BENG 186B Principles of Bioinstrumentation

Week 1 Review

Solutions

Selections from:

2015 Homework 1
2015 Homework 2

BENG 186B Winter 2015 HW #1 Solutions

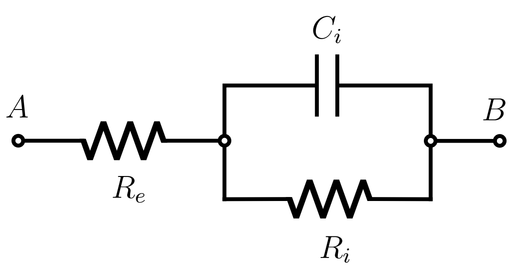
1. Assuming that the voltage range is 0 V to 5 V minus one LSB in all cases:

   (a) 8-bit ADC:
   2^8 = 256 distinct values (ranges from 0 to 255).
   $\text{LSB} = 5/256 = 19.5\,\text{mV}$

   (b) 12-bit ADC:
   2^{12} = 4096 distinct values (ranges from 0 to 4095).
   $\text{LSB} = 5/4096 = 1.22\,\text{mV}$

   (c) 16-bit ADC:
   2^{16} = 65536 distinct values (ranges from 0 to 65535).
   $\text{LSB} = 5/65536 = 0.0763\,\text{mV}$

2. Recall that the impedance of capacitors is given by $1/(j\omega C)$, where C is the capacitance and j is the imaginary number.



   (a)

$$Z = Z_{Re} + \frac{1}{1/Z_{Ri} + 1/Z_{Ci}} = R_e + \frac{1}{1/R_i + j\omega C_i} = R_e + \frac{R_i}{1 + j\omega R_i C_i} \tag{1}$$

   For algebraic simplicity, let $A = \omega R_i C_i$:

$$Z = R_e + \frac{R_i}{1 + jA} = R_e + \frac{R_i(1 - jA)}{(1 + jA)(1 - jA)} \tag{2}$$

$$\boxed{Z = R_e + \frac{R_i - jR_iA}{1 + A^2} = \underbrace{\frac{R_i + R_e(1 + A^2)}{1 + A^2}}_{a} - j\underbrace{\frac{R_iA}{1 + A^2}}_{b}} \tag{3}$$

   The rightmost side of the above equation is of the form $a + jb$, which allows for the calculation of phase and magnitude.

(b)

$$|Z| = \sqrt{a^2 + b^2} \tag{4}$$

$$|Z| = \sqrt{\left(\frac{R_i + R_e(1 + A^2)}{1 + A^2}\right)^2 + \left(-\frac{R_i A}{1 + A^2}\right)^2} \tag{5}$$

$$|Z| = \sqrt{\frac{{R_i}^2 + 2R_e(1 + A^2) + {R_e}^2(1 + A^2)^2 + {R_i}^2 A^2}{(1 + A^2)^2}} \tag{6}$$

$$|Z| = \sqrt{\frac{{R_e}^2(1 + A^2)^2 + {R_i}^2(1 + A^2) + 2R_e(1 + A^2)}{(1 + A^2)^2}} \tag{7}$$

$$\boxed{|Z| = \sqrt{\frac{{R_e}^2(1 + A^2) + {R_i}^2 + 2R_e}{1 + A^2}} = \sqrt{{R_e}^2 + \frac{{R_i}^2 + 2R_e}{1 + \omega^2 {R_i}^2 {C_i}^2}}} \tag{8}$$

(c)

$$\angle Z = \text{arctan}\left(\frac{b}{a}\right) \tag{9}$$

$$\angle Z = \text{arctan}\left(\frac{-(R_i A)/(1 + A^2)}{(R_i + R_e(1 + A^2))/(1 + A^2)}\right) \tag{10}$$

$$\angle Z = \text{arctan}\left(-\frac{R_i A}{R_i + R_e(1 + A^2)}\right) \tag{11}$$

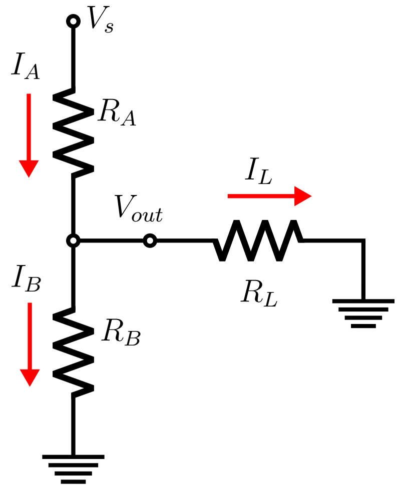
$$\boxed{\angle Z = \text{arctan}\left(-\frac{\omega {R_i}^2 C_i}{R_i + R_e(1 + \omega^2 {R_i}^2 {C_i}^2)}\right)} \tag{12}$$

(d) There are several possible ways to convert impedances into voltages:

- Use a Wheatstone bridge
- Use a voltage divider
- Use a current source

Building a current source is relatively complex compared to building a resistor network, but allows for simple mathematics using Ohm's law ($V = IZ$).

3. Voltage divider only:



(a) Apply KCL at the V_{out} node:

$$I_A = I_B + I_L \tag{13}$$

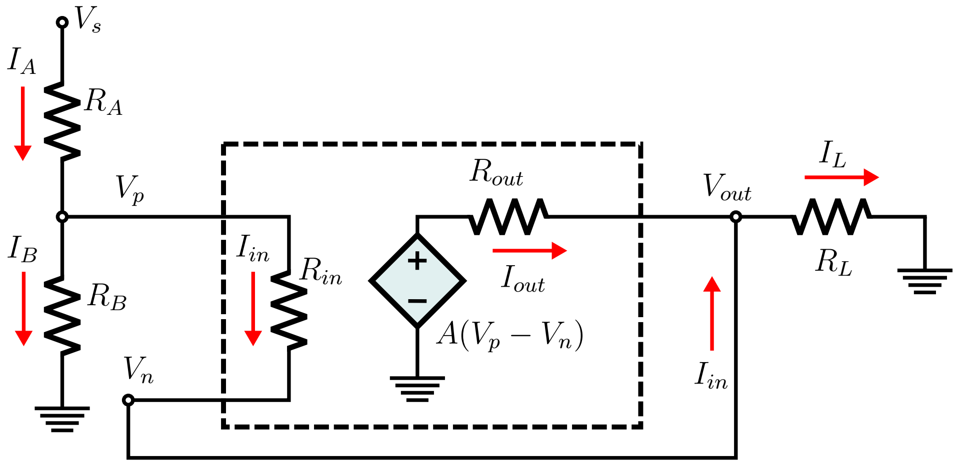
$$\frac{V_s - V_{out}}{R_A} = \frac{V_{out} - 0}{R_B} + \frac{V_{out} - 0}{R_L} \tag{14}$$

$$\frac{V_s}{R_A} = V_{out}\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_L}\right) \tag{15}$$

$$\boxed{V_{out} = \frac{V_s}{1 + R_A/R_B + R_A/R_L}} \tag{16}$$

(b) Since R_L is not known in advance, the scientists will not be able to calculate the correct values for R_A and R_B needed to get the desired V_{out}. Also, because R_L changes as the device operates, V_{out} will vary and very likely lead to device malfunction.

(c) Voltage divider with an amplifier:



Instead of using resistances R, we can use conductances $G = 1/R$ for algebraic simplicity. Apply KCL at the V_{out} node (recall that $V_n = V_{out}$):

$$I_L = I_{out} + I_{in} \tag{17}$$

$$G_L(V_{out} - 0) = G_{out}(A(V_p - V_n) - V_{out}) + G_{in}(V_p - V_n) \tag{18}$$

$$V_p(G_{out}A + G_{in}) = V_{out}(G_L + G_{out} + G_{out}A + G_{in}) \tag{19}$$

$$V_p = V_{out}\left(\frac{G_L + G_{out}}{G_{out}A + G_{in}} + 1\right) \tag{20}$$

Apply KCL at the V_p node:

$$I_A = I_B + I_{in} \tag{21}$$

$$G_A(V_s - V_p) = G_B(V_p - 0) + G_{in}(V_p - V_n) \tag{22}$$

$$V_p(G_A + G_B + G_{in}) = G_A V_s + G_{in} V_{out} \tag{23}$$

$$V_p = \frac{G_A V_s + G_{in} V_{out}}{G_A + G_B + G_{in}} \tag{24}$$

Combine the above equations to eliminate V_p:

$$V_{out}\left(\frac{G_L + G_{out}}{G_{out}A + G_{in}} + 1\right) = \frac{G_A V_s + G_{in} V_{out}}{G_A + G_B + G_{in}} \tag{25}$$

$$V_{out}\left(\frac{G_L + G_{out}}{G_{out}A + G_{in}} - \frac{G_{in}}{G_A + G_B + G_{in}} + 1\right) = V_s\left(\frac{G_A}{G_A + G_B + G_{in}}\right) \tag{26}$$

$$V_{out}\left(\frac{G_L + G_{out}}{G_{out}A + G_{in}} + \frac{G_A + G_B}{G_A + G_B + G_{in}}\right) = V_s\left(\frac{G_A}{G_A + G_B + G_{in}}\right) \tag{27}$$

Isolate V_{out}:

$$\boxed{V_{out} = V_s\left(\frac{G_A}{G_A + G_B + G_{in}}\right)\left(\frac{G_L + G_{out}}{G_{out}A + G_{in}} + \frac{G_A + G_B}{G_A + G_B + G_{in}}\right)^{-1}} \tag{28}$$

(d) Take the limit as $R_{in} \to \infty$ (equivalent to $G_{in} \to 0$) and $A \to \infty$:

$$\lim_{G_{in} \to 0} \frac{G_A}{G_A + G_B + G_{in}} = \frac{G_A}{G_A + G_B} = \frac{R_B}{R_A + R_B} \tag{29}$$

$$\lim_{G_{in} \to 0} \lim_{A \to \infty} \frac{G_L + G_{out}}{G_{out} A + G_{in}} = 0 \tag{30}$$

$$\lim_{G_{in} \to 0} \frac{G_A + G_B}{G_A + G_B + G_{in}} = 1 \tag{31}$$

The limits greatly simplify the full expression:

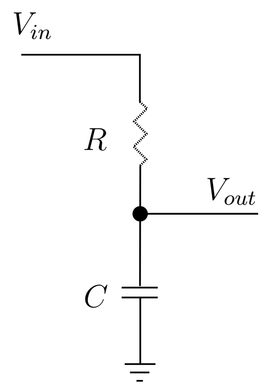
$$\boxed{V_{out} = V_s \left(\frac{R_B}{R_A + R_B} \right)} \tag{32}$$

Note that this is voltage divider equation, which does not depend on R_L. The added amplifier circuit effectively isolated the loading effect of R_L from the voltage divider.

(e) If we take the limit of (20) as $A \to \infty$, then $V_p = V_{out}$. In addition, recall that $V_n = V_{out}$, since V_{out} was connected to V_n in the circuit. It follows that $V_p = V_n$, and therefore no current flows through the input resistor R_{in}. This current is not dependent on R_L.

4. **Design Problem:**

- Because we want to reduce aliasing effects, we need to use a **low–pass filter** to remove high frequency components.



Here, V_{in} is the input, and V_{out} is connected to the ADC.

- Assume that the ADC has infinite input impedance, to avoid loading effects on the filter.
- The sampling rate is 100 kHz. By the Nyquist theorem, the maximum signal frequency should be less than 50 kHz.
- 50 kHz is the cutoff frequency of the first–order, passive RC filter:

$$2\pi f = \frac{1}{RC} \Rightarrow f = \frac{1}{2\pi RC} = 50\,\text{kHz}$$
$$RC = \frac{1}{2\pi \times 50\,\text{kHz}}$$

- We need to choose values for R and C to meet the 10 µA requirement. When the signal has zero frequency (a DC signal), the capacitor acts as an open circuit. Since the ADC has infinite input impedance, the input effectively carries no current.

  When the signal has infinite frequency, the capacitor acts as a short circuit. Of course, a signal of infinite frequency isn't physically possible, but represents the absolute worst case. Thus, the resistor in the filter would be the only thing limiting the input current.
- Since the input is at most 1 V, we can now compute what R should be:

$$V = IR \Rightarrow 1\,\text{V} = 10\,\mu\text{A} \times R \Rightarrow \boxed{R = 100\,\text{k}\Omega}$$

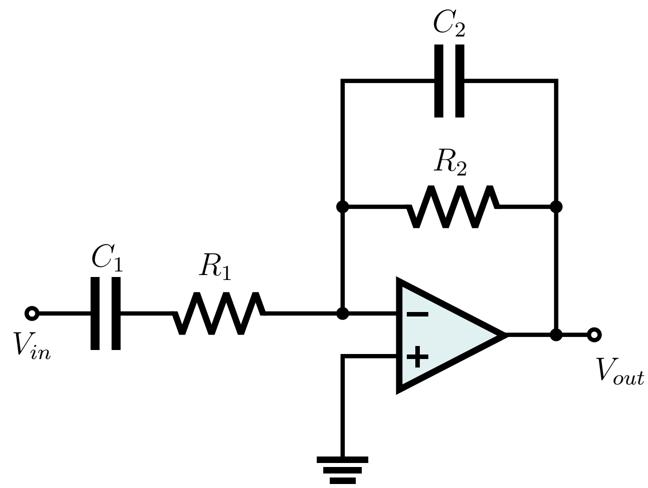
- We can now compute C:

$$100\,\text{k}\Omega \times C = \frac{1}{2\pi \times 50\,\text{kHz}} \Rightarrow \boxed{C = 31.83\,\text{pF}}$$

- **Sidebar:** Manufacturers typically do not produce components of arbitrary values. Instead they produce them using *preferred numbers* from an "E series" as component values (additional information can be found at `https://en.wikipedia.org/wiki/Preferred_number#E_series`). For example, C could be 33 pF (33 from the E12 series) instead of 31.83 pF. 33 pF capacitors are commercially available and inexpensive.

BEENG 186B Winter 2015 HW #2
Due *Thursday, January 29* at the beginning of class

1. Consider the following band–pass filter:

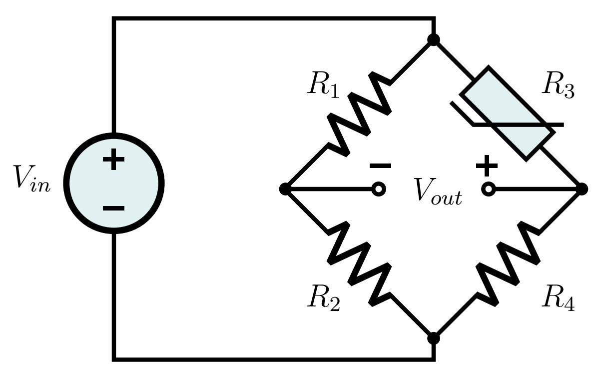


   (a) Find the transfer function $H(j\omega) = V_{out}/V_{in}$. Assume the op–amp is ideal.

   (b) Propose values for the circuit components such that the filter has a gain of 30 dB and a pass-band of 20 Hz to 20 kHz.

   (c) Draw a Bode plot of the filter response.

2. Consider the following circuit for a thermometer:



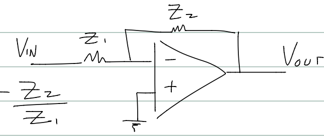
   R_3 represents a thermistor with a transfer function $R_3(T) = r_\infty e^{\beta T}$, where β and r_∞ are constants, and T is temperature.

   (a) Write the voltage output V_{out} of the system as a function of temperature T. Pay attention to the polarity of V_{out}.

   (b) What is the sensitivity of the thermistor's resistance with respect to temperature? In other words, calculate dR_3/dT.

   (c) What is the sensitivity of the voltage output V_{out} with respect to T?

(1.) Observation: Inverting Opamp So



$$\frac{V_{OUT}}{V_{IN}} = -\frac{Z_2}{Z_1}$$

$$H(s) = \frac{V_{OUT}}{V_{IN}} = \frac{(-1)\left(\underbrace{\frac{R_2\left(\frac{1}{sC_2}\right)}{R_2 + \frac{1}{sC_2}}}_{Z_2}\right)}{\underbrace{R_1 + \frac{1}{sC_1}}_{Z_1}} \Rightarrow H(s) = \frac{(-1)\; R_2 C_1 s}{(R_2 s C_2 + 1)(R_1 s C_1 + 1)}$$

$$H(s) = \frac{\frac{s}{\left(\frac{-1}{R_2 C_1}\right)}}{\left(\frac{s}{\left(\frac{1}{R_2 C_2}\right)} + 1\right)\left(\frac{s}{\left(\frac{1}{R_1 C_1}\right)} + 1\right)} \qquad H(jw) = \frac{\frac{jw}{\left(\frac{-1}{R_2 C_1}\right)}}{\left(\frac{jw}{\left(\frac{1}{R_2 C_2}\right)} + 1\right)\left(\frac{jw}{\left(\frac{1}{R_1 C_1}\right)} + 1\right)}$$

(b) Specifications: Gain: 30dB ; $30 = 20 \log_{10} G \Rightarrow G = 10^{\frac{3}{2}} = |H(jw)|$

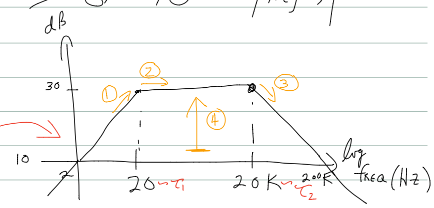
Cutoff Freq 1: $\omega_{C1} = 2\pi f_{C1} = 2\pi \cdot 20$ $\frac{\text{Radians}}{\text{Sec}}$

f_{C1} ↳ 20Hz

Cutoff Freq 2: $\omega_{C2} = 2\pi f_{C2} = 2\pi \cdot 2 \cdot 10^4$ $\frac{\text{Radians}}{\text{Sec}}$

f_{C2} ↳ $20 \cdot 10^3$ Hz

From Specs We Want Something That Looks Like This



Therefore, We Will Have An Eqn Which Looks Like: $H(jw) = \frac{(\text{Gain}) \cdot \tau_1 jw}{(1+\tau_1 jw)(1+\tau_2 jw)}$ (4) (1) (2) (3)

We Want τ_1 Because We Want 3dB Gain At ω_{C1}

Apply Lower Cut Off Freq First: 20 Hz. So $\omega_{C1} = 20 \cdot 2 \cdot \pi = \frac{1}{R_1 C_1} = \frac{1}{\tau_1}$

Since $C_1 = 80\mu F$ $R_1 = \frac{1}{40\pi \cdot 80 \cdot 10^{-6}} \Omega \approx 99 \Omega$

Apply Gain: $G \cdot \tau_1 = R_2 C_1 \Rightarrow 10^{\frac{3}{2}} \cdot R_1 C_1 = R_2 C_1 \Rightarrow 10^{\frac{3}{2}} = \frac{-R_2}{R_1}$

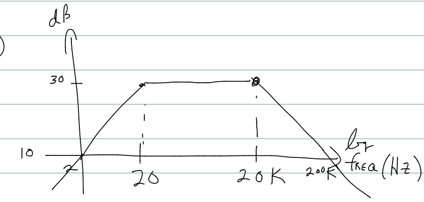
Since We Are Interested In Magnitude $\text{abs}\left(10^{\frac{3}{2}} = \frac{-R_2}{R_1}\right) = 10^{\frac{3}{2}} = \frac{R_2}{R_1}$ *

$$R_2 = (99.47\Omega)\, 10^{\frac{3}{2}} = 3.1456 \times 10^3 \Omega$$

Then Apply Upper Cut Off Freq: 20kHz. So $\omega_{C2} = 4\pi \cdot 10^4 = \frac{1}{R_2 C_2}$

$$C_2 = \frac{1}{\left(\frac{10^{\frac{3}{2}}}{99.47}\right) 10^4\, 4\pi} = 2.5298 \times 10^{-9} F$$

(c)



(2) (a)

$V_{OUT} = V_{OUT+} - V_{OUT-}$ $\quad V_{OUT+} = \dfrac{R_4}{R_3(T) + R_4}$

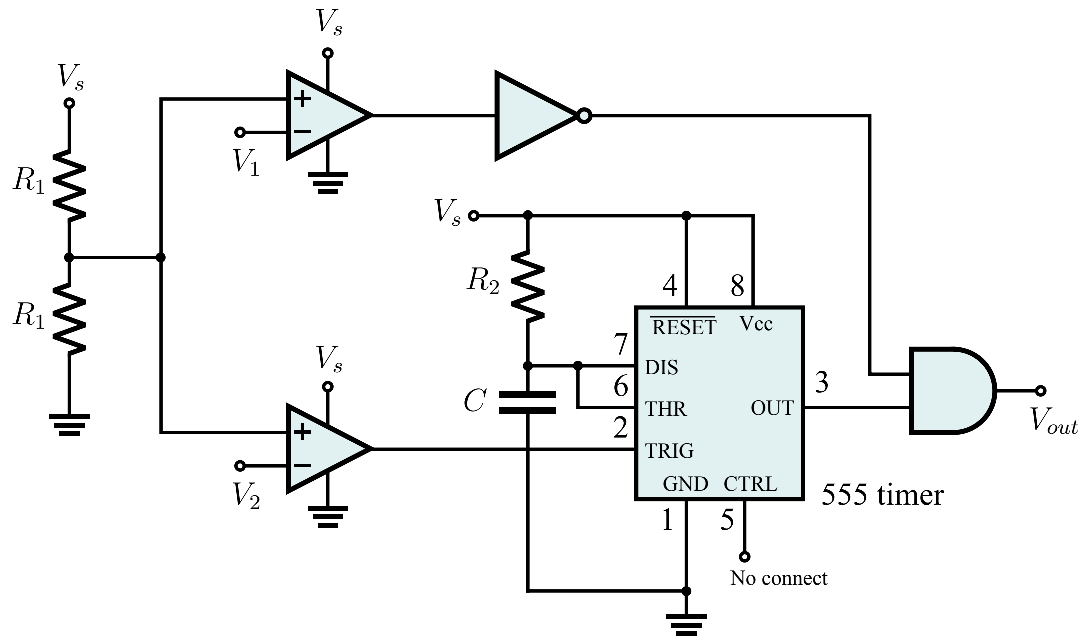
$V_{OUT-} = \dfrac{R_2}{R_1 + R_2}$

$$V_{OUT}(T) = \frac{R_4}{r_\infty e^{\beta T} + R_4} - \frac{R_2}{R_1 + R_2}$$

(b) $\dfrac{dR_3}{dT} = r_\infty \beta e^{\beta T}$

(c) $\dfrac{dV_{OUT}(T)}{dT} = (-1) \cdot r_\infty \beta e^{\beta T} \dfrac{R_4}{(r_\infty e^{\beta T} + R_4)^2}$

3. Consider the following circuit with $V_s = 5\,\mathrm{V}$, $R_1 = 1\,\mathrm{k\Omega}$, $R_2 = 25\,\mathrm{k\Omega}$, and $C = 9.1\,\mu\mathrm{F}$:



Two biosensors being worn by a patient output voltage signals V_1 and V_2. V_1 is 0 V for $t \leq 0$ s, then ramps linearly from 0 V to 5 V over the interval $0\,\mathrm{s} \leq t \leq 2\,\mathrm{s}$, and finally remains at 5 V for $t \geq 2$ s. V_2 is a square wave with an amplitude of 2.6 V (*i.e.*, pulsing from 0 V to 2.6 V), a period of 500 ms, and a duty cycle of 30 %. V_2 starts pulsing at $t = 0$ s.

(a) Sketch V_1, V_2 and V_{out} from $t = 0$ s to $t = 2$ s.

(b) What tolerance is required of the resistance value R_1 in order for this circuit to operate correctly for the given input signals. Explain.

4. **Design Problem: Portable Cytotoxicity Measurement Device**

Cytotoxic assays offer an effective means to measure the toxicity of a variety of compounds. These assays are performed by growing cells on a substrate, then exposing them to the compound. If the compound is toxic, the cells will begin to die and undergo lysis. Cell death is usually measured by chemically labeling the cell with fluorescent agents. As cells begin to die, the fluorescent signal will change. Therefore, cell death is directly correlated with fluorescence intensity. However, this method is cumbersome and expensive, requiring means to attach fluorescent markers to the target cells, and wavelength selective light sources and detectors.

An anti–bioterrorism agency tasked you with designing a label–free electronic method of testing the cytotoxicity of unknown compounds, anywhere in the field. For this purpose you make use of two samples of cells grown on two electrically conducting substrates in a Petri dish: a test sample to be exposed to the unknown compounds, and a control sample shielded from any compounds. You have to design two identical sensors, one for each sample, that couple electrically to the substrate in order to record impedance through the layer of cells.

You also need to design a decision circuit that will detect, based on relative impedance, if the cells in the test group exposed to the compound are dying compared to the control group.

This label-free electrical method is known as electric cell-substrate impedance sensing (ECIS). *Principle:* Healthy cell membranes are essentially non-conductive, with impedance limited by capacitance. When cells die, their membranes degrade, causing them to leak current and leading to a decrease in impedance, mostly a decrease in resistance.

Your design must meet these specifications:

- For each sensor, inject a current signal to probe the impedance by measuring voltage. Do this separately for the control group of cells, and the test group of cells.
- Don't expose cells to more than 10 μA of current for more than 10 μs at a time, otherwise you will kill them.
- This device needs to be portable (powered by batteries), as it will be used in remote locations.
- To save power, the device should only probe/sense cells once every 100 ms.
- The sensor output must be indicated by two LEDS (green and red). As long as the resistance of the test group is greater than one half the resistance of the control group, the green LED should remain lit. Otherwise, the red LED should be lit.

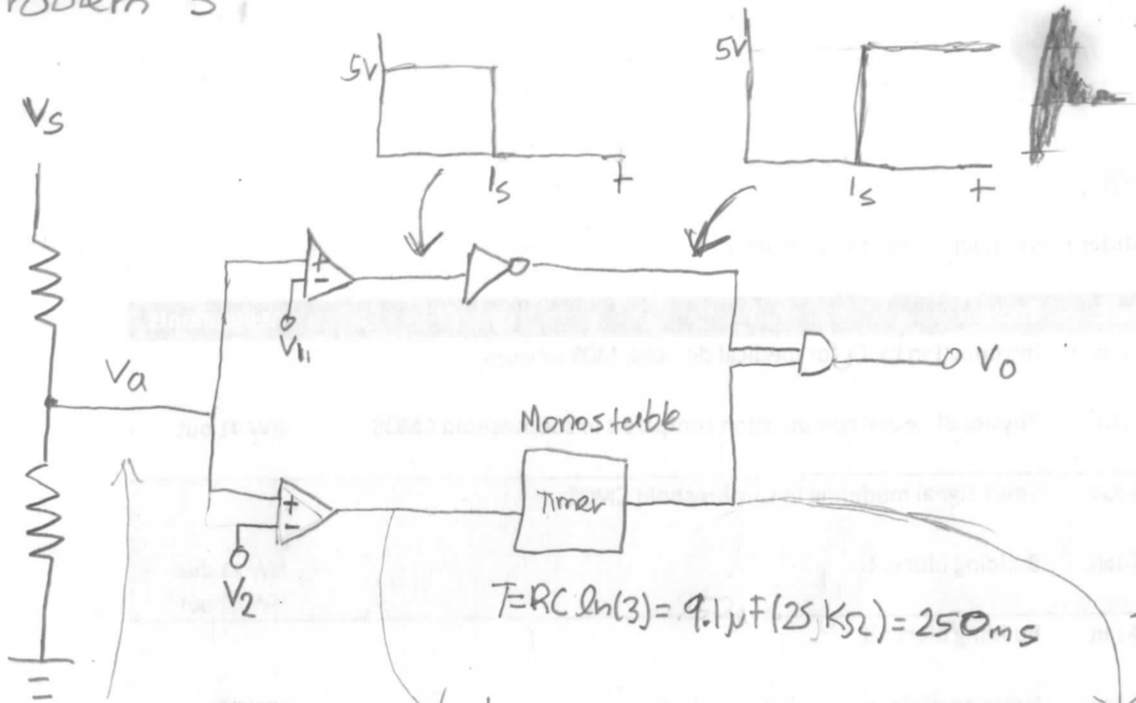
(a) Illustrate the design and briefly (no more than 3 to 4 sentences) describe the principles of operation of your system. Draw a conceptual system-level diagram outlining the major building blocks of the system, including sensors, decision circuit, and output display.

(b) Show a full circuit schematic for your instrument along with equations calculating the values of all resistors, capacitor and other circuit parts. Indicate part numbers for op-amps, timers and other active elements. Indicate the gain or sensitivity of each transducer part of the circuit.

(c) Sketch waveforms for the behavior of the measured quantities and corresponding voltages at different stages in the circuit to clarify the operation. Remember to label all axes and explain what your calculations are showing.

(d) **Bonus:** List some possible problems with your design/idea. How would you solve these problems?

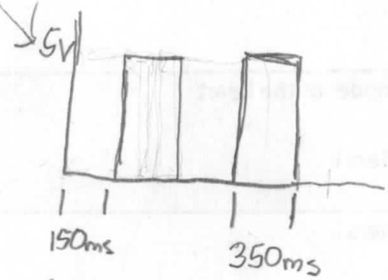
Problem 3

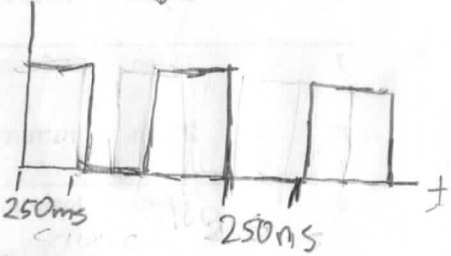


$T = RC \ln(3) = 9.1\,\mu F(25\,k\Omega) = 250\,ms$

Timer will trigger when V=0

$V_a = \frac{1}{2} V_s$





Square wave

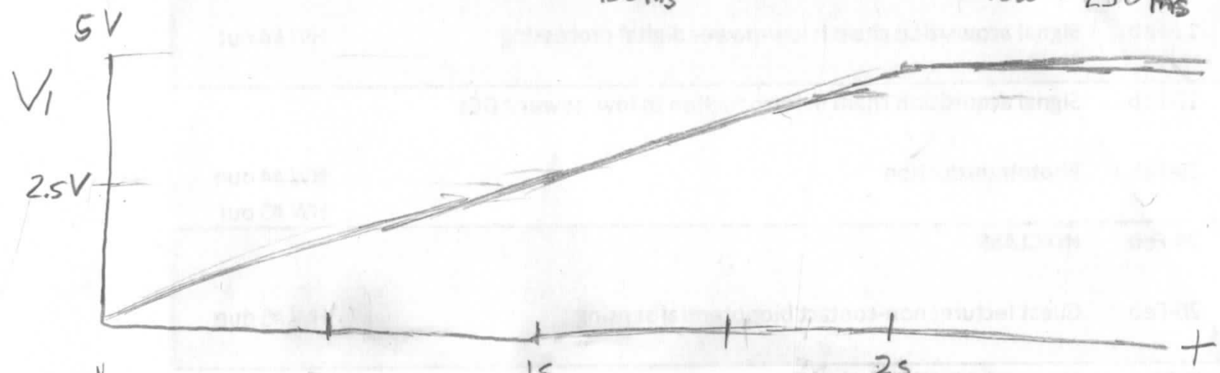
$T_{high} = 350ms$

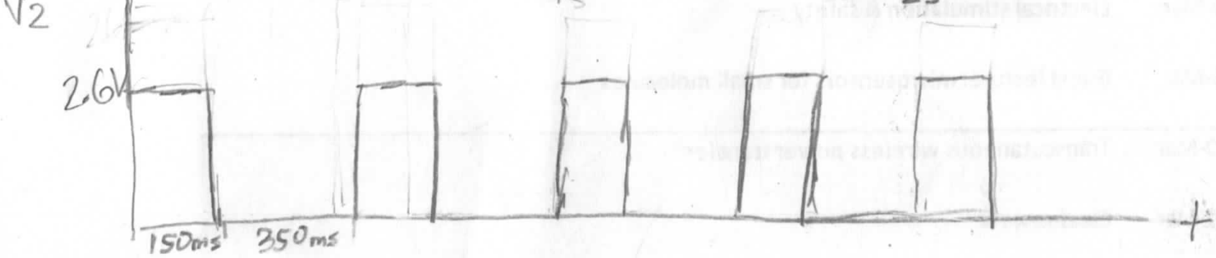
$T_{low} = 150ms$

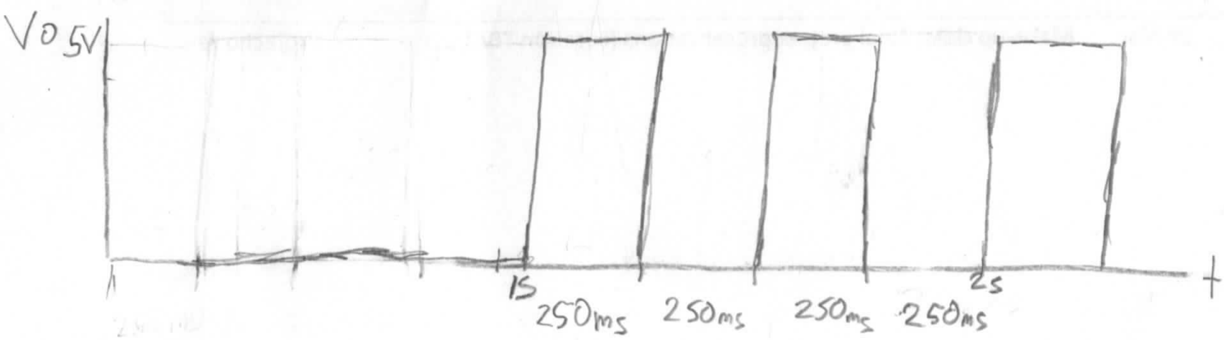
Square wave

$T_{high} = 250ms$

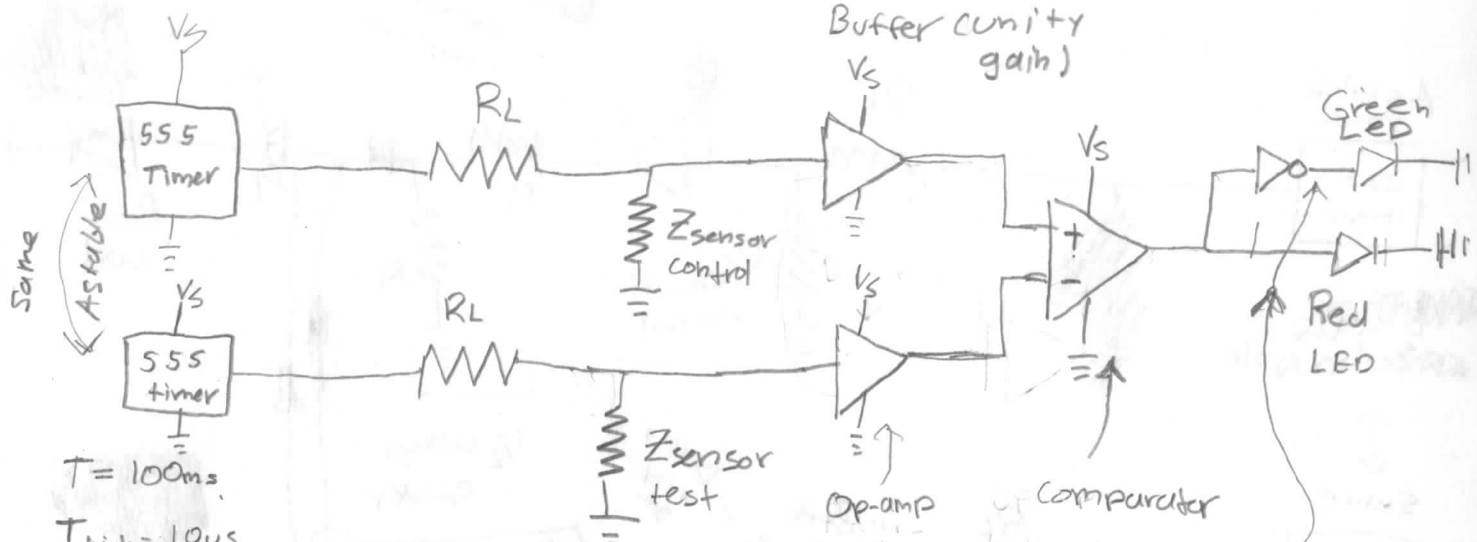
$T_{low} = 250ms$







Design Problem



$T = 100ms$

$T_{high} = 10\mu s$

$= .01\%$ Duty Cycle

gain: 2x (non-inverting op-amp)

assume: $R_L >>> Z_{sensor}$

So

$\frac{V_s}{R_L} \approx 10\mu A$

include the Diode to have any duty cycle

Can also add a monostable Timer before LED to increase pulse duration

can also use a voltage divider after the control output instead.

Z_{sensor} will decrease as cells die. So Voltage out will also decrease