BEENG 221 Mathematical Methods in Bioengineering

Fall 2014

Midterm

NAME: ____

- Open book, open notes.
- 80 minutes limit (end of class).
- No communication other than with instructor and TAs.
- No computers or internet, except for access to posted class materials.

Problem 1 (30 points): Short answer problems. Provide brief explanations (no lengthy derivations!) for each problem.

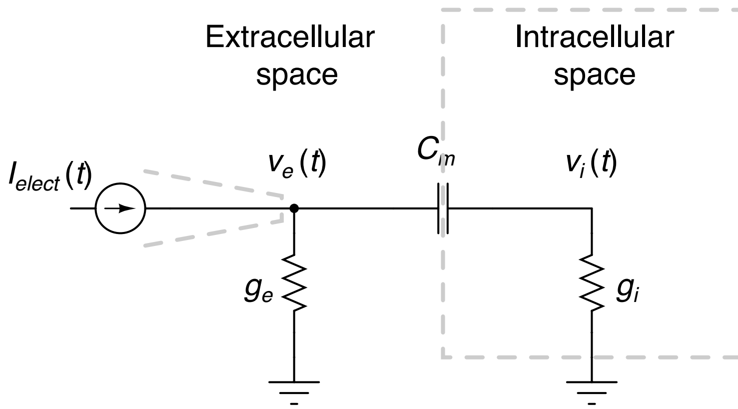
1. (5 points): A random walk process with random step size of standard deviation Δx and step time interval Δt is characterized with a diffusivity D. How does the diffusivity D change as the random step size increases by a factor two (*i.e.,* Δx becomes $2\Delta x$)?

2. (5 points): Conversely, how does the diffusivity D change as the step time interval increases by a factor two (*i.e.,* Δt becomes $2\Delta t$)?

3. (10 points): Find the radial frequency ω of wave oscillations of a free electron with energy E.

4. (10 points): Find the maximum time step Δt beyond which Euler numerical integration of the ODE $dx/dt = -x/\tau$ becomes unstable (*i.e.,* gives unbounded results).

Problem 2 (25 points): Consider an electrically excitable cell as shown below. The cell has membrane capacitance C_m and leak conductance g_i. The extracellular space has leak conductance g_e. Both intracellular and extracellular potentials are initially zero. At time zero, a constant current electrode current $I_{elect}(t) = I_0$ is injected into the extracellular space.



1. (5 points): Write the differential equations and initial conditions governing the dynamics of the intracellular and extracellular potentials, $v_i(t)$ and $v_e(t)$.

2. (15 points): Find the intracellular potential $v_i(t)$ over time. You may use Laplace or any method of your choice.

3. (5 points): Find the minimum level of electrode current I_0 to be injected in order for the intracellular potential $v_i(t)$ to reach the threshold V_{th} at which the cell generates an action potential. At what time does the action potential happen? Does the duration of the electrode current matter, and how?

Problem 3 (45 points): An athlete initially at rest starts to exercise. The body is covered with thermally insulating material. Underneath the skin ($x = 0$) is muscle tissue of thickness L, interfacing on the other end ($x = L$) with vasculature. The thermal conductivity of the muscle tissue is K_0, and the vasculature conducts heat to maintain the tissue interface at a constant temperature T_0. Specific heat of the muscle tissue is c, and mass density is ρ. Once starting to exercise ($t \geq 0$), the athlete burns calories (Joules) uniformly in the muscle tissue at constant rate, with heat generation $Q(x, t) = Q_0$.

1. (5 points): Write the partial differential equation governing temperature $u(x, t)$ in the muscle tissue. Express initial and boundary conditions.

2. (10 points): Solve for a particular solution $u_p(x)$ for the temperature in the tissue at steady state.

3. (5 points): Write the homogeneous problem, with homogeneous partial differential equation and boundary conditions. You may combine constants into a thermal diffusivity D.

4. (10 points): Find the eigenmode decomposition for the general solution of the homogenous problem.

5. (15 points): Solve for the temperature in the tissue over time $u(x,t)$ from initial conditions.