# BENG 186B Winter 2017

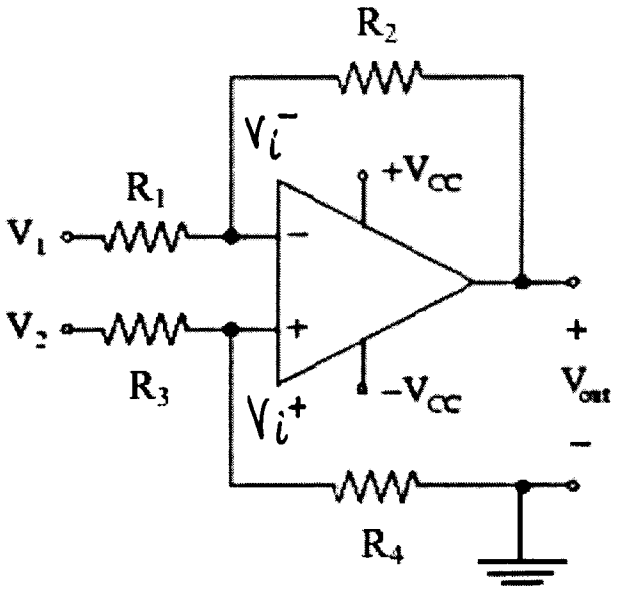
## Quiz 2

### Monday, February 13, 2017

*Last Name, First Name:* SOLUTIONS

- This quiz is closed book and closed notes. You may use a calculator for algebra and arithmetic.
- This quiz has 9 pages, including this cover sheet. Do not attach separate sheets. If you need more space, use the back of the pages.
- Circle or box your final answers and show your work on the pages provided.
- There are 4 problems. Points for each problem are given in **[brackets]**. There are 100 points total.
- You have 50 minutes to complete this quiz.

1. **[30 pts]** Consider the following circuit:




(a) Assume the operational amplifier is ideal and unsaturated. Derive an expression for $V_{out}$ in terms of $V_1$ and $V_2$.

Negative feedback $\Rightarrow$ $V_i^- = V_i^+$

KCL @ $V_i^+$ : $\dfrac{V_2 - V_i^+}{R_3} = \dfrac{V_i^+}{R_4}$ $\Rightarrow$ $V_i^+ = \dfrac{R_4}{R_3+R_4} \cdot V_2$

( voltage divider )

KCL @ $V_i^-$ : $\dfrac{V_1 - V_i^-}{R_1} = \dfrac{V_i^- - V_{out}}{R_2}$

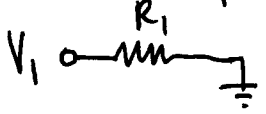
$\Rightarrow$ $V_{out} = -\dfrac{R_2}{R_1} \cdot V_1 + \left(1 + \dfrac{R_2}{R_1}\right) \cdot V_i^-$

$= -\dfrac{R_2}{R_1} \cdot V_1 + \dfrac{R_1+R_2}{R_1} \cdot \dfrac{R_4}{R_3+R_4} \cdot V_2$

(b) What are the input impedances at the two inputs, $V_1$ and $V_2$?

$V_1$ : $V_i^- = V_i^+$ does not change with input current

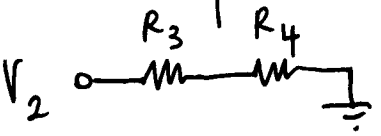
$\Rightarrow$ impedance on the $V_i^-$ node is zero




: $Z_{in_1} = R_1$

$V_2$ : $V_i^+$ takes no current

$\Rightarrow$ impedance on the $V_i^+$ node is $\infty$




: $Z_{in_2} = R_3 + R_4$

(c) Pick values for $R_1, R_2, R_3$, and $R_4$ to amplify the difference $(V_2 - V_1)$ 100-fold at the output.

$V_{out} = 100\ (V_2 - V_1)$ for :

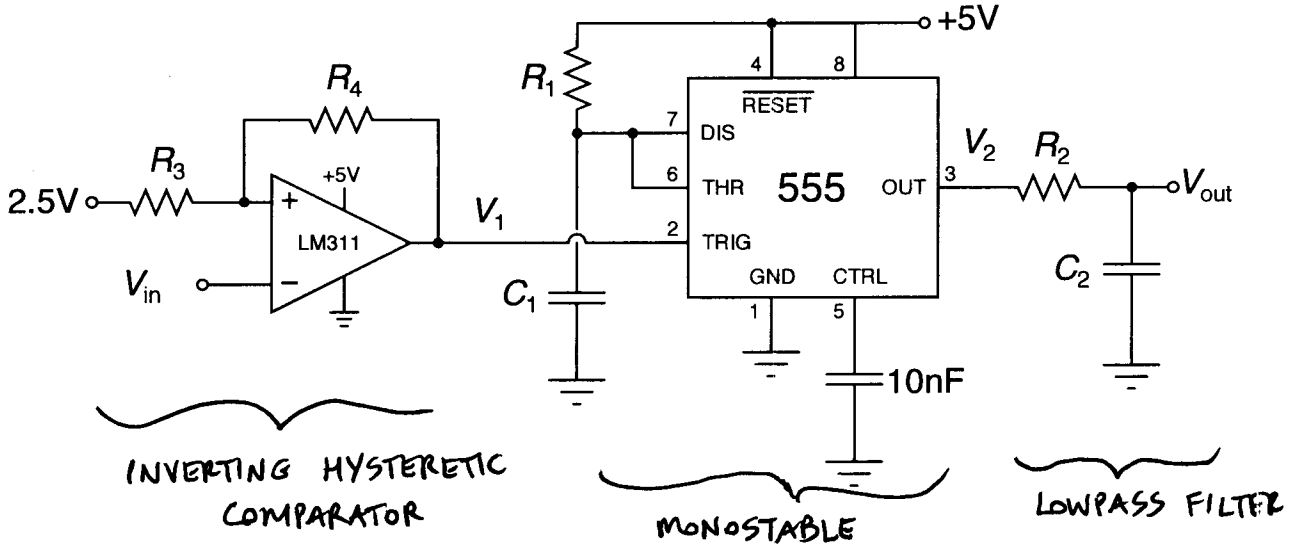
- $\frac{R_2}{R_1} = 100 \Rightarrow R_2 = 100\, R_1$
- $\frac{R_1 + R_2}{R_1} \cdot \frac{R_4}{R_3 + R_4} = 100$

$\Rightarrow \frac{R_4}{R_3} = \frac{100}{101 - 100} \Rightarrow R_4 = 100\, R_3$

For instance : $\begin{cases} R_1 = 1\,k\Omega & R_2 = 100\,k\Omega \\ R_3 = 1\,k\Omega & R_4 = 100\,k\Omega \end{cases}$

2. **[25 pts]** Consider the circuit below with components LM311 and 555 all assumed ideal. You are given a positive voltage supply of 5 V, and ground voltage (0 V). You may find these equations useful for the 555 timer ($\ln(3) \approx 1.1$ and $\ln(2) \approx 0.7$):

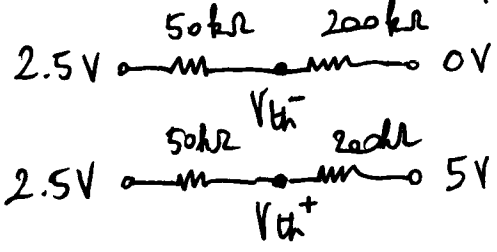
$$T = \ln(3) \times RC \quad T_{lo} = \ln(2) \times R_2C \quad T_{hi} = \ln(2) \times (R_1 + R_2)C$$




$R_1 = 470\,\text{k}\Omega$, $R_2 = 1\,\text{M}\Omega$, $R_3 = 50\,\text{k}\Omega$, $R_4 = 200\,\text{k}\Omega$, $C_1 = 200\,\text{nF}$, $C_2 = 10\,\mu\text{F}$.

For an ECG input waveform $V_{in}$, sketch the waveforms for the voltages $V_1$, $V_2$ and $V_{out}$ on the diagrams on the next page. Show your work below.

$V_1$ : Inverting hysteretic comparator, with two threshold references:




$V_{th}^- = 2\,\text{V}$ (voltage divider)

$V_{th}^+ = 3\,\text{V}$ (by symmetry)

$\Rightarrow$ $V_1$ switches to 0V when $V_{in}$ goes above $V_{th}^+ = 3\text{V}$;

$V_1$ switches to 5V when $V_{in}$ goes below $V_{th}^- = 2\text{V}$.

$V_2$: Monostable (single-shot), with pulse width:

$$T = \ln(3)\, R_1 C_1 = 1.1 \cdot 470\,k\Omega \cdot 200\,nF \approx 100\,ms$$

(103.4 ms)

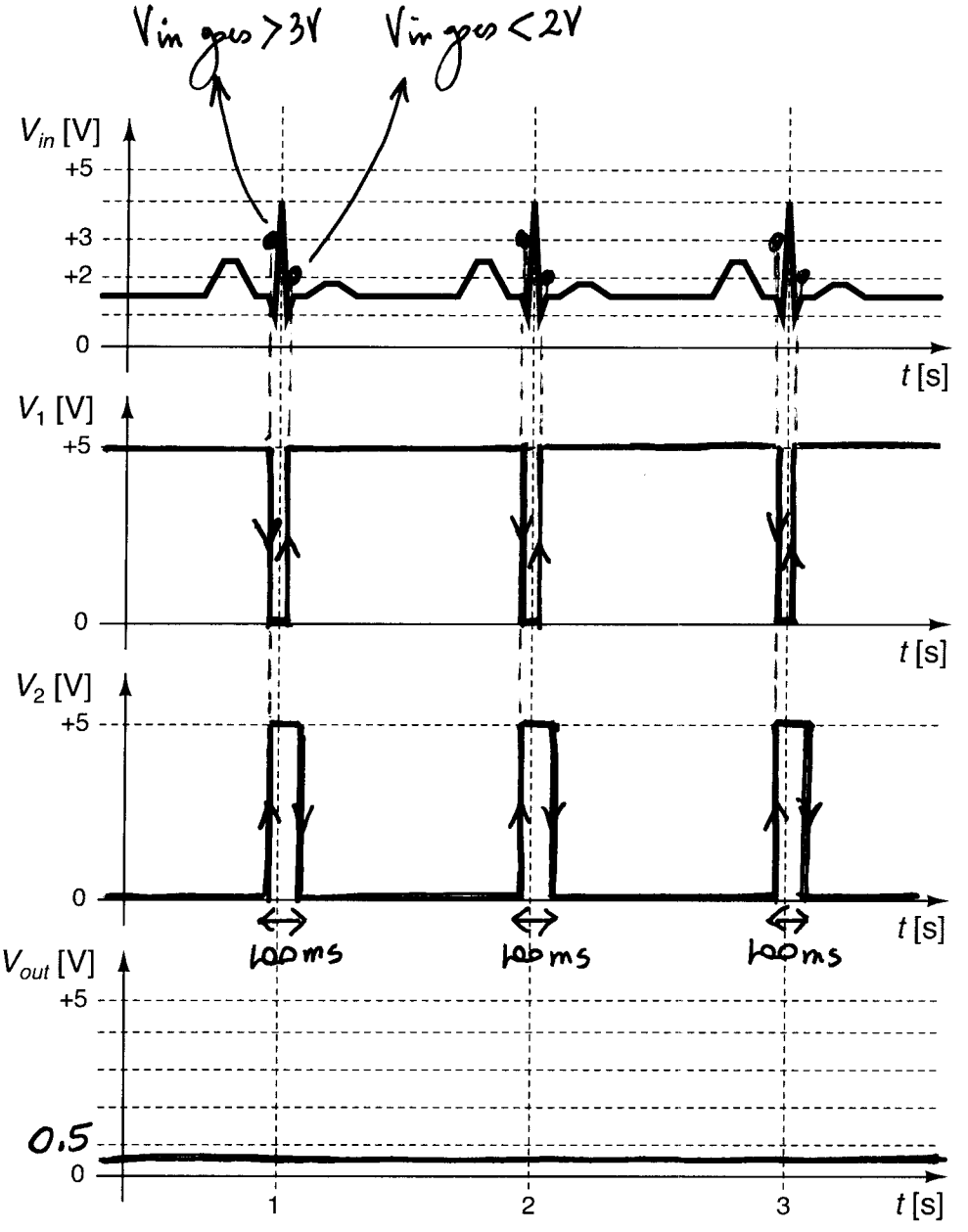
$\Rightarrow$ $\approx$ 100 ms 5V pulse whenever $V_1$ goes low (below 5V/3 = 1.67 V).

$V_{out}$: Low-pass filter, with cut-off frequency:

$$f_c = \frac{1}{2\pi R_2 C_2} = \frac{1}{2\pi \cdot 1M\Omega \cdot 10\mu F} = \frac{1}{63\,s} = 0.016\,Hz$$

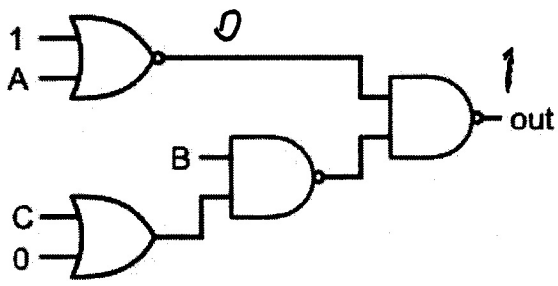
This cut-off frequency is much lower than the ECG heart rate (1 Hz), and so the output $V_{out}$ is approximately the AVERAGE of $V_2$:

$$V_{out} \approx ave(V_2) = \frac{100\,ms}{1\,s} \cdot 5V = 0.5V$$


Vin goes > 3V
Vin goes < 2V
$V_{in}$ [V]
+5
+3
+2
0
t [s]
$V_1$ [V]
+5
0
t [s]
$V_2$ [V]
+5
0
t [s]
100ms
100ms
100ms
$V_{out}$ [V]
+5
0.5
0
1
2
3
t [s]

3. **[20 pts]** Circle the **best answer (only one answer per question)**:

(a) [4 pts] Find the simplest logical expression for the output of the circuit shown at right:




i. NAND$(0, \text{NAND}(B, C))$

ii. NAND$(A, \text{NAND}(B, C))$

iii. NAND$(A, C)$

iv. 0

(v.) 1

(b) [4 pts] Voltage comparators operate using

i. no feedback

ii. positive feedback

iii. negative feedback

(iv.) i or ii

v. i or iii

(c) [4 pts] Your EEG signal will show the strongest alpha wave activity when you are

i. taking an exam.

(ii.) awake but not paying attention and starting to get sleepy during lecture.

iii. releasing your exam-related stress by working out at the gym.

iv. in a deep sleep when the quarter finally ends.

(d) [1 pt ea.] Indicate for each statement below whether it is true or false:

i. **TRUE** / **FALSE**: The Nernst potential for chloride is positive when the extracellular chloride concentration is higher than the intracellular chloride concentration.

ii. **TRUE** / **FALSE**: ECoG can measure higher amplitude signals than EEG.

iii. **TRUE** / **FALSE**: ECoG can measure higher frequency signals than EEG.

iv. **TRUE** / **FALSE**: EOG stands for electroosteogram and is clinically used to noninvasively provide a measure of patient bone density.

v. **TRUE** / **FALSE**: The potential measured from a current dipole is negative when $R^+ > R^-$.

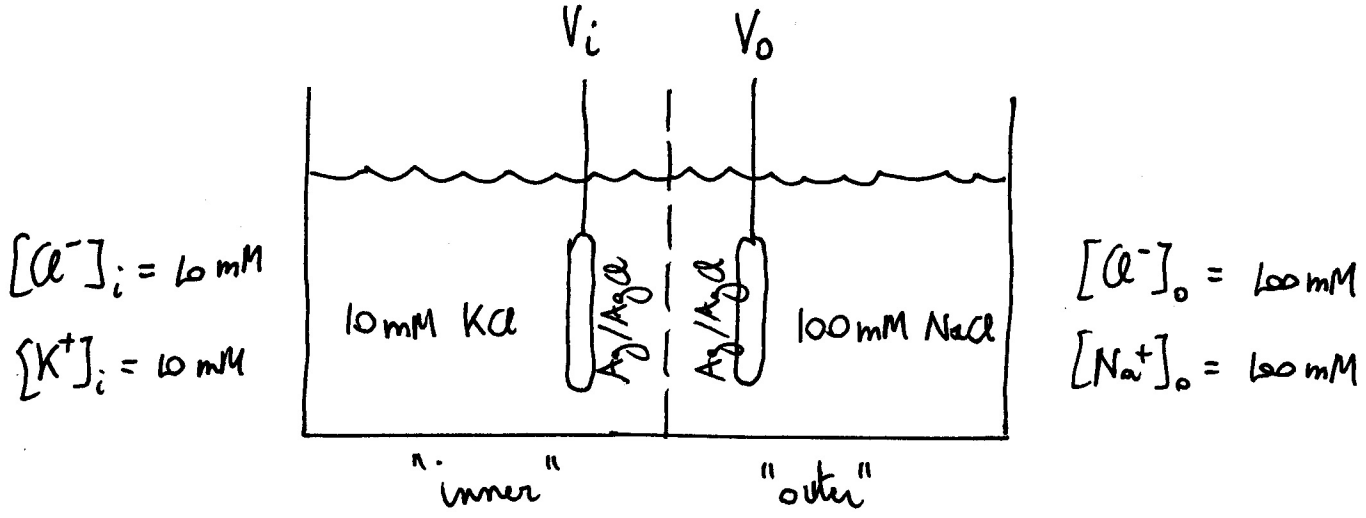
vi. **TRUE** / **FALSE**: The QRS wave of ECG indicates atrial depolarization.

vii. **TRUE** / **FALSE**: When an excitable cell is at its resting membrane potential, the membrane is completely impermeable to the movement of ions (all ion channels are closed).

viii. **TRUE** / **FALSE**: For differential measurement between two identical electrodes, the half-potentials add and contribute twice the electrode half-potential to the measurement.

4. **[25 pts]** Consider an electrochemical cell with an "inner" compartment containing 10 mM of KCl, and an "outer" compartment containing 100 mM of NaCl, at room temperature. The two compartments are separated by a membrane that is permeable to $Cl^-$ only. Two identical Ag/AgCl electrodes are inserted, one in each compartment, to measure the voltage difference $V_i - V_o$ between the inner and the outer compartment. This voltage is measured by a voltmeter with infinite input impedance. At room temperature $RT/F \ln(10) \approx 60$ mV.

   (a) Find the voltage $V_i - V_o$ between the electrodes measured by the voltmeter.




Identical Ag/AgCl electrodes ⇒ half potentials cancel out

Permeable to $Cl^-$ only ⇒ Nernst potential for $Cl^-$

$$\Rightarrow V_i - V_o = -\frac{RT}{F} \ln(10) \cdot \log_{10} \frac{[Cl^-]_o}{[Cl^-]_i} = -60\,\text{mV} \cdot (+1)$$

$$= -60\,\text{mV}$$

↑ valence for $Cl^- = -1$

(b) The membrane is replaced with one that is now equally permeable to all ion types: $P_{\mathrm{K}} = P_{\mathrm{Na}} = P_{\mathrm{Cl}}$. The GHK equation for the voltage across the membrane is:

$$V_m = 60\,\mathrm{mV} \times \log\left(\frac{P_{\mathrm{Na}}[\mathrm{Na}^+]_o + P_{\mathrm{K}}[\mathrm{K}^+]_o + P_{\mathrm{Cl}}[\mathrm{Cl}^-]_i}{P_{\mathrm{Na}}[\mathrm{Na}^+]_i + P_{\mathrm{K}}[\mathrm{K}^+]_i + P_{\mathrm{Cl}}[\mathrm{Cl}^-]_o}\right)$$

Find the new voltage $V_i - V_o$ between the electrodes measured by the voltmeter.

$(P = P_K = P_{Na} = P_{Cl})$

$$V_i - V_o = 60\,\mathrm{mV} \cdot \log_{10} \frac{P \cdot 100\,\mathrm{mM} + P \cdot 0 + P \cdot 10\,\mathrm{mM}}{P \cdot 0 + P \cdot 10\,\mathrm{mM} + P \cdot 100\,\mathrm{mM}}$$

$$= 60\,\mathrm{mV} \cdot \log_{10} 1 = 0\,\mathrm{V}$$