# **BENG 186B Principles of Bioinstrumentation**

### Week 4 Review

## Solutions

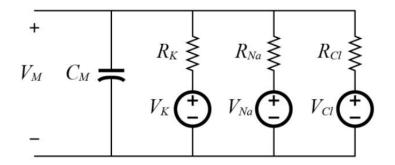
Selections from:

2015 Homework 3 2015 Homework 4

#### BENG 186B Winter 2015 HW #3

#### SOLUTIONS

1. Nernst Potentials: Consider the following circuit model for a cell.  $R_K = 2.7 \text{ k}\Omega$ ,  $R_{Na} = 30 \text{ k}\Omega$ ,  $R_{Cl} = 3.3 \text{ k}\Omega$ .



(a) With the intracellular and extracellular concentrations given in the table below, calculate the Nernst Potential for each of the ionic species:  $V_K$ ,  $V_{Na}$ , and  $V_{Cl}$ .

Ionic Species	Intracellular concentration	Extracellular concentration	
K+	397 mM	20 mM	
Na <sup>+</sup>	50 mM	437 mM	
Cl <sup>-</sup>	40 mM	556 mM	

(b) Using the circuit model with Nernst potentials  $V_K$ ,  $V_{Na}$ , and  $V_{Cl}$  and the resistances given above, find the membrane potential  $V_M$  at steady-state.

Hint: At DC steady-state, any capacitance reduces to an open circuit connection.

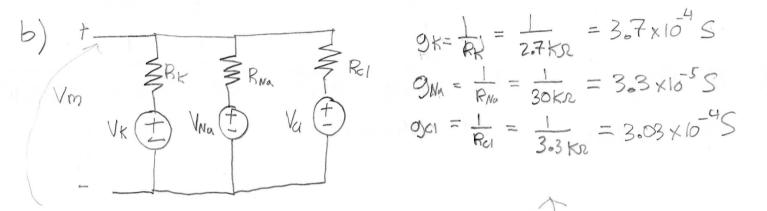
(c) Now find the equilibrium resting potential  $V_M$  using the Goldman-Hodgkin-Katz equation. Compare the two values of the membrane potential. Which value is more reasonable for a typical resting potential of a cell?

*Hint*: Membrane conductance (the reciprocal of membrane resistance) for any ion type is directly proportional to membrane permeability for that ion type.

(d) For a membrane capacitance  $C_m = 1 \,\mu\text{F}$ , find the time constant for the membrane potential  $V_M$  of the cell to recover from a transient and settle to its steady-state value.

Problem I a)  $V_{kt} = 62mV \log \frac{[K^{+}]_{out}}{[K^{+}]_{in}} = 62mV(\log \frac{20}{297}) = -77mV$   $V_{Na}^{+} = 62mV \log \frac{[Na^{+}]_{out}}{[Na^{+}]_{in}} = 62mV (\log \frac{437}{50}) = 56mV$   $V_{c\bar{l}} = -62mV \log \frac{[C\bar{l}]_{out}}{[C\bar{l}]_{out}} = -62mV (\log \frac{556}{40}) = -68mV$  $V_{c\bar{l}} = -62mV \log \frac{[C\bar{l}]_{out}}{[C\bar{l}]_{in}} = -62mV (\log \frac{556}{40}) = -68mV$ 

negative



KCI Q Vm  

$$\frac{V_{K} - V_{m}}{R_{K}} + \frac{V_{Na} - V_{m}}{R_{Na}} + \frac{V_{ai} - U_{m}}{R_{cl}} = 0$$

$$\frac{V_{K}}{R_{K}} + \frac{V_{Na}}{R_{Na}} + \frac{V_{ci}}{R_{ci}} = Vm\left(\frac{1}{R_{K}} + \frac{1}{R_{Na}} + \frac{1}{R_{ci}}\right)$$

$$\frac{V_{K}}{R_{K}} + \frac{V_{Na}}{R_{Na}} + \frac{V_{ci}}{R_{ci}} = Vm\left(\frac{1}{R_{K}} + \frac{1}{R_{Na}} + \frac{1}{R_{ci}}\right)$$

$$\frac{V_{K}}{R_{K}} + \frac{Q_{Na}}{R_{Na}} + \frac{Q_{ci}}{Q_{ci}} = Vm$$

$$\frac{Q_{K}}{Q_{K}} + \frac{Q_{Na}}{Q_{M}} + \frac{Q_{ci}}{Q_{ci}} = Vm$$

$$\frac{Q_{K}}{Q_{K}} + \frac{Q_{Ma}}{Q_{M}} + \frac{Q_{ci}}{Q_{ci}} = Vm$$

(3.7×104 + 3.3×105 + 3.03×104)

C) BY GHK equation

$$V_{m} = \frac{BT}{F} \cdot l_{n} \frac{REKT_{i} + P_{Na}ENa^{\dagger}}{REKT_{i}} + \frac{P_{i}ECT_{i}}{P_{i}ECT_{i}}$$

$$V_{m} = 27_{m}V_{h} \frac{(3_{0}7_{\times10}^{-4})(20) + (3_{0}3_{\times10}^{-5})(437) + (3_{0}03_{\times10}^{-4})(40)}{(3_{0}7_{\times10}^{-4})(397) + (3_{0}3_{\times10}^{-5})(50) + (3_{0}03_{\times10}^{-4})(556)}$$

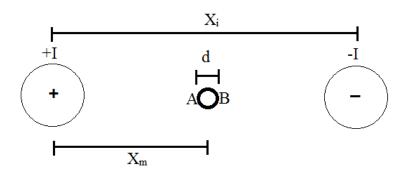
Both are relatively close and within the range of a typical resting potential

$$\frac{\partial}{\partial z} = RC$$

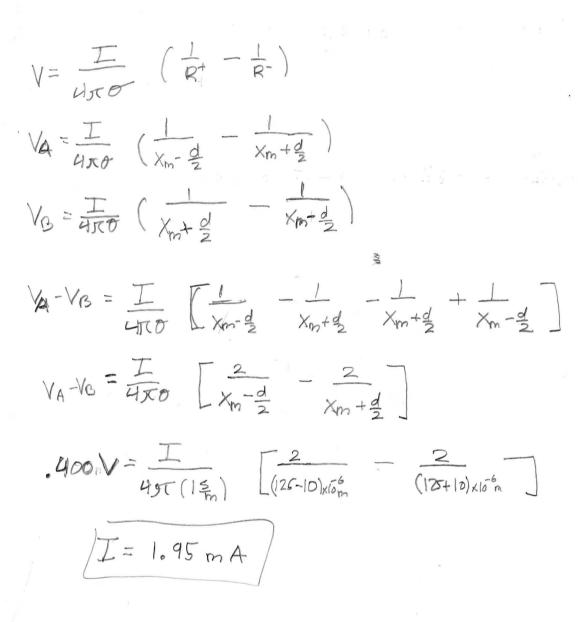
$$R = (9k + 9Nu + 9c1) = (3.7x104) + (3.3x105) + (203x105)R = 1.42 KD$$

2. Electroporation: Often it is necessary to insert genetic material into a cell, crossing its membrane. Electroporation is one means to open the membrane for insertion through the application of a high voltage. Here we study single cell electroporation, by injection of currents +I and -I through two nearby electrodes into the extracellular space, as shown in the figure below. The voltage across A and B, on both sides of the the cell, should not exceed 400 mV, otherwise the cell may die. The conductivity of the extracellular medium is  $\sigma = 1 \ \Omega^{-1} m^{-1}$ , the inter-electrode distance is  $X_i = 250 \ \mu$ m, the cell is midway between the electrodes centered at distance  $X_m = 125 \ \mu$ m from either electrode, and the cell diameter is  $d = 20 \ \mu$ m. Determine the maximum amplitude of the current I you can safely inject.

*Hint*: Express the voltage  $V_A - V_B$  across the cell as a difference between two biopotentials generated by the same current dipole.

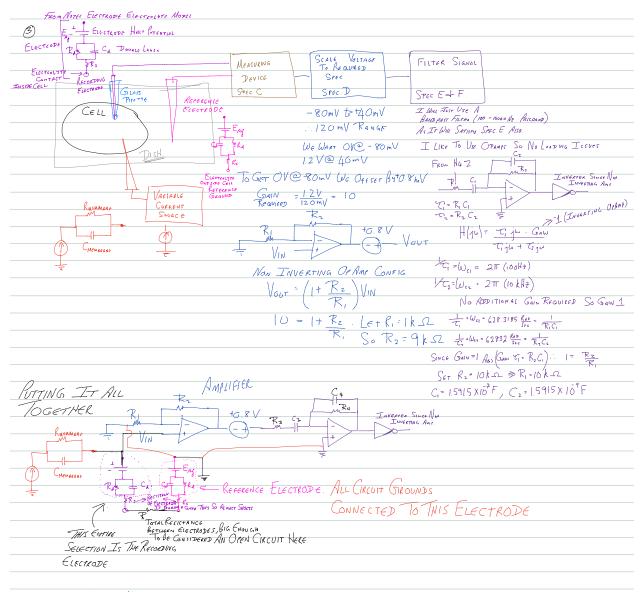


Problem2



- 3. **Intracellular electronic recording:** In electrophysiology there are several techniques used to measure various aspects of electric activity in single cells. One such technique is the *current clamp*, which injects a current into the cell and measures the resulting membrane voltage of the cell. Based on the design specifications below design a circuit which accomplishes a current clamp for intracellular voltage recording.
  - (a) You are given a variable current source that is set to the desired current clamp value.
  - (b) The reference electrode is located in the extracellular space, and the recording electrode reaches inside the cell (through a glass pipette penetrating the cell membrane).
  - (c) The signal you are trying to measure at the recording electrode is in the range of -80 mV to 40 mV.
  - (d) Amplify the signal so an external voltmeter measures 1.2 V at 40 mV and 0 V at -80 mV. The output voltage should vary linearly with input voltage. The input impedance of your amplifier should be near-infinite.
  - (e) 60 Hz line noise present on the electrode wires should be reduced in the amplifier output.
  - (f) The frequency range of interest in the voltage signal is 100 Hz to 10 kHz.

*Bonus*: Design a *voltage clamp* circuit to the same above specifications for the current clamp, except the signal you are measuring is now current into the recording electrode for a fixed voltage across the electrodes, and the current signal ranges between -250 nA to +250 nA, where the corresponding output voltage should range from -1 V to 1 V.



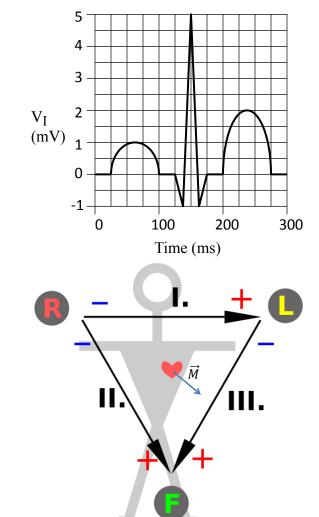
BONUS TO COME !!

#### **BENG 186B Winter 2015 HW #4**

Due Thursday February 19 at the beginning of class

### 1. Cardiac vectors:

(a) Given the following ECG of Lead I and knowing that the amplitude of the R-wave in the Lead II ECG is 10 mV, determine the magnitude and direction of the cardiac vector M, assuming it remains constant throughout the ECG. The length of each of the leads is 1 m.



(b) Neatly sketch the ECG for Lead II and Lead III. On each of the graphs, label the axes and show the values of ECG amplitude at the P, R, and T waves.

#### **Problem 1-Cardiac vector**

Magnitude and direction of the cardiac vector:

You can set up the following two equations, knowing the lead voltage of the peak of the R-wave for Leads I and II and knowing that they are 60 degrees apart. Note: a=1m

$$5mV = a * M * \cos(\theta)$$
$$10mV = a * M * \cos(60 - \theta)$$

2 equations, 2 unknowns

Using trig. Identities we can convert the second equation into:

$$10 = a * M * \cos(\theta) \cos(60) - \sin(\theta) \sin(60)$$

Setting both equations equal to M

$$M = \frac{5}{\cos(\theta)} \cos(\theta)$$
$$M = \frac{10}{\cos(\theta)\cos(60) - \sin(\theta)\sin(60)}$$
$$M = \frac{10}{\frac{1}{2} \cos(\theta) - \frac{\sqrt{3}}{2} \sin(\theta)}$$

Setting them equal to each other

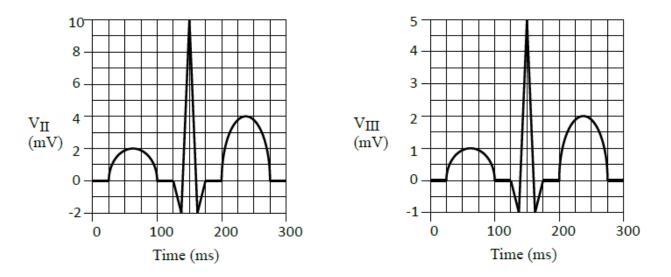
$$\frac{10}{\frac{1}{2} * \cos(\theta) - \frac{\sqrt{3}}{2} * \sin(\theta)} = \frac{5}{\cos(\theta)}$$
$$10 * \cos(\theta) = \frac{5}{2} * \cos(\theta) - \frac{5\sqrt{3}}{2} * \sin(\theta)$$
$$\frac{15}{2} * \cos(\theta) = \frac{5\sqrt{3}}{2} * \sin(\theta)$$
$$\frac{3}{\sqrt{3}} = \tan(\theta)$$
$$\theta = 60$$

From previous equation you get M=10mV/m

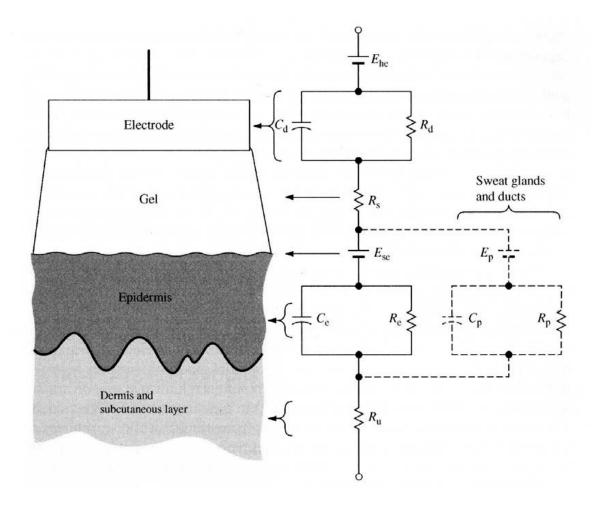
Therefore the cardiac vector has a magnitude of 10mV/m and is parallel to lead II.

Sketches:

Note that Lead III is the same as LeadII since it is also it is also 60 degrees from the cardiac vector.



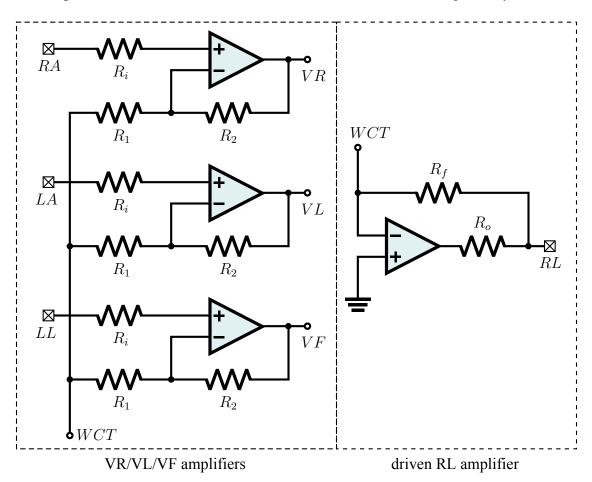
2. Electrode model: Consider the following skin–electrode model below:



- (a) Find the expression for the impedance of the circuit when the subject is not sweating, that is, disregarding the sweat glands and ducts contribution.
- (b) Find the expression for the impedance of the circuit when the subject is sweating, that is, including the sweat glands and ducts contribution.
- (c) Using the following parameter values sketch (or plot) the magnitude of the impedance as a function of frequency, from 0.1Hz to 100Hz on a log-log scale. Make sure to properly label your plot with values and units.

$E_{hc} = 200 \mathrm{mV}$	$R_s = 1 \mathrm{k}\Omega$	$C_d = 1  \mathrm{pF}$	$C_e = 10  \mathrm{pF}$	$C_p = 0$
$E_{sc} = 430 \mathrm{mV}$	$R_u = 100 \mathrm{k}\Omega$	$R_d = 1 \mathrm{M}\Omega$	$R_e = 10 \mathrm{M}\Omega$	$R_p = \infty$

2 @ Non-SWEAT GLAND Z  $\frac{Z_{first}}{2\pi} = \frac{C_a \| R_a + R_s + C_e \| R_e + R_u}{2\pi}$  $= \frac{R_a}{1 + j w C_a R_a} + \frac{R_s + R_e}{H_j w C_a R_a}$ B SWEATING CONDITION  $\frac{\mathcal{Z}_{\text{Torn}}}{\frac{\mathcal{Z}_{\text{Torn}}}{\text{Source}}} = \mathcal{R}_{\text{A}} \| \mathcal{L}_{\text{A}} + \mathcal{R}_{\text{S}} + \left[ \left( \mathcal{L}_{\text{A}} \| \mathcal{R}_{\text{A}} \right) \| \left( \mathcal{L}_{\text{A}} \| \mathcal{R}_{\text{A}} \right) \right] + \mathcal{R}_{\text{A}}$  $= \frac{P_{a}}{1+j\omega C_{a}R_{a}} + R_{c} + \frac{R_{c}R_{p}}{\overline{I_{c}+R_{p}}}$  $H_{j\omega}(C_{c}+C_{p})\frac{R_{c}R_{p}}{\overline{I_{c}+R_{p}}}$ C Tes: Zron - Rd + Rs + Re + Ra 200 - Itys Ca Rd + Rs + Re - + Ra + Ru  $= \frac{10^6 \Omega}{14 f^{\omega} (0^6 \pi \chi) (x 10^6 F)} + \frac{10^3 \pi}{14 f^{\omega} (0^6 \pi \chi) (x 10^6 F)} + \frac{10^3 \pi}{14 f^{\omega} (0^6 \pi) (10^6 F)}$ = 10° a + 10° a + 10° a + 10° a Hyu(10° a)(10° F) Hyu(10° Z Toral = ZToral Wer Das

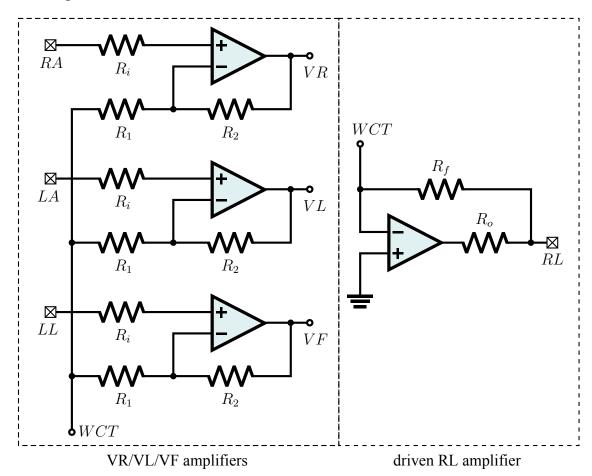


3. ECG amplifiers: Consider the four-electrode frontal lead ECG bioamplifier system below:

- (a) Derive the lead voltage outputs VR, VL, and VF in terms of the electrode voltages RA, LA, and LL, respectively. What are the voltage gains on each of these leads?
- (b) Derive the driven right leg voltage output RL in terms of the common mode of RA, LA, and LL. What is the common mode voltage gain driving the right leg?
- (c) Find the effective resistance from body to ground in terms of the RL electrode impedance  $R_{RL}$  and the resistances of the circuit.
- (d) What purpose do resistors  $R_i$  and  $R_o$  serve in this circuit?
- (e) **BONUS**: Using additional opamps and resistances, augment your circuit to produce six amplified leads I, II, III, aVR, aVL, and aVF from the VR, VL and VF outputs. Express the total voltage gain of the leads in terms of the resistances.

#### BENG 186B Winter 2015 HW #4 Solutions

3. ECG amplifiers: Consider the circuit below:



(a) All op-amps in the above circuit has negative feedback, so the voltage of the inverting and non-inverting inputs of each op-amp are equal. Because of this, the Wilson central terminal (WCT) must be at the same voltage as ground. This effectively makes the other three op-amps simply a non-inverting amplifier.

$$VR = \left(1 + \frac{R_2}{R_1}\right) RA$$

$$VL = \left(1 + \frac{R_2}{R_1}\right) LA$$

$$VF = \left(1 + \frac{R_2}{R_1}\right) LL$$
(1)

The voltage gain for each of the left three amplifiers is  $1 + R_2/R_1$ .

(b) We can apply KCL to the virtual ground junction in the DRL amplifier:

$$\frac{RA-0}{R_1} + \frac{LA-0}{R_1} + \frac{LL-0}{R_1} = \frac{0-RL}{R_f}$$
(2)

(Remember the ideal op-amp assumptions! The junctions between  $R_1$  and  $R_2$  in each of the amplifiers must be equal to their respective amplifier inputs.)

Rearranging:

$$RL = -\frac{R_f}{R_1} \left( RA + LA + LL \right) \tag{3}$$

If we note that the common-mode signal for RA, LA, and LL is their average:

$$V_{cm} = \frac{RA + LA + LL}{3} \tag{4}$$

We can further rewrite the RL transfer function:

$$RL = -\frac{3R_f}{R_1} \left(\frac{RA + LA + LL}{3}\right) \tag{5}$$

$$RL = -\frac{3R_f}{R_1} V_{cm} \tag{6}$$

The voltage gain here is  $-3R_f/R_1$ .

(c) Noise (such as 60 Hz power line noise) induces currents in the body,  $i_d$ . They flow through the impedance of the right leg electrode,  $R_{RL}$ . (They flow through RL since the other electrodes, RA, LA, and LL are connected to high–impedance inputs.) By Ohm's law, this gives rise to the common–mode signal:

$$i_d R_{RL} = V_{cm} - RL \tag{7}$$

However, recall the DRL amplifier transfer function:

$$RL = -\frac{3R_f}{R_1} V_{cm} \tag{8}$$

Combine these two:

$$i_d R_{RL} = V_{cm} + \frac{3R_f}{R_1} V_{cm}$$
 (9)

$$i_d R_{RL} = V_{cm} \left(\frac{3R_f}{R_1} + 1\right) \tag{10}$$

$$V_{cm} = i_d \underbrace{\left(\frac{R_{RL}}{3R_f/R_1 + 1}\right)}_{R_{cm}} \tag{11}$$

By comparison with Ohm's law, the effective resistance is therefore:

$$R_{\rm eff} = \frac{R_{RL}}{3R_f/R_1 + 1}$$
(12)

Suppose that we don't use a DRL and connected *RL* directly to ground. This results in:

$$i_d R_{RL} = V_{cm} - 0 \tag{13}$$

$$V_{cm} = i_d \underbrace{(R_{RL})}_{R_{\text{eff}}} \tag{14}$$

With a DRL, you can greatly reduce  $R_{RL}$  and minimize the generation of  $V_{cm}$  due to  $i_d$ .

- (d) R<sub>o</sub> serves as protection against an accidental short circuit at RL. It puts an upper limit on the current the op-amp can source or sink, determined by I<sub>max</sub> = (op-amp supply)/R<sub>o</sub>. Acceptable answers for R<sub>i</sub> can be "no purpose" (if we assume the op-amps are ideal), or to fix input bias currents that may be present (if we assume the op-amps are not ideal). Many modern op-amps have negligible input bias currents, so they may not be needed.
- (e) **BONUS**: The *VR*, *VL*, and *VF* are unipolar electrodes and are referenced with respect to *WCT*:

$$VR = RA - WCT$$
  $VL = LA - WCT$   $VF = LL - WCT$  (15)

Leads I, II, and III can be written in terms of the above equations:

$$I = LA - RA = (LA - WCT) - (RA - WCT) = VL - VR$$
(16)

$$II = LL - RA = (LL - WCT) - (RA - WCT) = VF - VR$$
<sup>(17)</sup>

$$III = LL - LA = (LL - WCT) - (LA - WCT) = VF - VL$$
(18)

The augmented leads can then be written in terms of VR, VL, and VF:

$$aVR = RA - \frac{1}{2}(LA + LL) \tag{19}$$

$$= (RA - WCT) - \frac{1}{2}[(LA - WCT) + (LL - WCT)]$$
(20)

$$= VR - \frac{1}{2}(VL + VF) \tag{21}$$

$$aVL = LA - \frac{1}{2}(RA + LL) \tag{22}$$

$$= (LA - WCT) - \frac{1}{2}[(RA - WCT) + (LL - WCT)]$$
(23)

$$= VL - \frac{1}{2}(VR + VF) \tag{24}$$

$$aVF = LL - \frac{1}{2}(RA + LA) \tag{25}$$

$$= (LL - WCT) - \frac{1}{2}[(RA - WCT) + (LA - WCT)]$$
(26)

$$= VL - \frac{1}{2}(VR + VL) \tag{27}$$

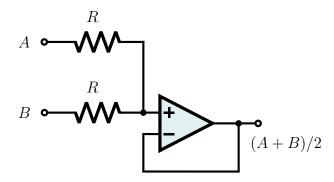
Since op-amps can perform arithmetic on voltage signals, we can translate these equations into physical circuit implementations. There are many ways to do this.

To compute leads I, II, and III, we can simply use differential amplifiers. For example, to compute lead I, we can connect VL into the (+) input and VR into the (-) input of a differential amplifier (with resistors set appropriately).

For the augmented leads, we need to calculate equations of the form:

$$C - \frac{1}{2}(A+B) \tag{28}$$

We can calculate (A + B)/2 with the following circuit:



(Convince yourself that this does in fact produce the above output, for any resistance value R!) This output can be fed into the (-) input and C can be fed into the (+) input of a differential amplifier to obtain the augmented leads.

Note that the bonus question asks to obtain the bipolar and the augmented leads from only VR, VL, and VF. Circuits that don't do this (*e.g.* introduce new leads and adding more instrumentation amplifiers) will not receive full credit (although they may receive partial bonus credit for correct implementations nonetheless).

- 4. **Design problem:** Your task is to design part of an internal (or implanted) automatic defibrillator (IAD) operating from a single 1.2 V battery. Your design has three parts.
  - (a) The IAD has three electrodes (ground and two lead I electrodes) available for the purposes of obtaining electrocardiograms. Design an amplifier so the IAD microcontroller can observe heart activity.
    - Assume that the electrodes include protection circuitry for overvoltage protection during IAD electric shock delivery.
    - This amplifier should have a gain of at least 1000.
    - The amplifier should filter out signals above 1 kHz.
    - The amplifier should use active grounding. The ground should not pass more than  $5 \,\mu A$  and should reduce the common mode signal by at least a factor of 1000.
  - (b) The IAD defibrillator is configured to perform cardioversion when needed by delivering a high-energy electrical shock to the heart. It is critical that such shock never be delivered during a T wave, or ventricular tachycardia may be induced which may be fatal. Design a circuit to block the IAD from delivering any shock for a duration of 400 ms after every R wave in the ECG, spanning the T wave with sufficient timing margin. The circuit should generate a digital control signal that goes high (1.2 V) for the duration of the block.
  - (c) The IAD includes an ultrasonic transmitter to notify an external monitor whenever a shock is being delivered. Design a circuit that generates a 500 kHz 1.2 V square wave driving an ultrasonic speaker whenever the IAD cardioverter is active. The cardioverter goes active when it is driven to 1.2V by the IAD microcontroller.

a) Need on instrumentation emplifier to obtain ECG.  
- Gain: 1000  
- Low pass filter: 
$$S_c = 1000 \text{ HZ}$$
  
- Active ground  $\rightarrow$  Driven Right Leg  
- Max correct = 5 mA  
- Connon-mode reduction = 1000  
- Jumping: 1.2 V  
Possible circuit:  
Va Rs Ry  
Lead I  
- RA O  
- Ra Ry  
Vef  
- Max controller  
- Stage  
- Differential  
- Stage  
- S

