Name (Last, First): SOLUTIONS

BENG 186B Winter 2014

Final

- This exam is closed book, closed notes. Calculators are OK.
- Circle and put your final answers in the space provided; show your work only on the pages provided. Include units.
- Do not attach separate sheets. If you need more space, use the back of the pages. This exam has 25 pages, including this page.
- Points for each problem are given in [brackets], 100 points total.

1	/16
2	/08
3	/10
4	/08
5	/12
6	/10
7	/12
8	/10
9	/10
10	/4
Total	/100

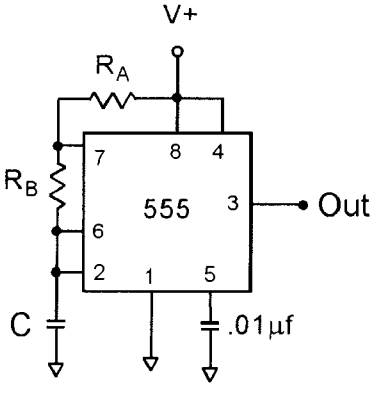
Useful schematics and equations:

The Goldman-Hodgkin-Katz (GHK) equation:

$$E = (60mV) \cdot \log_{10} \frac{P_K [K^+]_o + P_{Na} [Na^+]_o + P_{Cl} [Cl^-]_i}{P_K [K^+]_i + P_{Na} [Na^+]_i + P_{Cl} [Cl^-]_o}$$

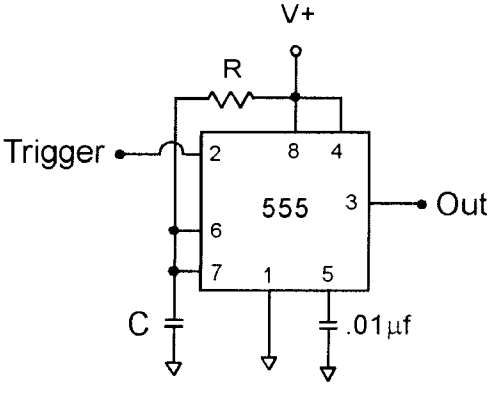
Common mode gain and differential gain:

$$v_o = A_d v_d + A_c v_{cm}$$
$$v_d = v_a - v_b$$
$$v_{cm} = (v_a + v_b) / 2$$



Oscillator

$T_{High} = 0.7 (R_A + R_B) C$; $T_{Low} = 0.7 R_B C$



One-shot

$T_{Pulse} = 1.1 R C$

1) **[16 pts]** Indicate whether each of the following statements is true or false.

[**TRUE** / **FALSE**] One of the steps in the process of analog-to-digital signal conversion is sampling.

[**TRUE** / **FALSE**] The minimum sampling frequency is defined by the Nyquist criterion.

[**TRUE** / **FALSE**] Most sensors capture digital signals, which are then converted to analog for processing.

[**TRUE** / **FALSE**] A voltage divider is an example of a zeroth-order system.

[**TRUE** / **FALSE**] The average power dissipated by capacitors and inductors in a circuit is zero.

[**TRUE** / **FALSE**] A diode bridge rectifier is a useful way to rectify DC voltages to AC voltages

[**TRUE** / **FALSE**] The Nernst equation is a special case of the Goldman-Hodgkin-Katz equation.

[**TRUE** / **FALSE**] The ionic concentrations of cells are passively maintained by two processes: diffusion and ionic pump activation.

[**TRUE** / **FALSE**] Non-polarizable electrodes are preferred for bioinstrumentation measurements.

[**TRUE** / **FALSE**] A common-mode rejection ratio of 10 watts/watts can be expressed as 10 dB.

[**TRUE** / **FALSE**] The second-order dynamics of a fluid-filled catheter are due to the inertia and diffusivity of the fluid inside the tube.

[**TRUE** / **FALSE**] Range-gated (pulsed) Doppler imaging operates by measuring the delays in wave propagation and the frequency difference between the emitted and reflected signals.

[**TRUE** / **FALSE**] The Clark electrode presents a membrane permeable to O_2, which allows it to measure the CO_2 concentration.

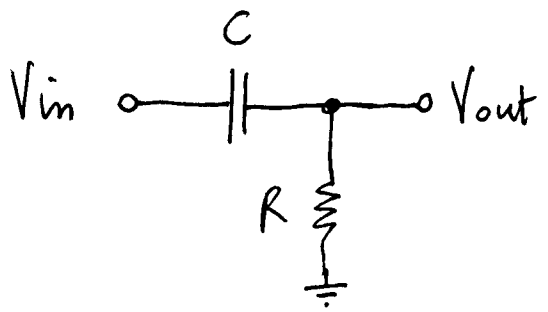
[**TRUE** / **FALSE**] A potentiostat can be used to measure the concentration of O_2 due to the linear relationship between concentration and electric current when a polarizing voltage is applied between Pt and Ag/AgCl electrodes in a salt buffer solution.

[**TRUE** / **FALSE**] The isosbestic wavelength defines where the maximum difference between the absorptivities of oxygenated hemoglobin and reduced hemoglobin occurs.

[**TRUE** / **FALSE**] Diodes can be an effective way of protecting your circuit from over-voltage.

2) **[8 pts]** Short answer, write your answer only in the space provided.

a) Draw a circuit that realizes a high-pass filter with 1 kHz cutoff frequency. Minimize the number of components, and label input and output. [2 pts]



$$f_c = \frac{1}{2\pi RC} = 1\ \text{kHz}$$

For instance:

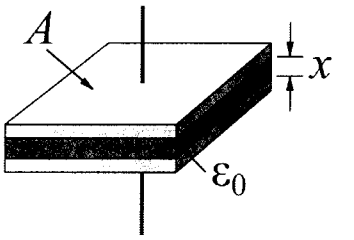
$C = 1\ \text{nF}$

$R = 1\ \text{M}\Omega / 2\pi \approx 159\ \text{k}\Omega$

b) For a signal with a range of 1.28 V and 4-bit representation, find the resolution of one least significant bit (1 LSB). [2 pts]

$$1\ \text{LSB} = \frac{1.28\ \text{V}}{2^4} = 0.08\ \text{V} = 80\ \text{mV}$$

c) For a capacitive sensor of transversal displacement as shown below, derive the sensitivity of the sensor as a function of ε_0, A, and x. [2 pts]



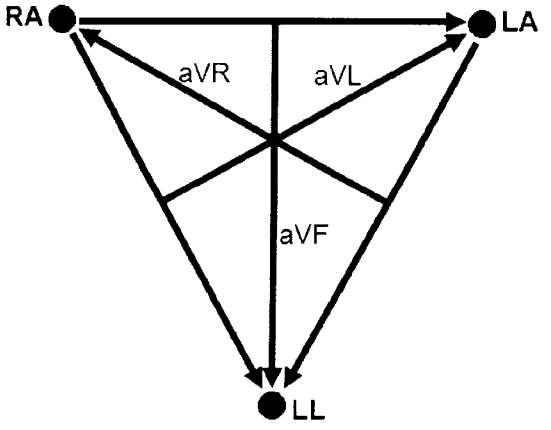
$$C = \varepsilon_0 \frac{A}{x}$$

$$\Downarrow$$

$$\text{Sensitivity} \quad \frac{dC}{dx} = -\varepsilon_0 A \frac{1}{x^2}$$

(This problem assumed $\varepsilon_r = 1$; Otherwise, replace ε_0 with $\varepsilon = \varepsilon_0 \varepsilon_r$.)

d) Given the Einthoven's triangle, write the most simplified expression for the cardiac leads in terms of the potentials RA, LA, and LL. [2 pts]

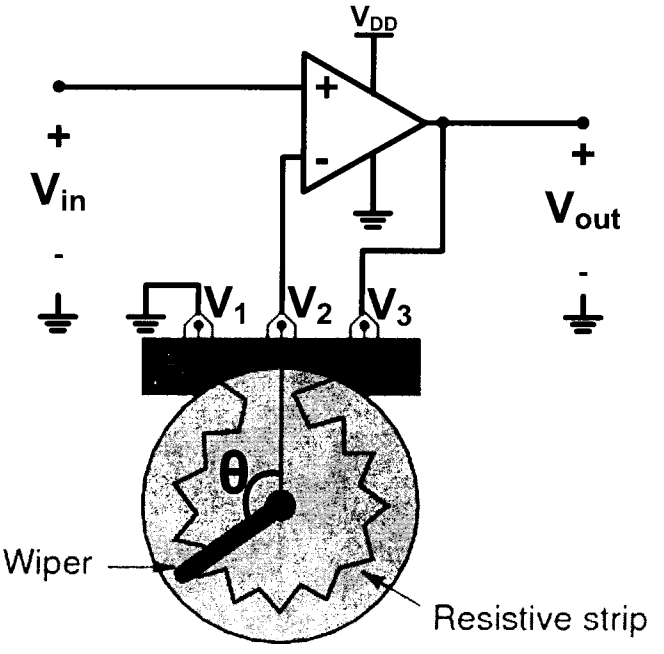


aVR = $RA - \frac{1}{2}(LL+LA)$

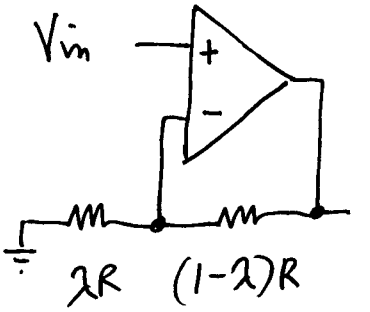
aVL = $LA - \frac{1}{2}(RA+LL)$

aVF = $LL - \frac{1}{2}(RA+LA)$

3) **[10 pts]** Consider the following circuit. The potentiometer has a total resistance of 10 kΩ and its angle θ ranges from 10° to 350°.



a) Find θ for a gain (V_{out}/V_{in}) of 4. [6 pts]



$$V_{out} = \left(1 + \frac{(1-\lambda)R}{\lambda R}\right) V_{in}$$

$R = 10\,k\Omega$

$$\lambda = \frac{\theta - 10^\circ}{340^\circ} \quad \text{for } 10^\circ \le \theta \le 350^\circ$$

$$\Rightarrow \text{Gain} \quad \frac{V_{out}}{V_{in}} = 1 + \frac{1-\lambda}{\lambda} = \frac{1}{\lambda} = 4$$

$$\Rightarrow \quad \lambda = \frac{\theta - 10^\circ}{340^\circ} = \frac{1}{4}$$

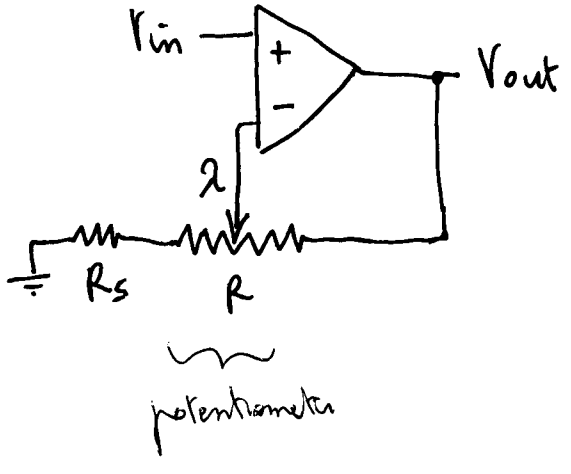
$$\Rightarrow \quad \theta = 10^\circ + \frac{340^\circ}{4} = 95^\circ$$

b) Find the minimum and maximum achievable gain of the circuit. Show how to change the circuit to limit the maximum gain to 10. [4 pts]

$$\text{Gain} = \frac{1}{\lambda} \Rightarrow \quad 1 \leq \text{Gain} \leq \infty$$

min. ($\lambda = 1$) max. ($\lambda = 0$)

Change the circuit by adding a resistance in series in the feedback path:



Max gain ($\lambda = 0$):

$$1 + \frac{R}{R_s} = 10$$

$$\Rightarrow R_s = \frac{1}{9} R \approx 1.1\ k\Omega$$

4) **[8 pts]** The resting potential of a cell separated from extracellular fluid by a membrane permeable to Na^+, K^+, and Cl^- is measured at 37 °C. Refer to the table for individual ion concentrations found during the experiment.

Ion	Intracellular concentration (mM)	Extracellular concentration (mM)
Na^+	10	145
K^+	140	5
Cl^-	4	110

a) Find the Nernst potential for each ion type. [3 pts]

$$E_{Na} = \underbrace{(60\,mV)}_{\frac{RT}{F}\ln(10) \text{ at } 37^\circ C \text{ (approximately!)}} \log_{10}\frac{145}{10} \approx 70\,mV$$

$$E_K = (60\,mV)\log_{10}\frac{5}{140} \approx -87\,mV$$

$$E_{Cl} = -(60\,mV)\log_{10}\frac{110}{4} \approx -86\,mV$$

b) When we apply a pharmacological blocker that makes the membrane impermeable to K^+, the resting potential is found to be 30 mV. What is the permeability of Cl^- relative to that of Na^+? [2 pts]

$P_K = 0$

$$\text{GHK:} \quad 30\,mV = (60\,mV)\log_{10}\frac{0.5 + P_{Na}\cdot 145 + P_{Cl}\cdot 4}{0.140 + P_{Na}\cdot 10 + P_{Cl}\cdot 110}$$

$$\Rightarrow \quad P_{Na}\cdot 145 + P_{Cl}\cdot 4 = 10^{\frac{1}{2}}\left(P_{Na}\cdot 10 + P_{Cl}\cdot 110\right)$$

$$\Rightarrow \quad \frac{P_{Cl}}{P_{Na}} = \frac{145 - 10^{\frac{1}{2}}\cdot 10}{10^{\frac{1}{2}}\cdot 110 - 4} = 0.33$$

c) Now assume that the cell membrane is only permeable to K^+ ions. How many K^+ ions would have to cross the membrane to lower the membrane potential by 70 mV, and in which direction? Assume the membrane capacitance of the cell to be 1 $\mu F/m^2$ and that the cell is a sphere with diameter of 10 μm. [3 pts]

Number of K^+ ions LEAVING the cell :

$$\frac{\Delta Q}{q} = \frac{C\,\Delta V}{q} = \frac{C_A \cdot A \cdot \Delta V}{q}$$

(valence = +1)

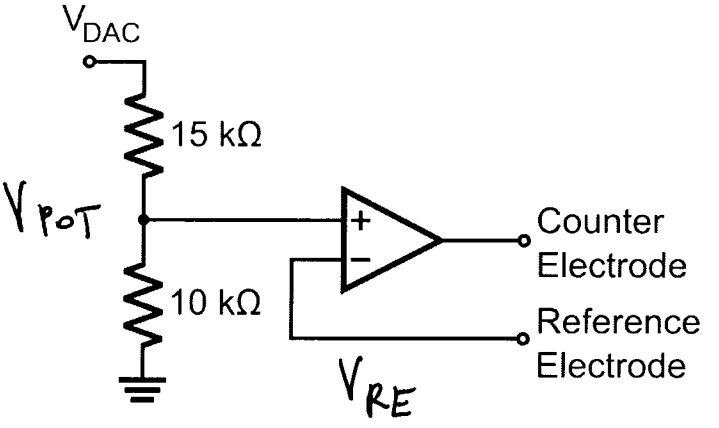
with
- $C_A = 1\,\mu F/m^2$
- $A = \pi D^2 = 314\,\mu m^2 = 3.14\,10^{-10}\,m^2$
- $\Delta V = 70\,mV = 0.07\,V$
- $q \approx 1.6\,10^{-19}\,C$

$$= \frac{10^{-6}\,\frac{F}{m^2}\,3.14\,10^{-10}\,m^2\,0.07\,V}{1.6\,10^{-19}\,C} \approx 137$$

($\times 10^4$ for $C_A = 1\,\mu F/cm^2$)

5) **[12 pts]** A crack team of bioengineers developed a glucose meter for their senior design project. You will now get to analyze their basic circuit design to see how it behaves.

a) The glucose test strip needs a bias voltage applied to its counter electrode to work. The team used the following circuit:



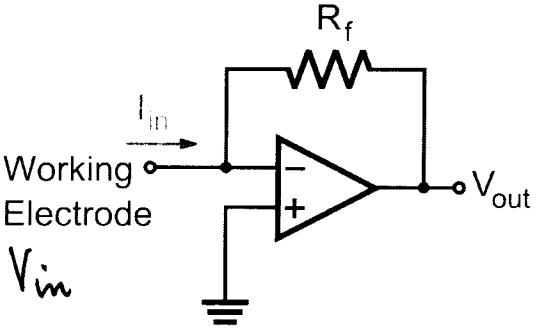
The op-amp is ideal, and there is a feedback path through the test strip between the counter and reference electrode. If V_{DAC} is equal to 1 V, what is the voltage at the reference electrode? (*Hint*: what can you say about the two op-amp inputs?) [2 pts]

Negative feedback through the ionic solution

$$\Rightarrow V_{RE} = V_{POT} = \frac{10k\Omega}{10k\Omega + 15k\Omega} \cdot V_{DAC}$$

$$= 0.4\ V$$

b) When the bias voltage and a glucose sample are applied to the test strip, it produces a small amount of current through its working electrode. The transimpedance amplifier shown below converts this current to a voltage:



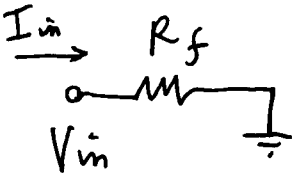
The op-amp is ideal. **Derive from first principles** the transfer function V_{out}/I_{in} for the above circuit. [2 pts]

Negative feedback through R_f $\Rightarrow$ $V_{in} = 0$ (virtual ground)

Ohm's law : $V_{in} - V_{out} = R_f \cdot I_{in}$

$$\Rightarrow \frac{V_{out}}{I_{in}} = -R_f$$

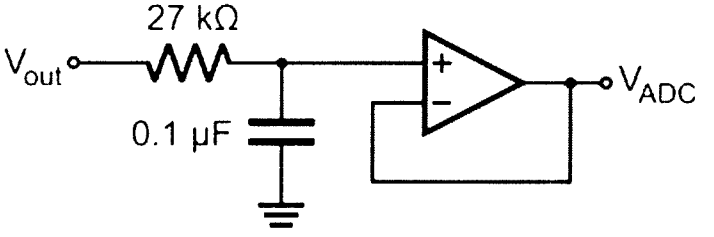
c) Why is this circuit better than a simple resistor? [2 pts]



$$\frac{V_{in}}{I_{in}} = R_f$$

The potentiostat (circuit b) keeps the input voltage constant (at ground) whereas the voltage depends on current with a simple resistor circuit.

d) The team also used the following circuit to filter the output of the transimpedance amplifier before it is read by the ADC:



The op-amp is ideal. **Derive from first principles** the transfer function for the above circuit. What kind of filter is this and what is the cutoff frequency? [4 pts]

R-C lowpass filter

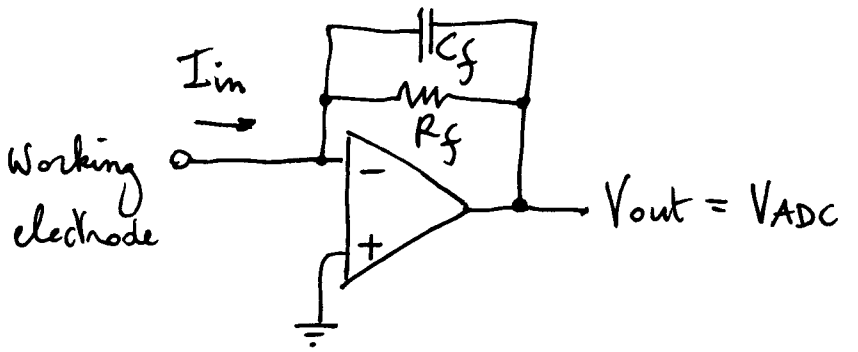
voltage buffer (gain = 1)

$$\frac{V_{ADC}(j\omega)}{V_{out}(j\omega)} = \frac{1}{1 + j\omega RC} \quad \text{with} \quad R = 27\,k\Omega, \quad C = 0.1\,\mu F$$

$\Rightarrow$ LOW-PASS with cutoff frequency

$$f_c = \frac{1}{2\pi RC} = \frac{1}{6.28 \cdot 27 \cdot 0.1\,ms} = 59\,Hz$$

e) **[BONUS 4 pts]** Show how to implement this function without this additional circuit by modifying the transimpedance amplifier.



$$\frac{V_{out}(j\omega)}{i_{in}(j\omega)} = -\frac{R_f}{1 + j\omega R_f C_f} \Rightarrow \text{Make } R_f C_f = RC \quad \left(C_f = \frac{R}{R_f} \cdot C\right)$$

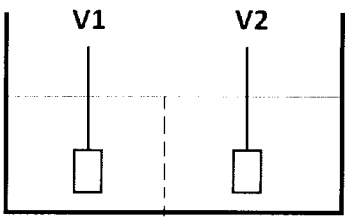
f) The team calibrated their circuit against standard glucose solutions to obtain the following transfer function:

$$B_{\text{ADC}} = -53.05 \frac{\text{L}}{\text{mmol}} \times [\text{Glucose}] - 47.5$$

[Glucose] is the glucose concentration in mmol/L, while B_{ADC} is the output reading in arbitrary units. What is the sensitivity of the device? [2 pts]

$$\text{Sensitivity} = \frac{d B_{ADC}}{d[\text{Glucose}]} = -53.05 \frac{L}{mmol}$$

6) **[10 pts]** The following set up is used to sense pH. The membrane separating chamber 1 and 2 is only permeable to H^+. Both chambers have Ag/AgCl electrodes with E_{hc}= 0.223 V, series resistance of 1 kΩ, double layer resistance of 100 kΩ, and double layer capacitance of 10 pF. The resistance of the H^+-permeable membrane is 2 MΩ.



a) What is the polarity of the voltage V1-V2 measured by a voltmeter of infinite impedance when chamber 1 contains a strong acid and chamber 2 contains a weak base. [1 pt]

$$V_1 - V_2 = (60mV)\log_{10}\frac{[H^+]_2}{[H^+]_1} < 0$$

$$\downarrow$$

$$[H^+]_1 \gg [H^+]_2$$

strong acid weak base

b) What is the expected V1-V2 when chamber 1 contains 10 mmol/L KCl solution and chamber 2 contains 1 mmol/L HCl solution? [2 pts]

KCl : salt (pH neutral) : $[H^+]_1 = 10^{-7}\ \frac{mol}{L}$

HCl : acid : $[H^+]_2 = [HCl] = 10^{-3}\ \frac{mol}{L}$

$$V_1 - V_2 = (60mV)\cdot\log_{10}\frac{[H^+]_2}{[H^+]_1} = (60mV)\cdot 4$$

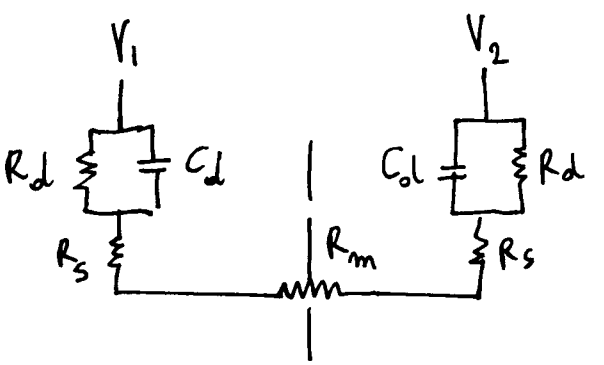
$$= 240\ mV$$

c) You now find out from the manufacturer of the electrodes that the two electrodes used are not identical and produce an open circuit voltage V1-V2 of 0.02 V when immersed in the same solution. How does this affect the expected voltage in part (b)? [1 pt]

Add 0.02V (difference in half-cell potentials)

$\Rightarrow V_1 - V_2 = 240\,mV + 20\,mV = 260\,mV$

d) Write the equivalent impedance between the electrodes as a function of radial frequency ω. [3 pts]



$R_s = 1\,k\Omega$

$R_d = 100\,k\Omega$

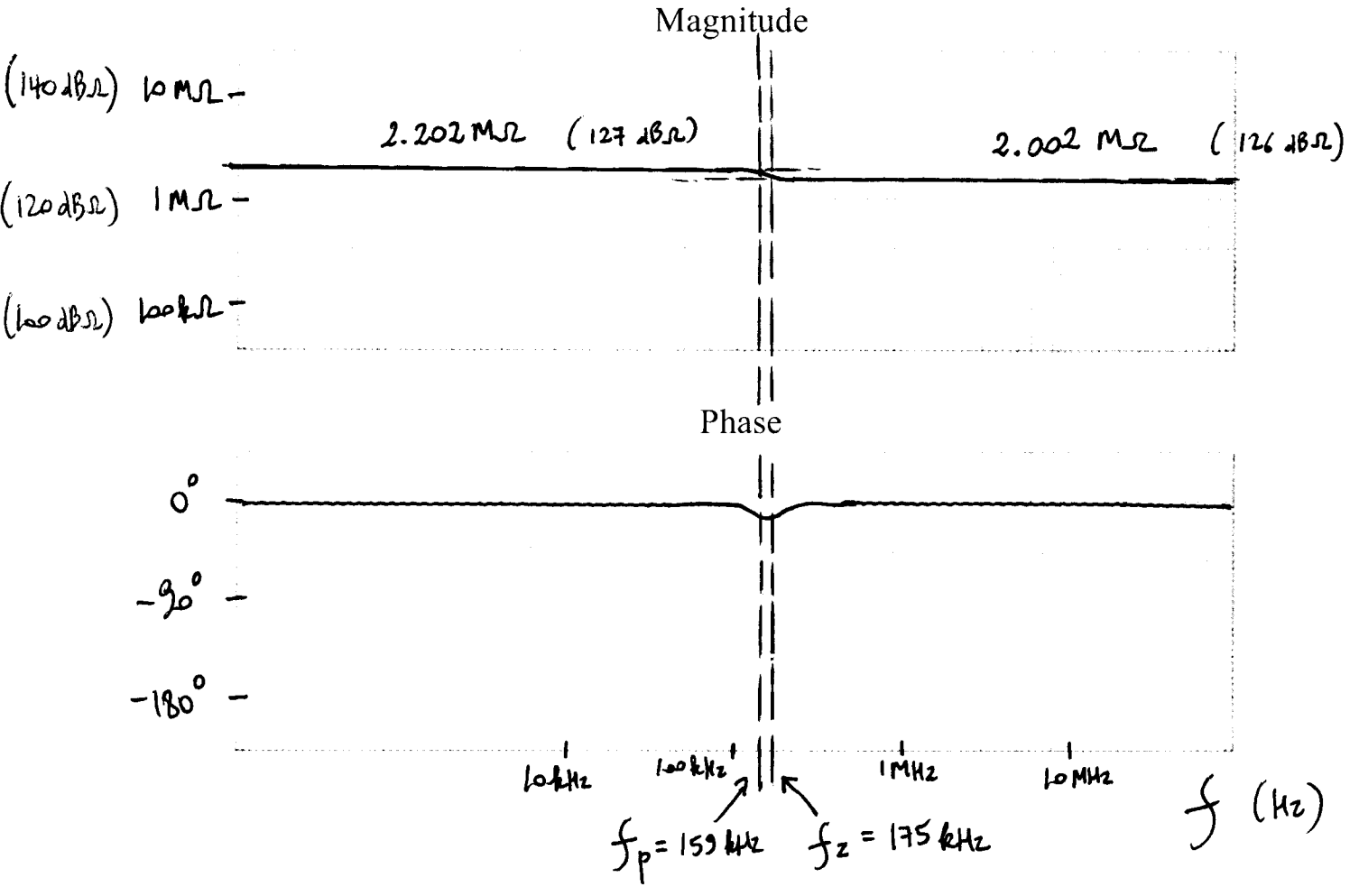
$C_d = 10\,pF$

$R_m = 2\,M\Omega$

$$Z_{eq}(j\omega) = R_m + 2R_s + 2\frac{R_d}{1 + j\omega R_d C_d}$$

$$= 2.002\,M\Omega + \frac{200\,k\Omega}{1 + j\omega(1\mu s)}$$

e) Plot the magnitude (in dB) and the phase (in degrees) of the impedance as a function of frequency f (in Hz) and specify the corner frequencies. [3 pts]



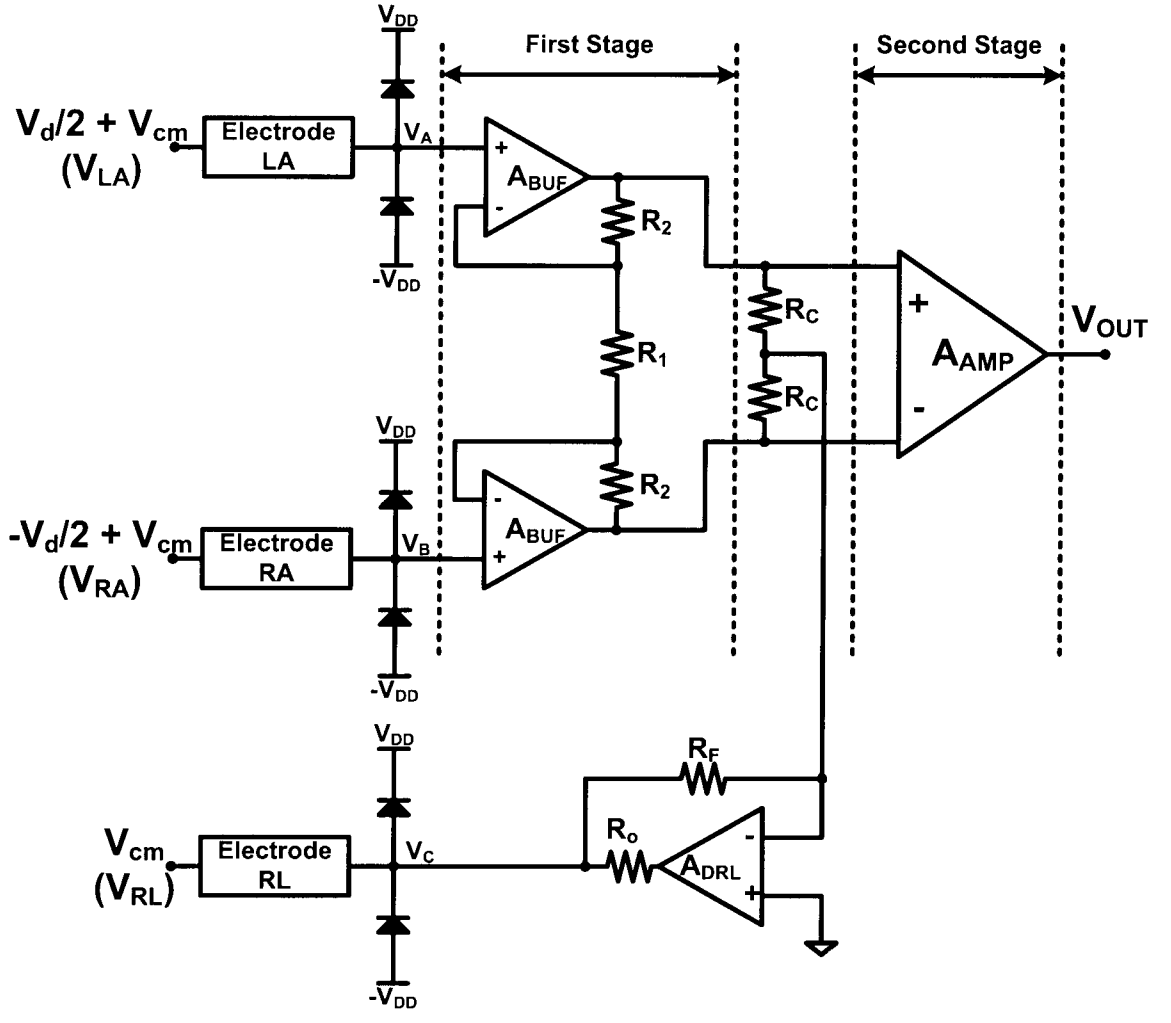
$$Z_{eq}(jw) = \frac{2.202\ M\Omega + jw\,(2.002\ M\Omega \cdot 1\mu s)}{1 + jw\,(1\mu s)}$$

$\Rightarrow$ POLE : $\omega_p = \frac{1}{1\mu s}$ or $f_p = \frac{1}{2\pi\,\mu s} = \frac{1}{6.28}$ MHz = 159 kHz

ZERO : $\omega_z = \frac{2.202}{2.002}\,\frac{1}{1\mu s}$ or $f_z = \frac{2.202}{2.002}\,\frac{1}{2\pi\,\mu s} = 1.1 \times 159$ kHz

= 175 kHz

7) **[12 pts]** An instrumentation amplifier (IA) with driven right leg active grounding common-mode suppression is shown below. The IA has resistances $R_1 = 10\ k\Omega$, $R_2 = 500\ k\Omega$, $R_C = 20\ k\Omega$, $R_F = 1\ M\Omega$, and $R_O = 1\ M\Omega$. The IA second stage A_{AMP} has differential gain of 100 V/V and common-mode gain of 0 V/V. Assume the amplifiers A_{BUF} are ideal.



a) Assume the protection diodes are ideal with turn-on voltage of 0.7 V. Find the resulting voltage range at V_A, V_B, and V_C. [2pts]

$$-V_{DD} - 0.7V \leq \begin{matrix} V_A \\ V_B \\ V_C \end{matrix} \leq V_{DD} + 0.7V$$

b) The body couples to a 60 Hz, 110 Vrms outlet with 1 pF capacitance. Find the rms amplitude of the displacement current entering the body. (*Hint*: for this part, you may assume the body is practically at ground potential) [2 pts]

$$i_A = j\omega C \, v_{outlet}$$

$$\Rightarrow i_{A\,rms} = \omega C \, v_{outlet\,rms} = 2\pi \, 60Hz \, 1pF \, 110V_{rms}$$

$$= 41.5 \, nA_{rms}$$

c) Electrode RL connects to the body with an impedance of 100 kΩ. For a 2 mVrms differential input V_d, find the signal-to-noise ratio SNR_{IN} in dB at each of the input nodes V_A and V_B for the displacement current found in part (b). [2 pts]

V_A and V_B : $\pm \underbrace{\frac{v_d}{2}}_{\text{SIGNAL}} + \underbrace{v_{cm}}_{\text{NOISE}} \Rightarrow SNR_{in}(dB) = 20\log_{10} \frac{v_{d\,rms}}{2\,v_{cm\,rms}}$

(same for both V_A and V_B)

- $\frac{1}{2} v_{d\,rms} = 1\ mV_{rms}$
- $v_{cm\,rms} = R_{RL\,eff} \cdot i_{A\,rms} = \frac{R_{RL}}{1 + \frac{2R_F}{R_C}} \cdot i_{A\,rms}$

$= \frac{100\,k\Omega}{101} \cdot 41.5\ nA_{rms} = 41.1\ mV_{rms}$

$\Rightarrow SNR_{in}(dB) = 20\log_{10} \frac{1\ mV_{rms}}{0.0411\ mV_{rms}} = 27.7\ dB$

d) Assume the electrode-body impedances of LA and RA are 90 kΩ and 110 kΩ, respectively. Find the common-mode rejection ratio CMRR and output signal-to-noise ratio SNR_{OUT} in dB. [2 pts]

$CMRR \approx \frac{|Z_{in}|}{|Z_{LA} - Z_{RA}|} = \frac{\infty}{\text{whatever}} = \infty \quad (\infty\ dB)$

$SNR_{out} = CMRR \cdot SNR_{in} = \infty \quad (\infty\ dB)$

e) Each protection diode has a parallel resistance of 400 MΩ. Now find the resulting CMRR and SNR_{OUT} in dB. [4 pts]

$$Z_{in} = 400\,M\Omega \parallel 400\,M\Omega = 200\,M\Omega$$

$$Z_{LA} - Z_{RA} = 90\,k\Omega - 110\,k\Omega = -20\,k\Omega$$

$$\Rightarrow CMRR \approx \frac{|Z_{in}|}{|Z_{LA} - Z_{RA}|} = \frac{200\,M\Omega}{20\,k\Omega} = 10{,}000 = 80\,dB$$

$$SNR_{out} = CMRR \cdot SNR_{in}$$

$$= 10{,}000 \cdot \frac{1\ mV_{rms}}{0.0411\ mV_{rms}}$$

$$= 2.43\,10^5$$

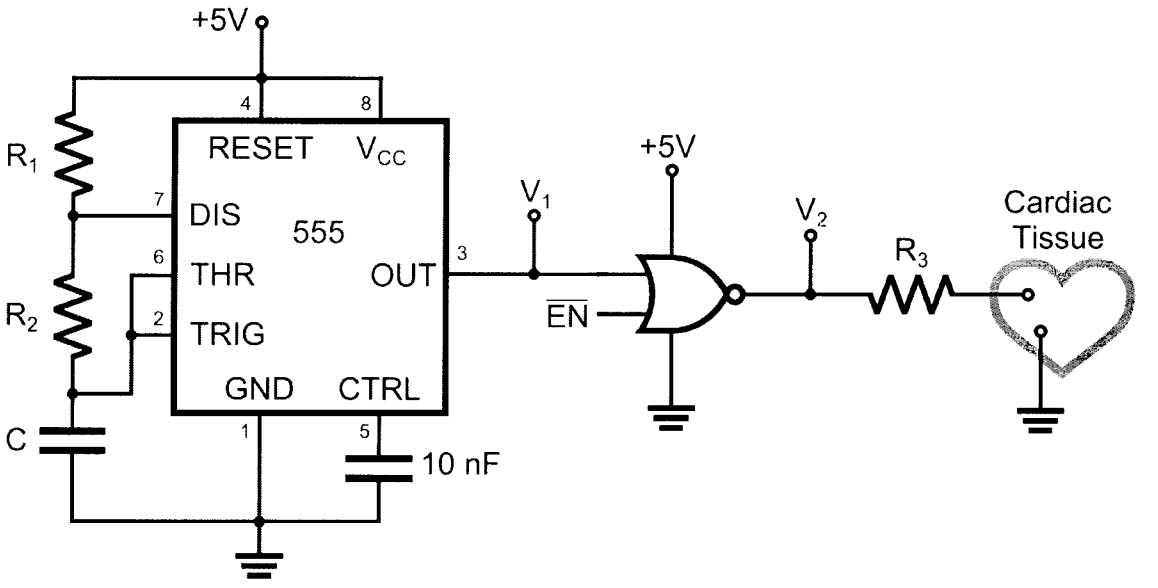
$$= 107.7\ dB$$

(Or simply: $SNR_{out}(dB) = CMRR(dB) + SNR_{in}(dB)$

$$= 80\,dB + 27.7\,dB$$

$$= 107.7\,dB\ )$$

8) **[10 pts]** The circuit below is used to generate a pacemaker drive signal. It is powered by a +5V battery, and has an "enable" input labeled $\overline{EN}$. The output of the circuit connects through an electrode to cardiac tissue. The values of the components are R_1 = 200 kΩ, R_2 = 500 kΩ, R_3 = 10 kΩ, and C = 1 μF.



a) Plot the waveform seen at V_1. [4 pts]

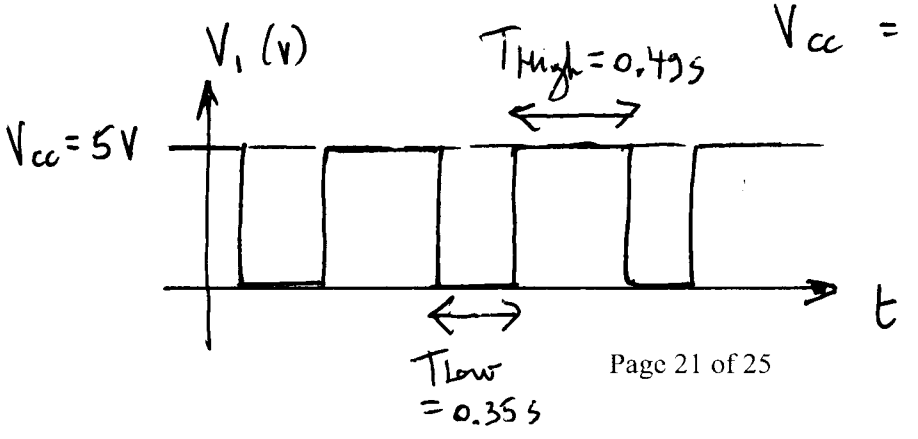
Astable multivibrator (oscillator) with $T_{High} = 0.7 (R_1 + R_2) C$

$= 0.49$ s

$T_{Low} = 0.7 R_2 C$

$= 0.35$ s

$V_{cc} = +5$ V

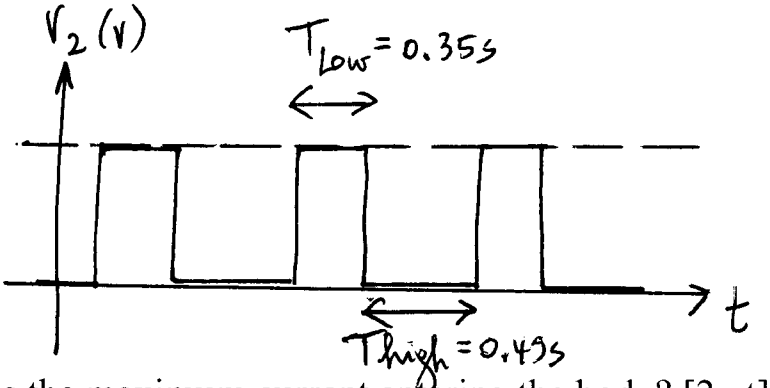


b) What is V_2 when $\overline{EN} = 1$? [2 pts]

NOR-gate: $\overline{EN} = 1 \Rightarrow V_2 = 0$ (ground, 0V)

c) What is V_2 when $\overline{EN} = 0$? [2 pts]

NOR-gate: $\overline{EN} = 0 \Rightarrow V_2 = \overline{V_1}$ (logic complement, logic inversion)



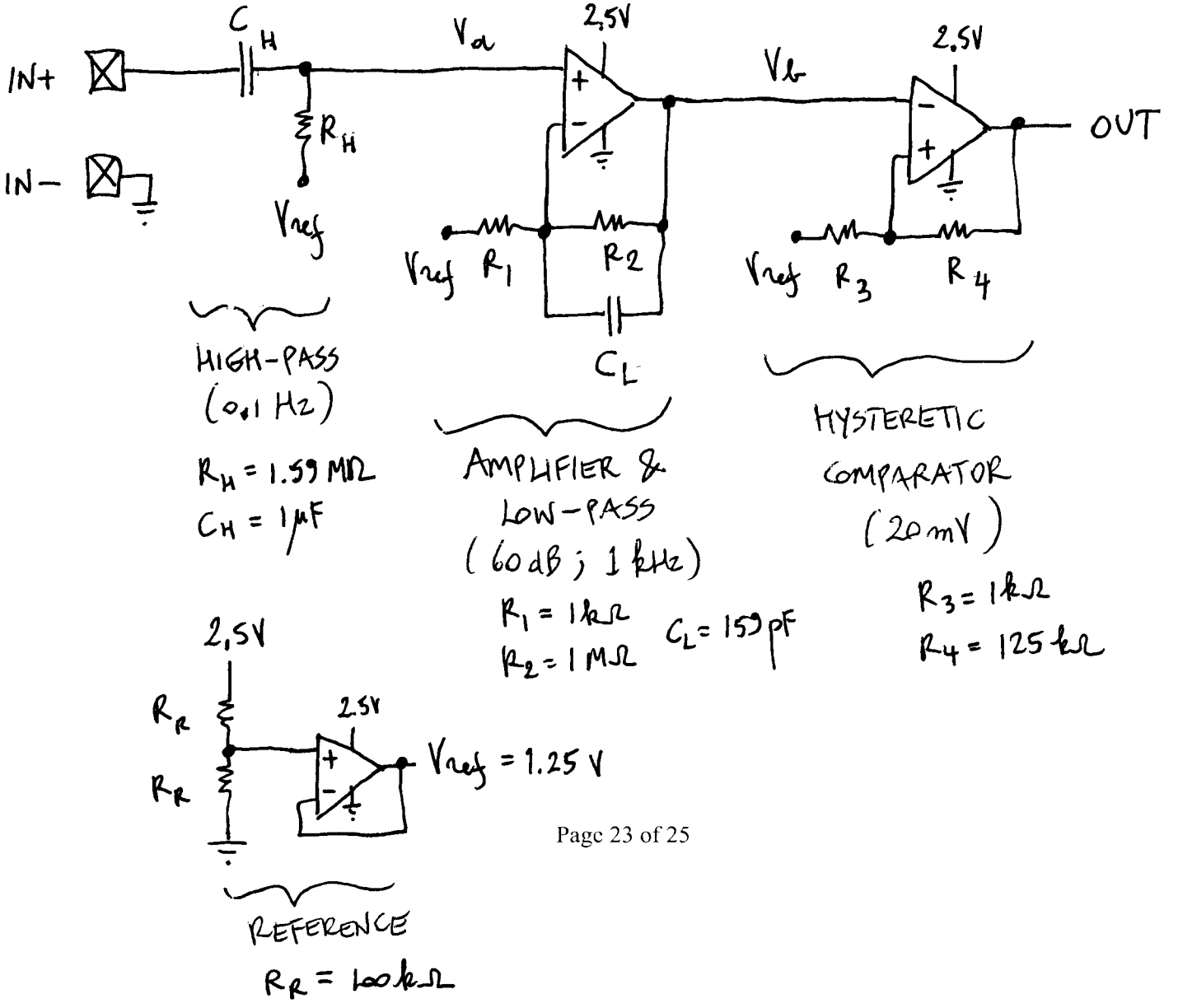
d) What is the maximum current entering the body? [2 pt]

$I_{max} = +5V / R_3 = 5V / 10k\Omega = 500\mu A$

9) **[10 pts]** Congratulations! You just landed an internship at a major bioinstrumentation company. Your first assignment is to design an infant ECG device with the following specifications:

- Band-pass filtering between 0.1 Hz and 1 kHz corner frequencies.
- Amplification of the 1 mVpp ECG differential signal by 60 dB.
- Level-crossing detection with 20 mV hysteresis to produce a square wave output signal at the heartbeat frequency.

You have two electrodes, a single 2.5 V battery, ideal op-amps, and any required resistors and capacitors. Be sure to label all components, inputs, outputs, and needed values.



- High-pass: $f_{cH} = \frac{1}{2\pi R_H C_H} = 0.1\ Hz$

$C_H = 1\mu F$

$R_H = \frac{1}{2\pi C_H\, 0.1Hz} = 1.59\ M\Omega$

- Amplifier (non-inverting) and low-pass:

Gain: $1 + \frac{R_2}{R_1} = 1{,}000$ (60 dB)

$R_1 = 1\ k\Omega$

$R_2 = 1\ M\Omega$

$f_{cL} \approx \frac{1}{2\pi R_2 C_L} = 1\ kHz$

$C_L = \frac{1}{2\pi R_2\, 1kHz} = 159\ pF$

- Hysteretic comparator (inverting — OK!):

Hysteresis: $\frac{R_3}{R_3 + R_4} \cdot 2.5V = 20\ mV$

$R_3 = 1\ k\Omega$

$R_4 \approx 125\ R_3 = 125\ k\Omega$

10) **[4 pts]** Questions from guest lectures. Please write your answer only in the space provided.

Wireless non-contact ECG and EEG

a. Name two characteristics of the circuit that are compromised due to the extremely high impedance of the non-contact sensors. [1 pt]

- Signal gain (sensitivity)
- Common-mode rejection

b. What kind of artifacts is still the major challenge for this type of electrode? [1 pt]

Motion artifacts

Wireless medical technology in global health

a. Name two applications of wireless technology in health care. [1 pt]

- Continuous remote monitoring of vital signals
- Access to clinical data and information anywhere

b. Which developed country presents the highest health expenditure per capita? [1 pt]

The United States