BENG 186B Winter 2013

Quiz 2

February 15, 2013

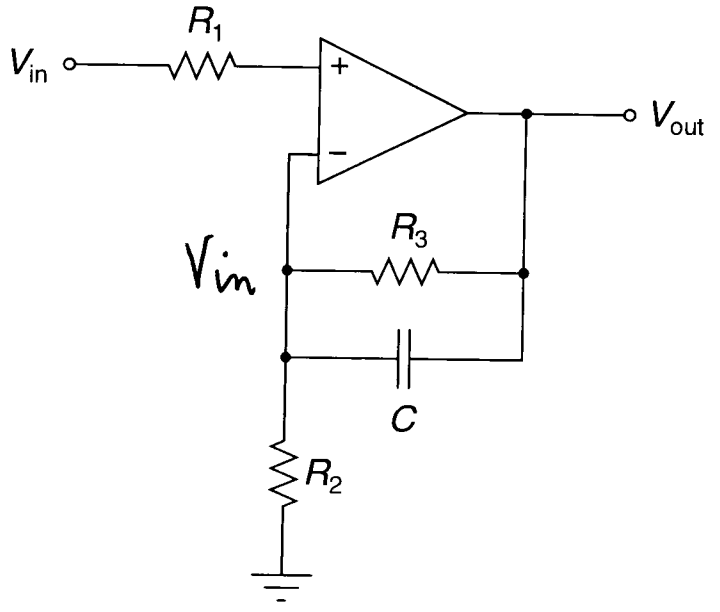
NAME (Last, First):

SOLUTIONS

- This quiz is closed book and closed notes. You may use a calculator for algebra.
- Circle your final answers in the space provided; show your work only on the pages provided.
- Do not attach separate sheets. If you need more space, use the back of the pages.
- Points for each problem are given in [brackets], 100 points total. The quiz is 50 minutes long.

1	/25
2	/25
3	/20
4	/30
Total	/100

1. [**25 pts**] Consider the following active circuit with voltage input V_{in} and voltage output V_{out}. You may assume that the op-amp is ideal.



a. Derive the transfer function $H(j\omega) = V_{out}(j\omega)/V_{in}(j\omega)$. [10 pts]

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{R_2 + \left(R_3 \parallel \frac{1}{j\omega C}\right)}{R_2} = 1 + \frac{R_3 \cdot \frac{1}{j\omega C}}{R_2\left(R_3 + \frac{1}{j\omega C}\right)}$$

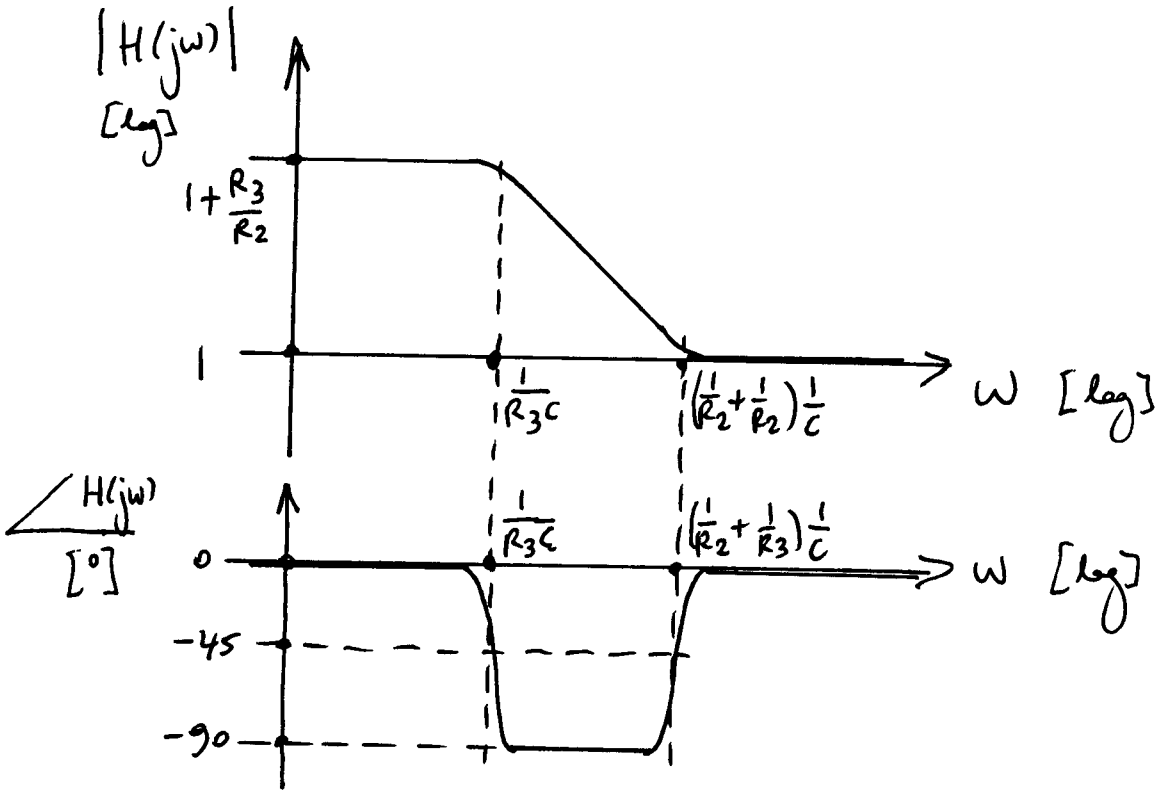
$$= 1 + \frac{R_3}{R_2} \cdot \frac{1}{1 + j\omega R_3 C}$$

b. Identify the pole(s) and/or the zero(s) of the transfer function $H(j\omega)$, and sketch the Bode plot (log amplitude and phase of $H(j\omega)$ as a function of log radial frequency ω). [8 pts]

$$H(j\omega) = 1 + \frac{R_3}{R_2} \cdot \frac{1}{1 + j\omega R_3 C} = \frac{\left(1 + \frac{R_3}{R_2}\right) + j\omega R_3 C}{1 + j\omega R_3 C}$$

$$\Rightarrow \quad \text{pole at } j\omega = -\frac{1}{R_3 C}$$

$$\text{zero at } j\omega = -\frac{1 + \frac{R_3}{R_2}}{R_3 C} = -\left(\frac{1}{R_2} + \frac{1}{R_3}\right)\frac{1}{C}$$



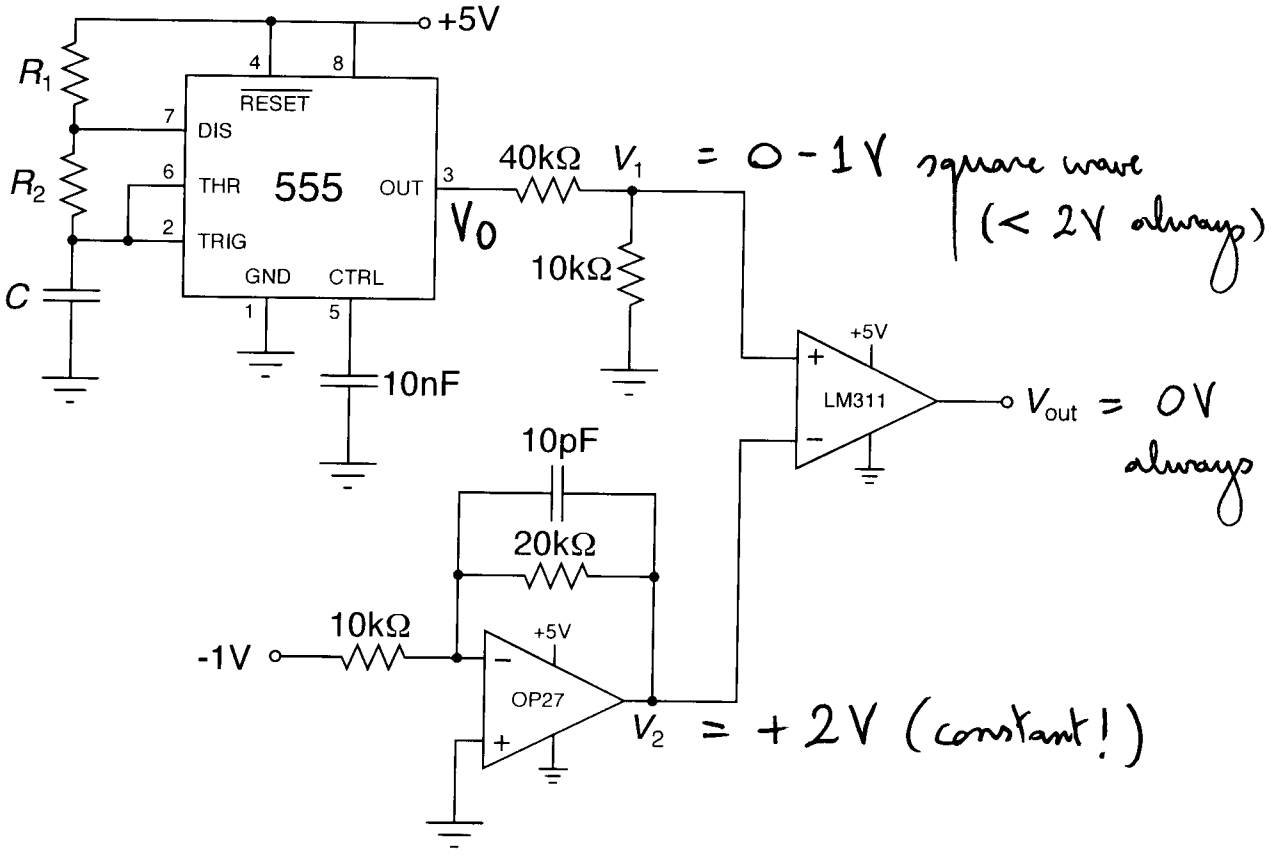
c. Derive the input impedance $Z_{in}(j\omega)$. [4 pts]

$Z_{in} = \infty$ since the OPAMP does not draw input current (ideal OPAMP).

d. Derive the output impedance $Z_{out}(j\omega)$. [3 pts]

$Z_{out} = 0$ since V_{out} is maintained by the ideal OPAMP regardless of output current (as long as it does not saturate).

2. **[25 pts]** The following circuit generates a voltage output V_{out} as shown below. The components OP27, LM311, 555 are assumed ideal, each with positive supply at +5 V and negative supply at GND (0 V). $R_1 = 2\ \text{M}\Omega$, $R_2 = 1\ \text{M}\Omega$, and $C = 100$ nF. The equations for the 555 are: $T_{high} = 0.7\ (R_1 + R_2)\ C$ and $T_{low} = 0.7\ R_2\ C$.



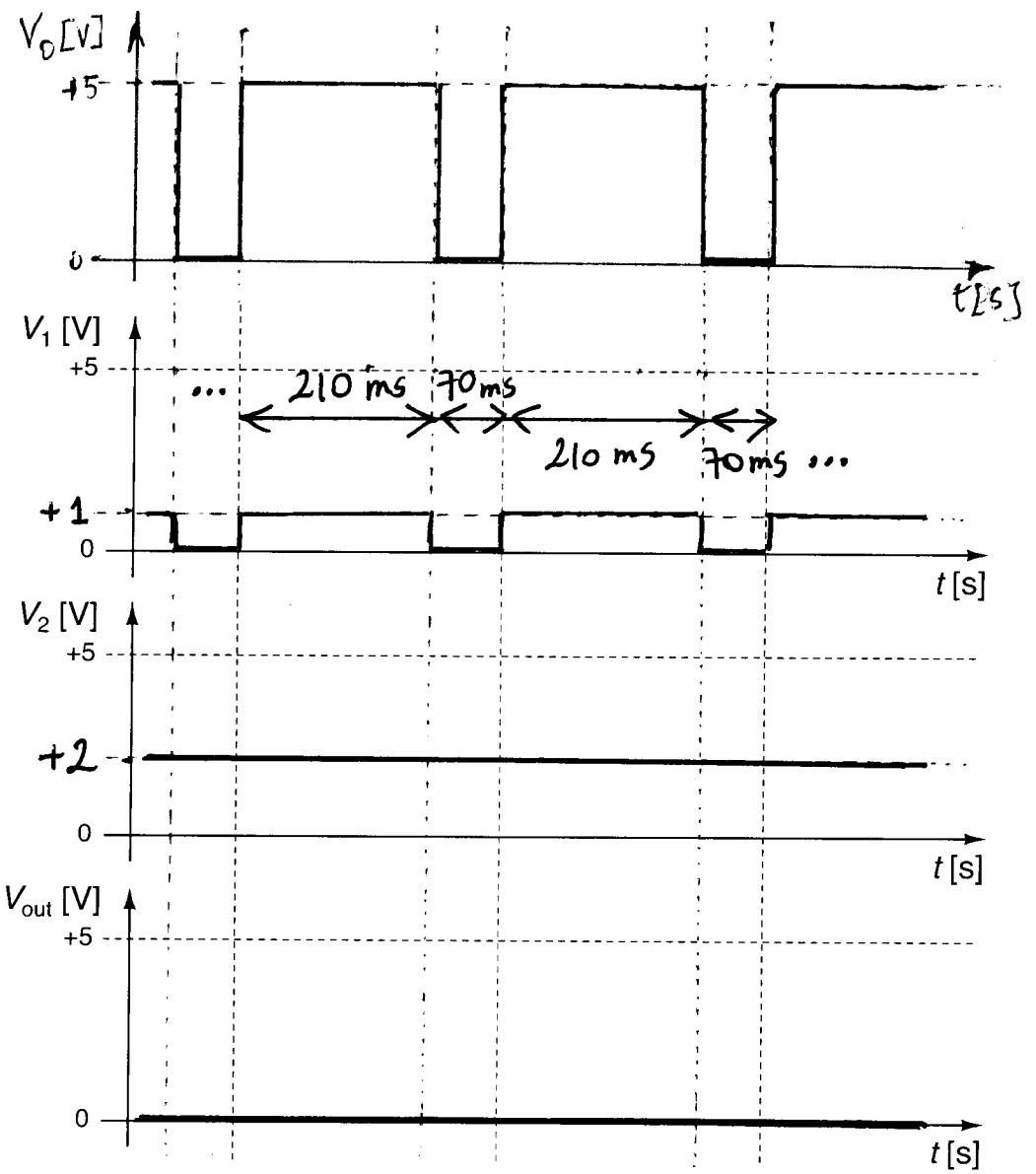
Sketch the waveforms for the voltages V_1, V_2 and V_{out} on the diagrams on the next page. Show your work below.

V_0 (555 out) is a square wave between 0V and +5V, with $T_{high} = 0.7(R_1+R_2)C =$ 210 ms, and $T_{low} = 0.7\ R_2 C =$ 70 ms.

$V_1 = \dfrac{10\ \text{k}\Omega}{10\ \text{k}\Omega + 40\ \text{k}\Omega} \cdot V_0 = \dfrac{1}{5} V_0$: the same square wave, but between 0V and +1V.

V_2 is $-1V$ amplified with an inverting gain of $-\frac{20k\Omega}{10k\Omega} = -2$, or $V_2 = +2V$. The 10pF capacitor is just an open circuit for DC voltages.

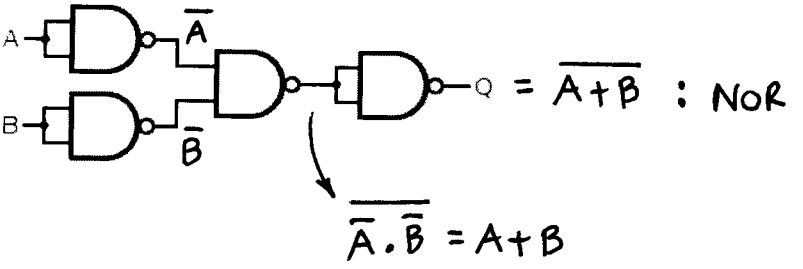
V_{out} remains at 0V since $V_1 < V_2$ always.



3. [**20 pts**] Circle the best answer (circle only one choice for each question a. and b., and circle TRUE or FALSE for each part of question c. and d.).

   a. [4 pts] In an active circuit using an ideal opamp, the voltage at the non-inverting input can be considered virtually identical to the voltage at the inverting input:
      i. When there is no feedback
      ii. When there is positive feedback
      (iii.) When there is negative feedback
      iv. When the output is saturated
      v. Through hysteresis

   b. [4 pts] The following digital circuit comprised of only NAND gates is functionally equivalent to the following gate:



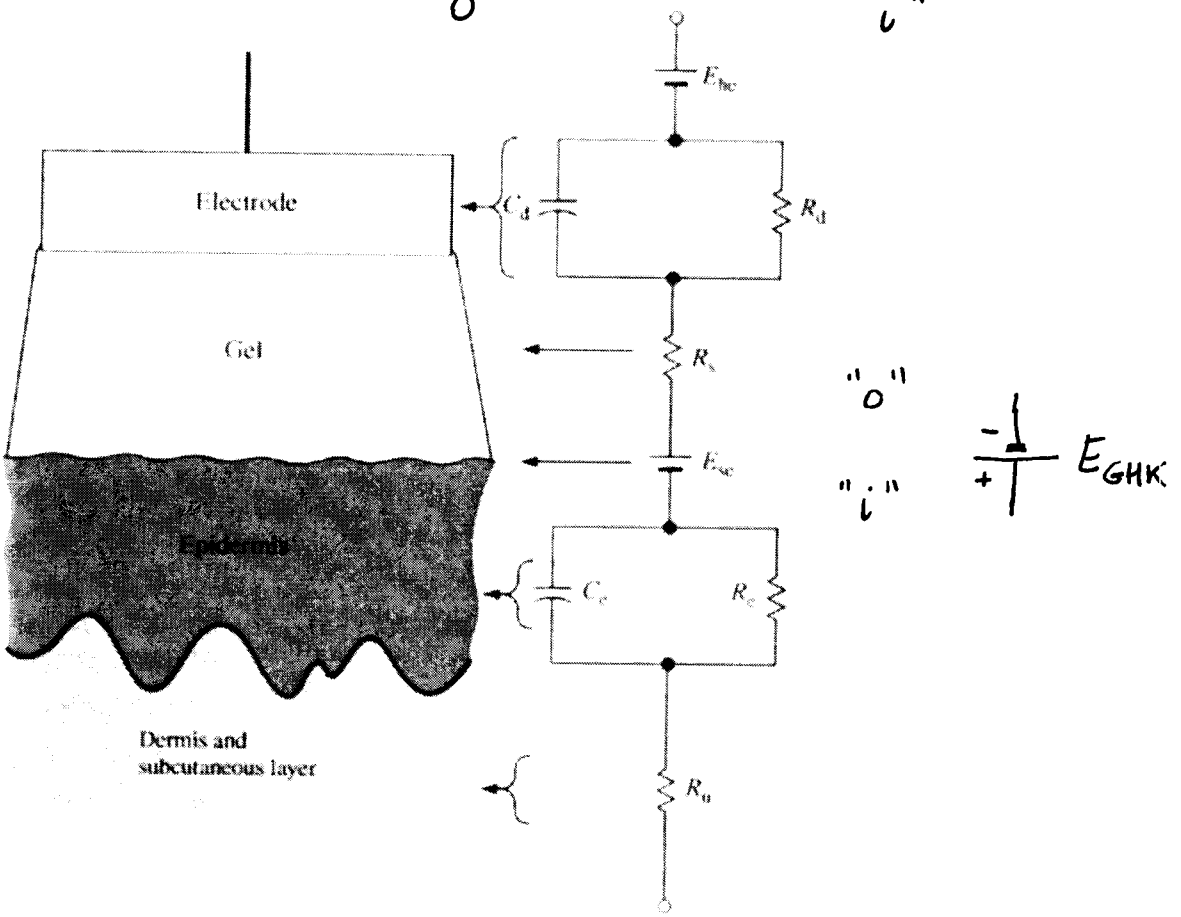
      i. AND
      ii. OR
      (iii.) NOR
      iv. XOR
      v. XNOR

   c. [6 pts] Indicate whether each of the following statements regarding electrocardiography are true or false:
      [(TRUE) / FALSE] Atrial depolarization occurs before ventricular depolarization in the ECG cycle.
      [TRUE (FALSE)] The AV node triggers the onset of ECG cycle.
      [TRUE (FALSE)] Ventricular repolarization produces the highest voltage amplitude in a typical ECG.

   d. [6 pts] Indicate whether each of the following statements regarding half-cell potentials are true or false:
      [TRUE (FALSE)] The Ag/AgCl electrode is polarizable.
      [TRUE (FALSE)] The half-cell potential is independent of ionic concentrations in the solution.
      [(TRUE) / FALSE] In a passive electrical circuit connected to two electrodes immersed in ionic solution, current flows from the electrode with highest half-cell potential to that with lowest.

4. [**30 pts**] Consider the circuit model for the electrode-skin interface below. The Ag/AgCl electrode has a half cell potential of +223 mV, and the ion concentrations in the gel and epidermis are given in the following table:

	Gel (outside skin)	Epidermis (inside skin)
$[Na^+]$	10 mmol/L	10 mmol/L
$[K^+]$	100 mmol/L	1 mmol/L
$[Cl^-]$	110 mmol/L	11 mmol/L



The Goldman-Hodgkin-Katz (GHK) equation:

$$E = (60mV) \cdot \log_{10} \frac{P_K[K^+]_o + P_{Na}[Na^+]_o + P_{Cl}[Cl^-]_i}{P_K[K^+]_i + P_{Na}[Na^+]_i + P_{Cl}[Cl^-]_o}$$

a. Find the Nernst potentials for Na^+, K^+ and Cl^- across the gel-epidermis interface. [4 pts]

$$E_{Na} = (60\,mV)\log_{10}\frac{[Na^+]_o}{[Na^+]_i} = 60\,mV \cdot \log_{10}\frac{10}{10} = 0$$

$$E_K = (60\,mV)\log_{10}\frac{[K^+]_o}{[K^+]_i} = 60\,mV \cdot \log_{10}\frac{100}{1} = 120\,mV$$

$$E_{Cl} = (-60\,mV)\log_{10}\frac{[Cl^-]_o}{[Cl^-]_i} = -60\,mV \cdot \log_{10}\frac{110}{11} = -60\,mV$$

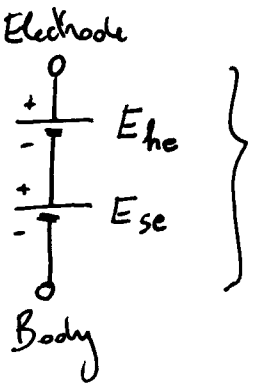
b. Find the equilibrium potential E_{se} across the gel-epidermis interface, assuming the boundary separating the epidermis from the gel is equally permeable to all ion types. [4 pts]

$$E_{se} = -E_{GHK} = -(60\,mV)\log_{10}\frac{[K^+]_o + [Na^+]_o + [Cl^-]_i}{[K^+]_i + [Na^+]_i + [Cl^-]_o}$$

"i" is epidermis
"o" is gel

$$= -60\,mV\log_{10}\frac{121}{121} = 0$$

c. Derive the voltage between the electrode and the subcutaneous body at rest. [4 pts]

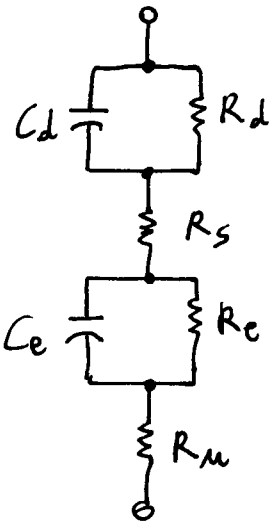


$$E_{he} + E_{se} = +223\,mV + 0$$

$$= +223\,mV$$

d. Find the general expression for the impedance $Z(j\omega)$ of the electrode-skin interface, based on the AC equivalent circuit. What is the order of the system? Identify all poles (and zeros). [10 pts]

see e.



$$Z(j\omega) \equiv \left(R_d \parallel \frac{1}{j\omega C_d}\right) + \left(R_e \parallel \frac{1}{j\omega C_e}\right) + R_s + R_m$$

$$= \frac{R_d}{1 + j\omega R_d C_d} + \frac{R_e}{1 + j\omega R_e C_e} + R_s + R_m$$

SECOND-ORDER SYSTEM

Two poles: $j\omega = -\frac{1}{R_d C_d}$, and $j\omega = -\frac{1}{R_e C_e}$

Two zeros:

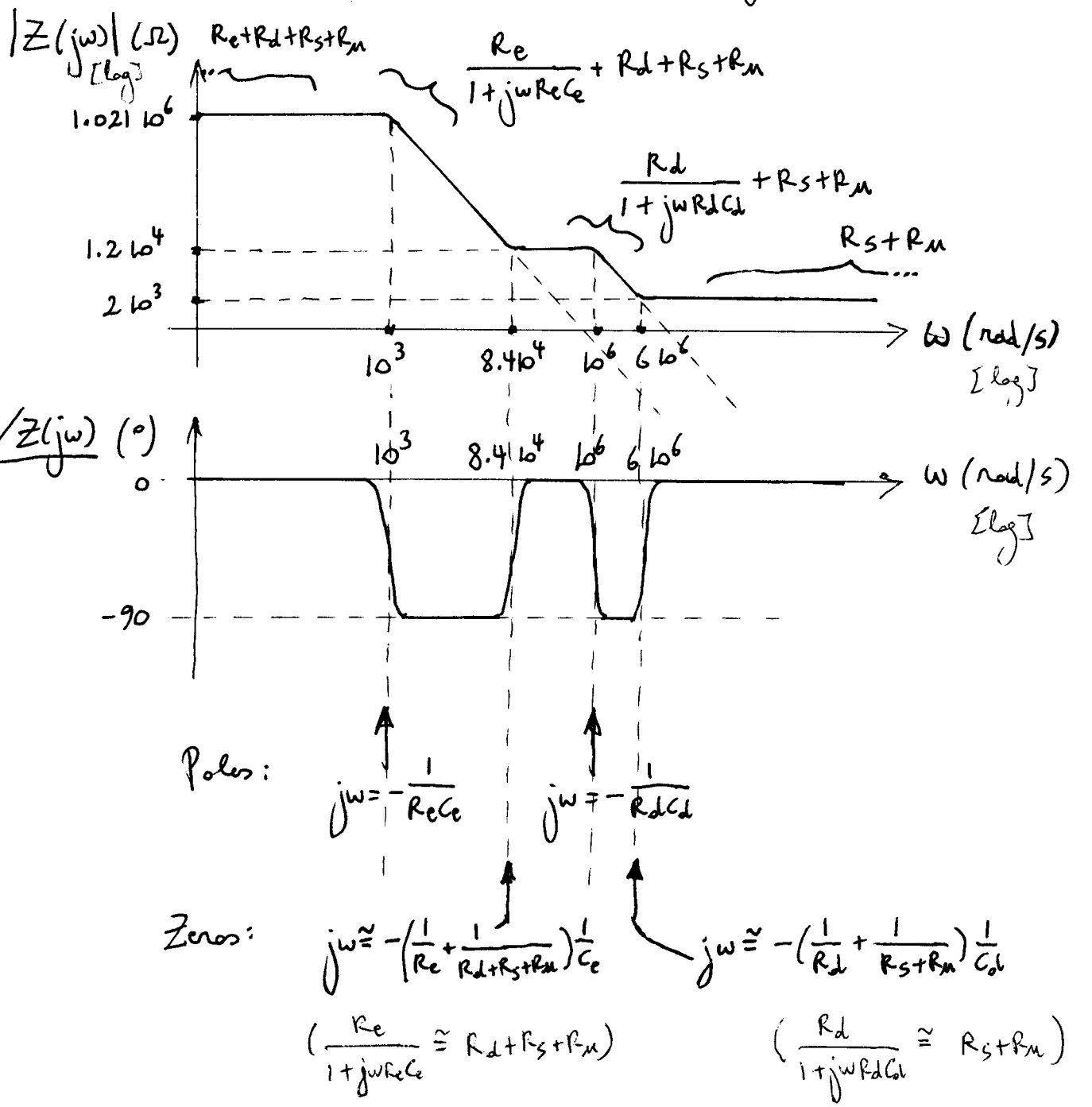
$$Z(j\omega) = \frac{R_d(1+j\omega R_e C_e) + R_e(1+j\omega R_d C_d) + (R_s+R_m)(1+j\omega R_d C_d)(1+j\omega R_e C_e)}{(1+j\omega R_d C_d)(1+j\omega R_e C_e)}$$

$$= \frac{\left[\underbrace{R_d+R_e+R_s+R_m}_{c} + j\omega\underbrace{\left(R_d R_e(C_d+C_e)+(R_s+R_m)(R_d C_d+R_e C_e)\right)}_{b} - \omega^2\underbrace{(R_s+R_m)R_d R_e C_d C_e}_{a}\right]}{(1+j\omega R_d C_d)(1+j\omega R_e C_e)}$$

zeros: $j\omega = -\frac{b \pm \sqrt{b^2-4ac}}{2a}$ $\quad \left(a(j\omega)^2 + b(j\omega) + c = 0\right)$

e. Sketch the Bode plot of the impedance $Z(j\omega)$ (log amplitude and phase as a function of log radial frequency ω) for $R_s = R_u = 1$ kΩ, $R_d = 10$ kΩ, $R_e = 1$ MΩ, $C_d = 100$ pF, and $C_e = 1$ nF. Indicate units and key values on your axes. [8 pts]

$Z(0) = R_d + R_e + R_s + R_u = 1.012\ M\Omega$

$Z(\infty) = R_s + R_u = 2\ k\Omega$

poles: $j\omega = -\frac{1}{R_d C_d} = 10^6$ rad/s, and $j\omega = -\frac{1}{R_e C_e} = 10^3$ rad/s