_1$, $R_2$ and $R_3$ are constant and all also equal to $R$ so that the bridge is perfectly balanced. You may assume the opamp is ideal.

(a) **[2 pts]** Find the node voltages $V^+$ and $V^-$ for a fixed source voltage $V_s$.

(b) **[6 pts]** Find the output voltage $V_{out}$.
(c) [4 pt] Find the sensitivity. Does it depend on the stress $\sigma$, and why?

(d) [2 pt] Find the output impedance on the $V_{out}$ node.
3. **[16 pts]** Now analyze the following voltage-in, current-out filter circuit:

(a) **[1 pts]** What is the impedance of the ideal voltage source driving the input of the filter?

(b) **[1 pts]** What is the impedance of the ideal current meter loading the output of the filter?

(c) **[3 pts]** Find the transfer function \( H(j\omega) = I_{out}(j\omega)/V_{in}(j\omega) \). Does it depend on frequency?
(d) [3 pts] Find the input impedance $Z_{in}(j\omega)$ of the filter.

(e) [2 pts] Find the output impedance $Z_{out}(j\omega)$ of the filter.
(f) [6 pt] Sketch the Bode plot of the output impedance \( Z_{out}(j\omega) \) from 0.1 Hz to 10 kHz, for \( R = 100 \, k\Omega \), \( L = 23 \, nH \), and \( C = 16 \, nF \). Be sure to indicate all units.
4. **[8 pts]** For each of the two different active circuits below, sketch $V_{out}$ versus $V_{in}$ on the diagram on the right. Show your reasoning.

(a) **[4 pts]**

(b) **[4 pt]**
5. **[6 pts]** The cardiac vector $\mathbf{M}$ in the frontal plane points vertically downwards with amplitude 1 mV/m during the R wave of the ECG, shifts 30 degrees sideways pointing towards the left leg with amplitude 0.5 mV/m during the T wave, and -30 degrees sideways pointing towards the right leg with amplitude 0.2 mV/m during the P wave, as shown below. The distance between the patient’s shoulders is 1 m, and the patient’s heart beats at 1 s intervals.

(a) **[3 pts]** Sketch one cycle of the ECG voltage as a function of time, recorded along lead I. Indicate units on your axes.

(b) **[3 pts]** Again, sketch one cycle of the ECG waveform, but now recorded along lead aVL.
6. **[8 pts]** Design a simple deep brain stimulator (DBS) that continuously generates a 130 Hz periodic square wave voltage output signal $V_{out}(t)$ with pulse height 5 V and duty cycle 0.1 (i.e., the output should be 0 V 90% of the time, and 5 V 10% of the time, alternating between these two every period). Include also short-circuit protection at the voltage output, drawing no more than 10 mA from an accidental short to ground. You may use any active devices that you learned in class, any combination of resistors and capacitors, and a single 5 V battery. You may also assume that all active devices include protection diodes to the power supplies at their input and output terminals.
7. [10 pts] Physiological signals as indicators of cardiovascular health are monitored using a variety of bioinstruments, that can be entirely non-invasive. Here we consider two such signals of interest: blood velocity, and oxygen saturation.

(a) [4 pts] Blood velocity is measured non-invasively using a Doppler transducer, as shown below. Find the transmit and receive angles $\theta_s$ and $\theta_r$ that maximize the sensitivity. Are these angles practical? What angles are usually adopted in practice? Explain.
(b) [6 pts] A pulse oximeter is used to measure oxygen saturation in the blood non-invasively by monitoring absorbances at different optical wavelengths. Given spectral measurements of the absorptivities \( a_o(\lambda) \) and \( a_r(\lambda) \) of oxygenated and reduced hemoglobin at two distinct wavelengths \( \lambda_1 \) and \( \lambda_2 \), find a general expression for the ratio of absorbances measured in the vessel at these two wavelengths, \( A(\lambda_1) / A(\lambda_2) \), as a function of \( \text{SO}_2 \). Under what conditions is the relationship linear, and how do you maximize sensitivity?
8. **[8 pts]** A Clark electrode is used to measure concentration of oxygen in the blood stream. The Clark “electrode” actually contains two electrodes: one Pt electrode, and one Ag/AgCl electrode, that are both immersed in the reference chamber of the Clark chemical cell. The reference chamber contains a saturated solution of KCl, and is separated from the sample chamber by an oxygen-permeable membrane. Blood passes through the sample chamber at a constant rate.

(a) **[4 pts]** Complete the diagram below to realize an oxygen-to-voltage transducer that outputs a voltage $V_{out}$ proportional to oxygen concentration $[O_2]$ in the blood. In addition to the transimpedance amplifier (TIA) circuit shown, you are given a Clark chemical cell with Pt and Ag/AgCl electrode terminals, and with blood passing through at known flow rate $\Phi$. You are also given a 0.7 V battery, in addition to the $\pm 3$ V power supplies to the TIA.
(b) [3 pts] Find the sensitivity of the oxygen-to-voltage transducer for $R_1 = R_2 = 1 \, \text{M}\Omega$ and $R_3 = 1 \, \text{k}\Omega$.

(c) [1 pts] Find the value of $C$ in order to filter out noise contaminating the output $V_{out}$ at frequencies 1 kHz and higher.
9. [10 pts] Consider the instrumentation amplifier (IA) and driven right leg (DRL) system below with resistances \( R_1 = R_f = 495 \text{k}\Omega \), \( R_2 = R_3 = R_a = 10 \text{k}\Omega \), \( R_4 = 100 \text{k}\Omega \), and with electrode resistances \( R_{RA} = 110 \text{k}\Omega \), \( R_{LA} = 90 \text{k}\Omega \), and \( R_{RL} = 100 \text{k}\Omega \). All opamp input impedances are \( R_{in} = 1 \text{G}\Omega \); otherwise, you may assume all active and passive components are ideal.

(a) [3 pts] Which lead of the ECG does this IA measure? Find the differential gain \( A_d \), and effective common-mode rejection ratio \( \text{CMRR}_{\text{eff}} \).
(b) [5 pts] For a peak current $i_d = 10 \mu A$ entering the body, find the peak common-mode voltage $v_{cm}$, and the corresponding peak common-mode voltage error at the IA output $V_{out}$.

(c) [2 pts] Find the value for $R_o$ to limit the short-circuit current on the DRL electrode to no more than $10 \mu A$ for a $\pm 3.3$ V double rail voltage supply.
10. **[4 pts] Guest lectures**

   (a) *Real-world EEG brain-computer interfaces (BCI)*

   i. [1 pt] List at least one of the factors that have limited practical commercial use of BCI.

   ii. [1 pt] What signal do you observe in the EEG on the occipital lobe when presenting a 15 Hz periodic flashing stimulus in the visual field of the subject?

(b) *Global health and wearables*

   i. [1 pt] Explain what is meant by “reverse tropicalization.”

   ii. [1 pt] What is the least number of electrodes that a wearable sensor needs to have in order to be able to monitor the full 6-lead frontal ECG?