BENG 186B Principles of Bioinstrumentation

Week 7 Review

Solutions

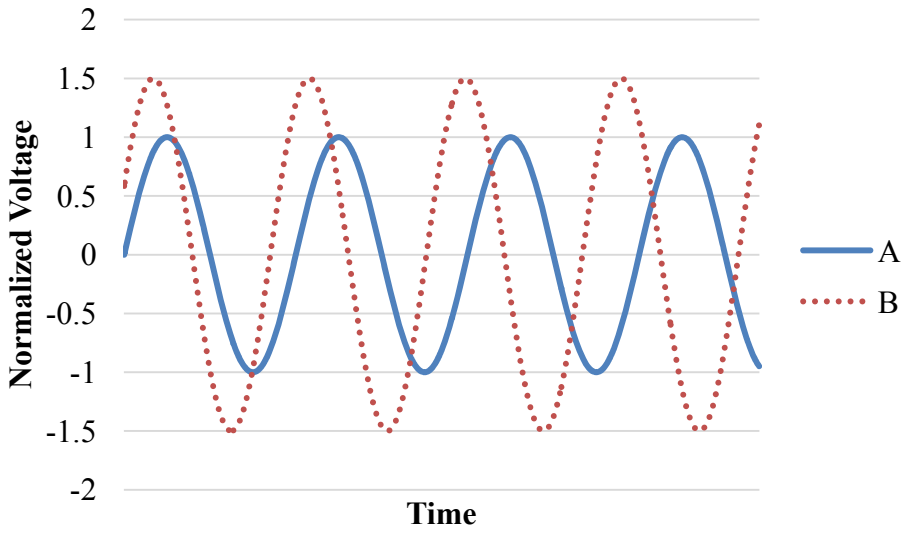
Selections from:

2015 Homework 5
2015 Homework 6

BENG 186B Winter 2015 HW #5

Due *Thursday February 26* at the beginning of class

1. One of the major problems during dialysis after bleeding out is the introduction of air bubbles as those can lead to embolism, which can result in cardiac arrest depending on where the embolism lodges. A safety check within the machine is to incorporate a bubble detector to warn clinicians on staff if bubbles have formed. As an engineer testing such a detection system, you receive the following signals from the system driven with a signal generator to mechanically stimulate test samples under two conditions: with and without bubbles. In each case the mechanical actuation by the signal generator couples to a membrane with compliance $C_d = 0.001$ Pa^{-1} in fluidic contact with the test sample.



   (a) Assuming that the amplitude of the signal generator voltage is kept the same in both tests with and without bubbles, and the signal generator is tuned to operate at the natural frequency in each case, determine which of the signals A and B correspond to the test with or without bubbles respectively. Explain why.

   (b) Determine the compliance of bubbles (C_b) with the parameters from the given plot. Assuming that C_b is directly proportional to the total volume of air inside the test system, explain how you can use the bubble detector signal to calculate the bubble density.

BENG 186B Winter 2015 HW #5 Solutions

1. **Bubble detector**:
   a. **Frequency rationale**:
   $f_n = \frac{\omega_n}{2\pi}$, where $\omega_n = \frac{1}{\sqrt{LC}}$, $C_{no\ bub} = C_0$, and $C_{bub} = C_0 + C_b$
   Hence, $C_{no\ bub} < C_{bub}$
   Therefore, $\omega_{no\ bub} > \omega_{bub}$
   From the plot, $\omega_{n_A} < \omega_{n_B}$
   Therefore $\omega_{n_A} = \omega_{bub}$ and $\omega_{n_B} = \omega_{no\ bub}$
   Amplitude rationale:
   $\left|\frac{V_{out}}{V_{source}}\right| = \frac{1}{2\xi}$, where $\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$, $C_{no\ bub} = C_0$, and $C_{bub} = C_0 + C_b$
   Hence, $C_{no\ bub} < C_{bub}$ and $\xi_{no\ bub} < \xi_{bub}$
   Therefore $\left|\frac{V_{out}}{V_{source}}\right|_{no\ bub} > \left|\frac{V_{out}}{V_{source}}\right|_{bub}$
   From the plot $\left|\frac{V_{out}}{V_{source}}\right|_A < \left|\frac{V_{out}}{V_{source}}\right|_B$
   Therefore $\left|\frac{V_{out}}{V_{source}}\right|_A = \left|\frac{V_{out}}{V_{source}}\right|_{bub}$ and $\left|\frac{V_{out}}{V_{source}}\right|_B = \left|\frac{V_{out}}{V_{source}}\right|_{no\ bub}$
   b. $\left|\frac{V_{out}}{V_{source}}\right| = \frac{1}{2\xi} = \frac{1}{R}\sqrt{\frac{L}{C}}$
   Bubbles: $\left|\frac{V_{out}}{V_{source}}\right| = \frac{1}{R}\sqrt{\frac{L}{C_0+C_b}}$
   No bubbles: $\left|\frac{1.5V_{out}}{V_{source}}\right| = \frac{1}{R}\sqrt{\frac{L}{C_0}}$
   Divide and cancel terms, $\frac{1}{1.5} = \sqrt{\frac{C_0}{C_0+C_b}}$, so $C_b = \frac{5}{4}C_0$
   Substitute $C_0 = 0.001\ Pa^{-1}$ so $C_b = 0.00125\ Pa^{-1}$
   Relationship with bubble density
   Divide and cancel terms, $\frac{V_{out_A}}{V_{out_B}} = \sqrt{\frac{C_0}{C_0+C_b}}$, so $C_b = C_0\left(\left(\frac{V_{out_B}}{V_{out_A}}\right)^2 - 1\right)$
   Assuming $C_b = k[volume_{air}]$ where k is constant and since C_0 is also constant, then $[volume_{air}]$ depends only on $\left(\frac{V_{out_B}}{V_{ot_A}}\right)^2$ and assuming that the volume of blood within the dialysis machine is fixed, the bubble density can be calculated.

2. For many biological problems, cell sorting is an important task of differentiating and categorizing different cell types within a heterogeneous mixture. Utilizing the Hall Effect phenomena, you design a system where you can separate cells of interest by using charged antibodies that attach specifically to the antigens of your cells. Since you have two cell types to separate, you design negatively and positively charged antibodies.

   Due to the separation of charges, the tube through which the cells flow acts like a parallel plate capacitor with a distance of 5 mm. The cells are flowing at 100 mL/s through the tube with a relative permittivity of 80. You induce the Hall Effect after generating a magnetic field of 0.25 T.

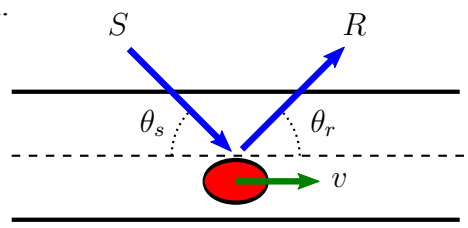
   (a) Draw the system with the corresponding vectors for the generated magnetic field, flow of cells, and electric field.

   (b) Assuming each cell to have a valance of either $+1$ or -1, calculate the concentration of cells of either type covering the walls of the tube.

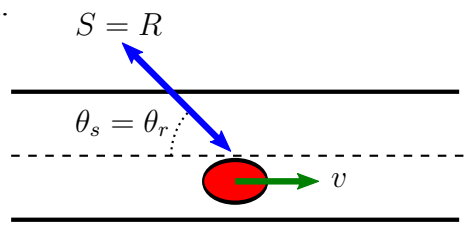
3. You have a device that can measure blood velocity using ultrasonic pulses. This requires the user to position the source and receiver correctly.

   (a) Given the following different setups, determine the expected frequency shifts when using an ultrasound sound system to measure the blood velocity.

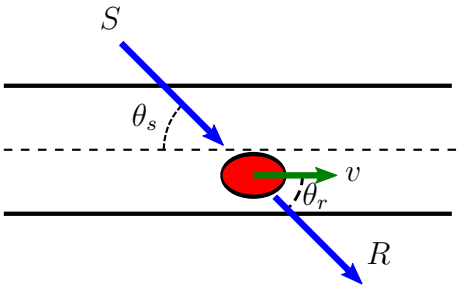
i.



ii.



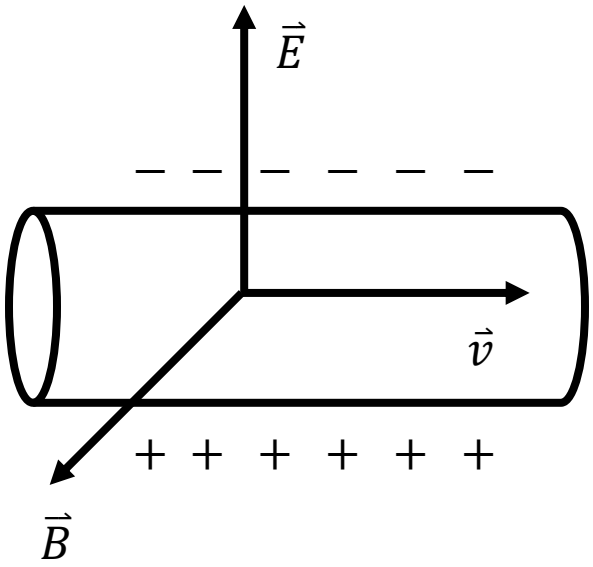
iii.



   (b) To calculate the Doppler shift, you need to assume the direction of the blood velocity (shown directed to the right in the above examples). If your assumption is wrong, how will that reflect in your calculated Doppler shift?

2. Cell sorter

a. Note that $\vec{v} \times \vec{B}$ and $\vec{E}$ is the result of the cross product, but the direction of $\vec{E}$ should be opposite of the potential whose polarity is determined by the direction of $\vec{B}$.



b. From the Hall effect, $V = \int_0^L \vec{v} \times \vec{B} \cdot d\vec{l}$.

This can be treated like a parallel plate capacitor, $C = \varepsilon_0 \varepsilon_r \frac{A}{d} = \frac{q}{V}$ where ε_0 is the electric constant ($\approx 8.854 \times 10^{-12}\ F/m$), ε_r is the relative permittivity, A is the surface area of the plates, d is the distance between the plates, q is charge, and V is voltage.

Rearranging leads to $\frac{q}{A} = \varepsilon_0 \varepsilon_r \frac{V}{d} = \varepsilon_0 \varepsilon_r \vec{v}\vec{B}$

Substitute, $\frac{q}{A} = 8.854 \times 10^{-12} \frac{F}{m} * 80 * \frac{100\frac{cm^3}{s}}{\pi(0.25cm)^2} * \frac{m}{100cm} * 0.25 \frac{V \cdot s}{m^2}$

Resulting in a charge density of $\frac{q}{A} = 9.019 \times 10^{-10} \frac{V \cdot F}{m^2} = 9.019 \times 10^{-10} \frac{C}{m^2}$

Each cell is either -1 or +1 charge (coulomb), which means cell density is very low at $9.019 \times 10^{-10} \frac{cells}{m^2}$

Also acceptable:

$1\ electron\ (-1) \approx -1.602 \times 10^{-19}\ C$ and $1\ proton\ (+1) \approx 1.602 \times 10^{-19}\ C$

Cell density is about $5.629 \times 10^{9} \frac{cells}{m^2}$ on both sides of the tube.

3. Doppler shifts

a. Different orientations

i. $\Delta f_1 = f_O - f_S = -f_S \frac{v}{c} cos\theta_S$

$\Delta f_2 = f_R - f_O = f_O \frac{v}{c} cos\theta_R$

$\Delta f = \Delta f_1 + \Delta f_2 = -f_s \frac{v}{c} cos\theta_S + \frac{v}{c} cos\theta_R \left(f_S - f_S \frac{v}{c} cos\theta_S\right)$

$\Delta f = f_s \frac{v}{c} \left(-cos\theta_S + cos\theta_R \left(1 - \frac{v}{c} cos\theta_S\right)\right)$

Also acceptable:

Since $\frac{v}{c} \ll 1$, then $\Delta f = f_s \frac{v}{c}(-cos\theta_S + cos\theta_R)$

ii. $\Delta f_1 = f_O - f_S = -f_S \frac{v}{c} cos\theta$

$\Delta f_2 = f_R - f_O = -f_O \frac{v}{c} cos\theta$

$\Delta f = \Delta f_1 + \Delta f_2 = -\frac{v}{c} cos\theta (f_S + f_O) = -\frac{v}{c} cos\theta \left(f_S + f_S - f_S \frac{v}{c} cos\theta\right)$

$\Delta f = -2 f_S \frac{v}{c} cos\theta + f_S \left(\frac{v}{c}\right)^2 \cos^2\theta$

iii. Same as (i)

b. The Doppler shift can be (+) or (-) depending on if the direction of the observer is moving towards or away from each other respectively. If the assumption of the direction is wrong, the resulting calculated Doppler shift will have an opposite sign of what is expected.

4. **Design problem**: Design an automated sphygmomanometer that includes a digital readout of systolic and diastolic pressure and heart rate.

   You have these components available:

   (a) Cuff with arbitrary digital control over applied pressure;

   (b) Sonic transducer contacting skin over artery inside the cuff, with digital readout of the acquired sound waveform;

   (c) Alphanumeric display;

   (d) Programmable microcontroller;

   (e) Start button and sleep button;

   (f) Power supply with power on/off switch.

   Sketch a diagram of the system, and outline the algorithm running on the microcontroller to control the cuff, acquire the sound signal, and compute and display the systolic pressure, diastolic pressure, and heart rate.

   You do not need to write actual code, but be quantitative in your description with sufficient information for a programmer to be able to complete the design. You may use timing diagrams and equations to define the sequence of control variables, the corresponding measured quantities, and the computation of the output variables.

   Define important parameters in the algorithm, and give numerical values based on the physiological range that you expect.

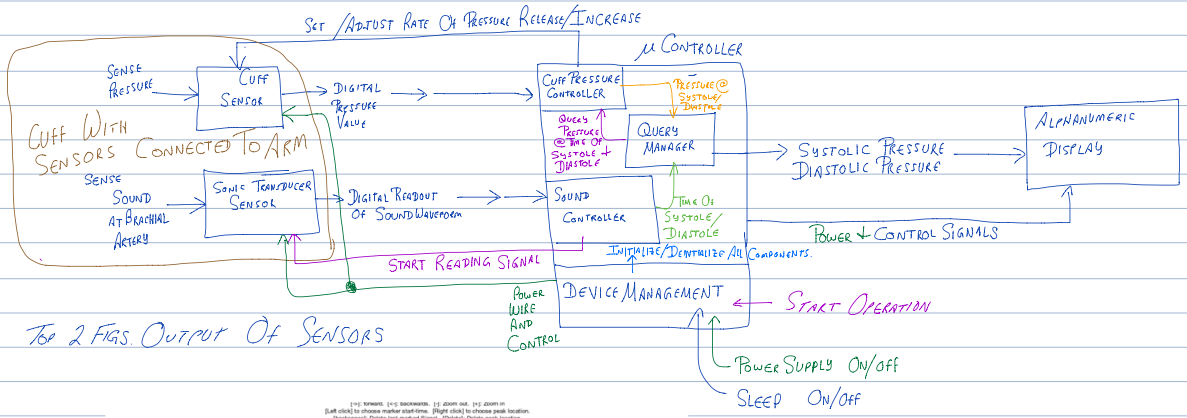
GRADING COMPONENTS

(4) (i) SKETCH -5 POINTS

(ii) OUTLINE OF ALGORITHM → CONTROL CUFF -5 POINTS; ACQUIRE SOUND SIGNAL -5 POINTS; COMPUTE & DISPLAY PRESSURE (SYSTOLIC & DIASTOLIC) -10 POINTS; HEART RATE -5 POINTS

(iii) IMPORTANT PARAMETERS -5 POINTS

(iv) NUMERICAL VALUES -5 POINTS



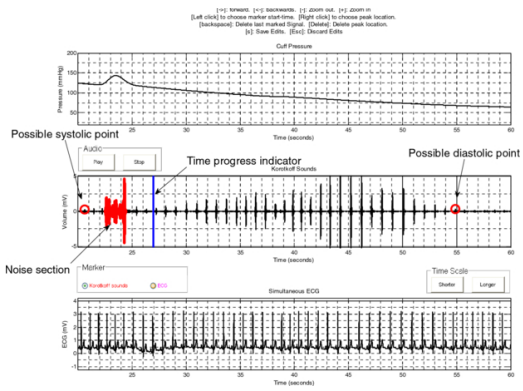


Figure from: J Abdul Sukor et al 2012 Physiol. Meas. 33 465.

ALGORITHM FOR OPERATION

→ WHILE THE SYSTEM IS ON AND START OPERATION IS ENABLED.

→→ USING THE μCONTROLLER SET A PRESSURE SENSOR SAMPLING RATE (MAX HEART RATE 210 BEATS/MIN ⇒ 4.20 SAMPLE@ / BEATS/MIN = 7 BEATS/SEC = 7 Hz)

→→ SET THE SAMPLING RATE OF THE SOUND SENSOR TO (MAX SOUND FREQ = 600Hz ⇒ SAMPLE @ 12kHz. FROM A FREQ ANALYSIS OF KOROTKOFF SOUNDS BY PAUL HENKEREN, MIT M.S THESIS.

→→ START INFLATING THE CUFF PAST THE PHYSIOLOGICAL PRESSURE LIMIT (TO APPROX 200 mmHg). ONCE AT LIMIT THE μCONTROLLER RELEASES PRESSURE SLOWLY

→→ THE μCONTROLLER DEFLATES THE CUFF WHILE SENSOR REPORTS PRESSURE & TIME UNTIL A LOWER PHYSIOLOGICAL THRESHOLD IS REACHED (APPROX 20 mmHg. IT MAY BE OK TO FULLY DEFLATE)

→→→ THE SOUND TRANSDUCER TRANSMITS VOLTAGE READING, AS SHOWN IN THE FIG. ABOVE, TO THE μCONTROLLER

→→ ONCE THE LOWER PRESSURE THRESHOLD IS REACHED TURN OFF THE SENSORS.

→→ THE SOUND CONTROLLER DETERMINES THE TIME OF SYSTOLE & DIASTOLE & SENDS THOSE TIMES TO THE QUERY MANAGER

↳ CONTROLLER ALGORITHM DETERMINES TIME OF SYSTOLE FROM VOLTAGE CORRESPONDING TO 1ST KOROTKHOFF SOUND, AND TIME OF DIASTOLE FROM 5TH KOROTKHOFF SOUND.

→→ THE QUERY MANAGER GETS THE PRESSURES FROM THE CUFF PRESSURE CONTROLLER AT THE TIMES ASCERTAINED FROM THE SOUND CONTROLLER

→→ ONCE THE DATA IS RECIEVED BY THE QUERY MANAGER FROM THE CUFF PRESSURE CONTROLLER, THE DATA IS SENT TO THE DISPLAY.

→→ AFTER THE INFO IS DISPLAYED FOR SOME TIME THE μCONTROLLER PUTS ALL COMPONENTS TO SLEEP & START IS DISABLED, THE μCONTROLLER THEN WAITS FOR ANOTHER START ENABLE

BENG 186B Winter 2015 HW #6

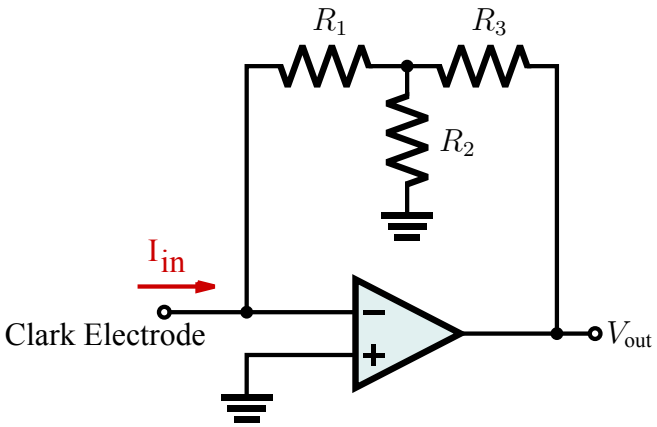
Due *Thursday March 12 at the beginning of class*

1. The lab technicians are running a routine checkup on a patient's blood sample using some electrodes connected to an aterial gas meter. The meter measures the electrodes and calculates pH, PCO_2, and $[O_2]$. Unfortunately, the meters are broken. Fortunately, you come in to save the day as the biomedical engineer with a multimeter to measure the electrodes directly.

   (a) You measure a voltage of -50 mV across the pH probe. The probe uses a saturated HCl concentration of 100 mM. What is the blood sample's pH?

   (b) You calibrate the PCO_2 electrode with a standard (pH = 7.4, PCO_2 = 40 mmHg). Then, you put a sample of the blood from part (a) in the PCO_2 electrode with 25 mM $NaHCO_3$. What is the sample's PCO_2?

   (c) For measuring PO_2, you construct a transimpedance amplifier to convert the current output from the Clark electrode to a voltage:



   You set $R_1 = 1\,\text{M}\Omega$, $R_2 = 50\,\text{k}\Omega$, and $R_3 = 5\,\text{M}\Omega$. Then, you flow the blood sample through a Clark electrode at 50 mL/s which has 0.7 V applied across it. You measure a voltage of 5.5 V at V_{out}. What is the sample's $[O_2]$?

1) From lecture notes (#15 pg. 3)

$$V = 62\,mV \cdot (pH_{HCl} - pH_{sample})$$

↑ 6 for 0.1 $\frac{mol}{L}$ HCl in glass bulb.

$$-50\,mV = 62\,mV\,(6 - pH)$$

$$\boxed{pH = 6.8}$$

2) From lecture notes (#15 pg. 5)

$$\log PCO_2 = -pH + K$$

$$\log\left(\frac{40\ mmHg}{1\ mmHg}\right) = -7.4 + K \Rightarrow K = 9.002$$

$$\log\left(\frac{PCO_2}{1\ mmHg}\right) = -6.8 + 9.002 \Rightarrow \boxed{PCO_2 = 160\ mmHg}$$

Okay if different number used — application of equations is more important.

3) Effective feedback resistance:

$$R_{eff} \approx \frac{R_1 R_3}{R_2} \quad \text{if} \quad R_1, R_3 \gg R_2 \quad \left(\begin{matrix} R_1 = 1M\Omega \\ R_3 = 5\,M\Omega \end{matrix} \gg R_2 = 50k\Omega\right)$$

(Lecture notes #15 pg. 8) $R_{eff} = 100\,M\Omega$

Transimpedance amp:

$V = -IR$

$5.5\,V = -I(100\,M\Omega)$

$I = -55\,nA$

(polarity irrelevant to question)

$$55\,nA = 4 \cdot F \cdot [O_2] \cdot (50\,mL/s)$$

(lecture notes #15 pg. 7)

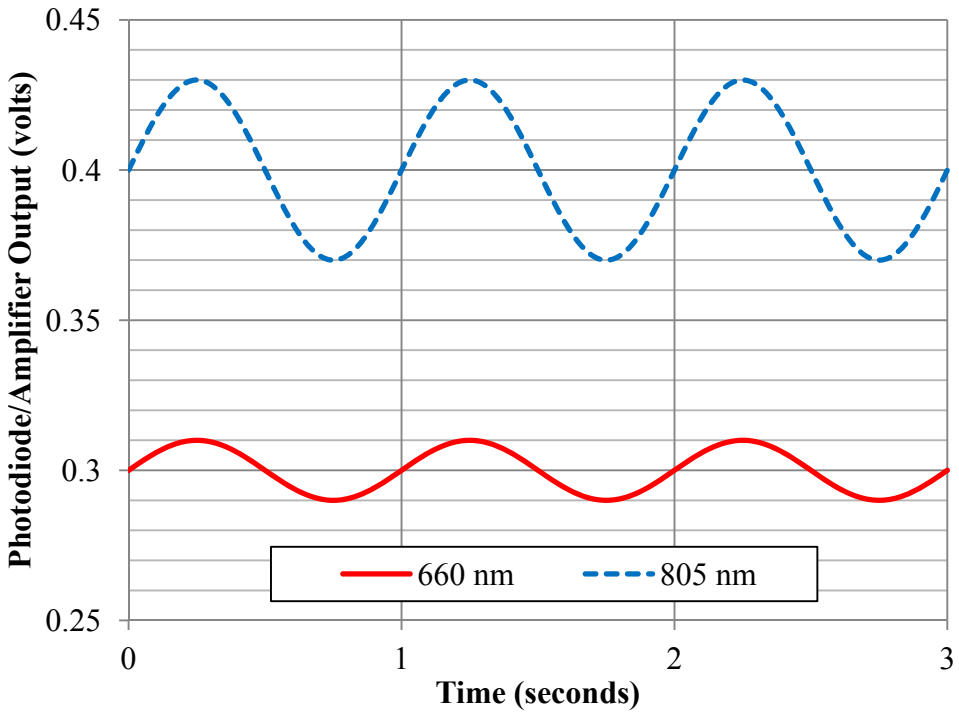
$$\boxed{[O_2] = 2.8\,pM}$$

2. Pulse oximeters work by passing red light (660 nm) and infrared light (805 nm) through a body part (such as a finger). The absorbance is measured using a photodiode and an amplifier. The SpO_2 can be calculated from the ratio of the absorbances.

   (a) Why do pulse oximeters use two different wavelengths to measure SpO_2? Be sure to mention the isobestic wavelength's role in computing SpO_2.

   (b) What is the origin of the DC component of the photodiode signal? What is the origin of the AC component?

   (c) Shown below is a sample of the raw pulse oximeter data:



   The SpO_2 can be estimated by:

$$SpO_2 = \left(110 - 25 \times \frac{\text{Abs}_{660\,\text{nm}}}{\text{Abs}_{805\,\text{nm}}}\right)\%$$

   What is the patient's SpO_2, according to the raw pulse oximeter data? *Hint:* Normalize the AC components against their corresponding DC components first. The AC component is the peak–to–peak voltage and the DC component is the average voltage.

   (d) What is the patient's heart rate, according to the raw pulse oximeter data?

2)

a) Pulse oximetry is determined via Beer's Law:

$$A(\lambda_i) = W \cdot L \cdot \alpha_i$$

The 805 nm light is the isobestic wavelength, meaning it has the same α_i as when 660 nm is used. This allows us to determine absorbance independent of W and L.

b) DC → skin, veins, muscle, fat, steady blood flow
Ac → arteries, pulsating blood flow

c) Normalize:

660 nm	805 nm
AC: 0.2 V	0.6 V
DC: 0.3 V	0.4 V
⇓	⇓
2/3	3/2

$$SpO_2 = 110 - 25 \times \frac{2/3}{3/2}$$

$$\boxed{SpO_2 = 98.9\ \%}$$

d) Heart rate

$$= \frac{1 \text{ pulse}}{\text{sec}} = \boxed{60 \text{ bpm}}$$

by inspection of pulse oximeter graph.

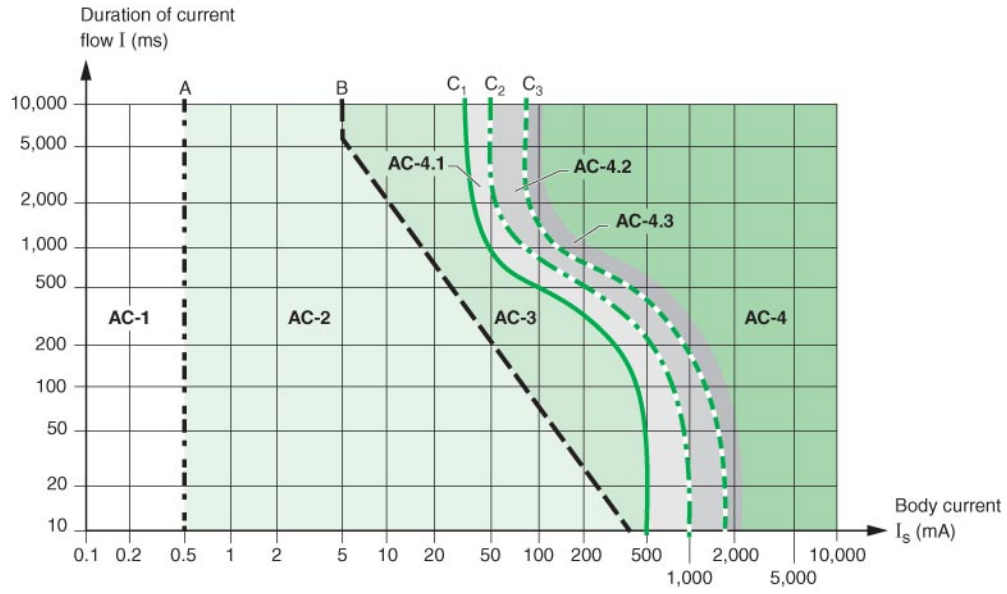
3. Cardiomyocytes have a typical membrane time constant of 2 ms and starts to fibrillate at above 100 mA.

   (a) Suppose an unfortunate person gets struck with 200 kV lightning. The current enters and exits the body through skin ($R_{\text{skin}} = 100\,\text{k}\Omega$). How much current will flow through the victim? Assume the internal body resistance is negligible compared with the skin resistance.

   (b) If 20% of the lightning strike's current mentioned in part (a) passes through the heart, how long can the victim withstand the strike before suffering cardiac arrest?

   (c) Lightning strikes aren't the only electrical hazard to worry about. Ground faults can make medical equipment dangerous to use without any outward signs of a problem.

   You are designing a 12–lead ECG powered by a 120 VAC supply. Because you are a competent engineer, you add resistors in series with each ECG electrode lead wire as part of the device's overall safety design. Using the graph below, specify what resistor value you should use so that any ground fault currents are imperceptible.



AC-1 imperceptible

AC-2 perceptible but no muscle reaction

AC-3 muscle contraction with reversible effects

AC-4 possible irreversible effects

- **AC-4.1** up to 5% probability of heart fibrillation
- **AC-4.2** 5-50% probability of fibrillation
- **AC-4.3** over 50% probability of fibrillation

Graph source: International Electrotechnical Commission (IEC) standard 60479

3)

a) $V = IR$

skin entry resistance, skin exit resistance

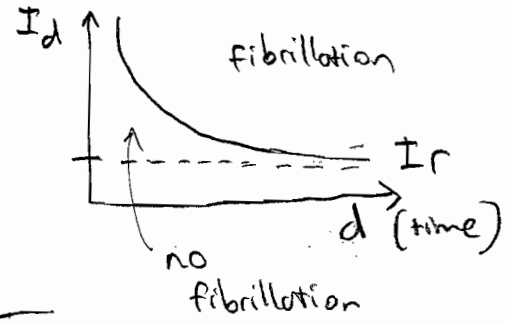
$200\ kV = I\,(100\ k\Omega + 100\ k\Omega)$

$\boxed{I = 1\ A}$

b) 20% of 1 A = 200 mA

Use rheobase equation to determine threshold:

$$I_d = I_r \frac{1}{1 - e^{-d/z}}$$



$z = 2\ ms$ (given)

$$200\ mA = 100\ mA \frac{1}{1 - e^{-d/(2ms)}}$$

fibrillation current for any duration d

$\Rightarrow \boxed{d = 1.386\ ms}$

c) For imperceptible ground fault currents, look at region AC-1 on graph.

Max current for AC-1: 0.5 mA

Max possible Voltage: 120 VAC

$V = IR$

$120\ V = (0.5\ mA)\,R \Rightarrow \boxed{R = 240\ k\Omega}$

4. **Design problem:** A group of athletes would like to know their metabolic rates during exercise. A non–invasive way to estimate this is to measure the CO_2 and O_2 concentration in their breath as they train.

   Your task is to design a circuit to interface a Severinghaus electrode (measures PCO_2) and a Clark electrode (measures PO_2) to a microcontroller. This microcontroller is already loaded with software that computes the metabolic rate based on PCO_2 and PO_2. You have:

   - The Severinghaus electrode. Its usable output range is $-250\,\text{mV}$ to $250\,\text{mV}$.
   - The Clark electrode. Its usable output current is $0\,\mu\text{A}$ to $100\,\mu\text{A}$. This electrode needs $0.7\,\text{V}$ applied to it.
   - Op–amps, and any resistors/capacitors as needed.
   - A stable $\pm 5\,\text{V}$ power supply.
   - A microcontroller with a built–in ADC. The ADC can only measure positive voltage.

   The circuit needs to filter out signals above $10\,\text{Hz}$ from both sensors. The ADC should read $0\,\text{V}$ to $5\,\text{V}$ linearly with respect to each electrode's usable output. You must include numerical values in your design. **BONUS**: Design your circuit in such a way that a $-5\,\text{V}$ supply is not required.

4) Design problem: possible solution: