

ALCOHOL DIFFUSION IN THE STOMACH LINING

Introduction

When ethyl alcohol is consumed, the liquid first enters the stomach. Alcohol in the stomach can be absorbed through the stomach lining into the bloodstream, or it can continue on to the small intestine, where it can again be absorbed into the bloodstream. Intoxication occurs after absorption into the bloodstream, due to the presence of alcohol in the blood.

Although alcohol can really be absorbed into the bloodstream at any point along the gastrointestinal digestive tract from the mouth during ingestion to the small intestine, the majority of the alcohol enters the bloodstream via either the stomach lining or the small intestine walls (Chemcases, 2003). Approximately 20% of ethyl alcohol consumed is absorbed in the stomach and 80% is absorbed in the small intestine (Chemcases, 2003), while a negligible amount is absorbed in the remainder of the digestive tract. The difference between absorption in the stomach and the intestine is due to the difference in surface areas that limits diffusion rates (.045 m² is the surface area of the stomach, 600 m² is the surface area of the small intestine). Because more alcohol is absorbed in the small intestine than in the stomach, most alcohol absorption studies have traditionally studied, simulated, or experimentally measured the changing rate of alcohol concentration in the blood or the breath and compared that to the rate of gastric emptying into the small intestine (Watkins and Adler, 1993). The rate of alcohol absorption overall is dependent on the rate of gastric emptying since alcohol absorption occurs primarily from the small bowel (Holt et al., 1980). Thus, most studies testing breathalyzers typically use this relationship to determine the accuracy of the breathalyzer, a device that measures breath alcohol content (BrAC), which is in turn a fraction of blood alcohol content (BAC), which provides a quantification of intoxication.

Because the typical profile of alcohol in the blood following a short period of drinking then stopping resembles a bell curve, certain driving under the influence (DUI) cases may be fought against by criminal defense attorneys who can prove that although BAC levels were above the legal limit at time of breathalyzer testing by the police officer, BAC levels were not about the legal limit at the time that the driver was actually driving the vehicle (CalDUI, 2010). It is difficult to pinpoint the exact BAC back in time, and to determine exactly where the person was on the bell curve at the time of driving. However, this process would involve considering exactly what drink was consumed, exactly what time drinking occurred, and quantifying alcohol absorption into the bloodstream through both the stomach and the small intestine.

In this problem, we intend to model the absorption of alcohol through the stomach lining in order to determine how much of blood alcohol content is due to absorption in the stomach alone. This approach further allows us to model the changing concentration of alcohol absorbed in stomach as a person's drinking behavior changes over time.

Problem Statement

Students A, B, and C decide to go out for a night of drinking in Pacific Beach after they turn in their last math assignment of the quarter. Student A goes to the bars and downs drink after drink until his stomach is full. He then realizes he needs to drive, and stops drinking immediately. Student B decides to go all out, and can be found constantly slowly drinking beer from a CamelBak on his back all night. Student C's drinking pattern is consistent throughout the night. He drinks beer, then drinks water, drinks beer, then drinks water, drinks beer, and so forth, throughout the night.

Model the concentration of alcohol at the outer edge of the stomach over time for each of the students. Determine the concentration profile over time at the center of the stomach, at the inner lining of the stomach, and the outer lining of the stomach. How long must Student A wait until he can safely drive the others home? You may define your own criteria for being able to safely drive.

Problem Set Up

Diffusion in the stomach walls can be most simply modeled by using Fick's 2nd Law of Diffusion in one dimension in Cartesian coordinates where C is concentration, t is time, x is position, D is diffusivity, and Q denotes a driving force (Equation 1).

$$(1) \quad \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}$$

However, because the stomach can be assumed to be roughly spherical, Fick's 2nd Law of Diffusion in three dimensions in spherical coordinates would be more accurate (Equation 2).

$$(2) \quad \frac{\partial C}{\partial t} = D \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial C}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial C}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 C}{\partial \phi^2} \right]$$

In order to simplify the problem, we will assume constant mixing in the stomach in order to approximate this as a one dimensional problem (Equation 3).

$$(3) \quad \frac{\partial C}{\partial t} = D \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial C}{\partial r} \right) \right] + Q$$

For simplicity, we assume that the stomach is shaped as a sphere and can be modeled as a sphere. The stomach normally holds a volume of about 1 liter (Hypertextbook). A sphere with a radius of about 6 cm would have this volume. The stomach wall has a thickness of 0.51 ± 0.11 cm (Rapaccini, et al. 1988). Therefore, we approximated the stomach as a sphere with an outer radius of 6.5 cm and an inner radius of 6.0 cm to account for a 0.5 cm thick wall.

Another difficulty in setting up the problem is keeping the concentration the same throughout the stomach. Note here that we are only considering diffusion through the stomach wall itself. However, there is volume/space inside the stomach itself! In order to accomplish this, we assume that the diffusivity in the volume of the stomach, or in the stomach chamber from the center of the stomach to the inner edge of the wall is extremely high: 10^{15} cm²/s. This forces the inside of the stomach to be well mixed and does not allow the concentration to vary with position inside the stomach volume. Finally, we can approximate the diffusivity of alcohol in the stomach wall to be 10^{-4} cm²/s.

Alcohol diffusion through the stomach wall of Student A is modeled in both Cartesian coordinates and spherical coordinates for comparison. Alcohol diffusion through the stomach walls of Student B and Student C are modeled in spherical coordinates.

Boundary Conditions

The stomach walls of students A, B, and C all have the same boundary conditions (Equation 4):

$$(4) \quad \begin{aligned} \frac{dC}{dr} \Big|_{r=0} &= 0 \\ C(r=0) &= 0 \end{aligned}$$

The first boundary condition makes it so the concentration profile is continuous and differentiable at the center of the sphere. The second boundary condition forces the concentration to go to 0 as the alcohol leaves the stomach and enters the bloodstream. Here it is assumed that the volume of the bloodstream is much larger than the stomach. This is not a completely unreasonable assumption given that the stomach volume is approximately 1 L and total blood volume is approximately 5 L.

Initial Conditions and Driving Forces

Student A

Student A drank until his stomach was full of beer at time = 0. The initial conditions for this problem set the concentration to 0.05 alcohol by volume (ABV) inside the stomach volume ($0 < r < 6$ cm) and to 0 in the stomach wall ($6 \text{ cm} < r < 6.5$ cm). There is no driving force.

Student B

Student B started off with no beer in his stomach. The initial conditions for this problem set the concentration to 0 everywhere. The person started drinking at a slow, constant rate. The driving force is modeled as a constant function applied to inside of the stomach volume only ($0 < r < 6$ cm). Once the stomach is full of beer, the concentration of alcohol in the stomach cannot further surpass the concentration of alcohol in beer.

Student C

Student C also drank until his stomach was full of beer at time = 0. The initial conditions for this problem set the concentration to 0.05 ABV inside the stomach volume ($0 < r < 6$ cm) and to 0 in the stomach wall ($6 \text{ cm} < r < 6.5$ cm). The person had a drink of beer, then a drink of water, then a drink of beer, and a drink of water, and so forth. The driving force can be modeled by a sine squared function applied to inside of the stomach volume only ($0 < r < 6$ cm). Each time he drinks beer, he is on the increasing part of the function. Each time he drinks water, he is on the decreasing part of the function.

Analytical Solution (Separation of Variables)

$$\frac{\partial C}{\partial t} = D \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial C}{\partial r}$$

$$C(r,t) = P(r)T(t)$$

$$P \frac{\partial T}{\partial t} = DT \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial P}{\partial r}$$

$$\frac{1}{DT} \frac{\partial T}{\partial t} = \frac{1}{Pr^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial P}{\partial r} \right)$$

$$\frac{1}{DT} \frac{\partial T}{\partial t} = \frac{1}{Pr^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial P}{\partial r} \right) = -\lambda^2$$

Both sides of the above equation are independent. Thus for them to be equal, they must both be equal to a constant. Now solve each side separately:

$$\frac{\partial T}{\partial t} = -\lambda^2 DT \Rightarrow T(t) = Ae^{-\lambda^2 Dt}$$

$$\frac{\partial}{\partial r} \left(r^2 \frac{\partial P}{\partial r} \right) = -\lambda^2 r^2 P$$

Substitution with x:

$$x = \lambda r \Rightarrow dx = \lambda dr$$

$$\frac{\lambda \partial}{\partial x} \left(\frac{x^2}{\lambda^2} \lambda \frac{\partial P}{\partial r} \right) = -x^2 P$$

$$x^2 \ddot{P} + 2x \dot{P} + x^2 P = 0$$

Let k = 0:

$$x^2 \ddot{P} + 2x \dot{P} + [x^2 + k(k+1)]P = 0$$

The solutions are Bessel functions, but k = 0. Thus,

$$P(x) = B_1 \frac{\sin x}{x} + B_2 \frac{\cos x}{x}$$

$$P(r) = B_1 \frac{\sin \lambda r}{\lambda r} + B_2 \frac{\cos \lambda r}{\lambda r}$$

$$C(r,t) = \left(C_1 \frac{\sin \lambda r}{\lambda r} + C_2 \frac{\cos \lambda r}{\lambda r} \right) \cdot e^{-\lambda^2 Dt}$$

Apply Boundary Conditions:

$$\frac{\cos \lambda r}{\lambda r} (r=0) \text{ is discontinuous } \therefore C_2 = 0$$

$$C(R,t) = 0 = C_1 \frac{\sin \lambda R}{\lambda R} e^{-\lambda^2 Dt}$$

$$\lambda R = n\pi \Rightarrow \lambda = \frac{n\pi}{R} \text{ for } n = 1, 2, 3, \dots$$

$$C(r,0) = \begin{cases} C_0, & r < R - w \\ 0, & r > R - w \end{cases}$$

For $r < R - w$:

$$C_0 = \sum_{n=1}^{\infty} C_n \frac{\sin \frac{n\pi r}{R}}{\frac{n\pi r}{R}} e^0$$

This is not a simple problem to solve analytically! The solution can be better estimated with a numeral approximation approach as shown in the following section.

Numerical Solution (MATLAB pdepe)

The MATLAB built in pdepe solver was used to numerically approximate the solutions modeling Students A, B, and C. Code and plots are attached for each of the three cases. Student A was modeled in both spherical and Cartesian coordinates in order to show the effect that the coordinate systems have on the concentration profiles. Students B and C were modeled only in spherical coordinates.

MATLAB Boundary Conditions and Initial Conditions

Driving forces and initial conditions were implemented in MATLAB using if/else statements. Homogeneous boundary conditions were implemented in MATLAB using typical pdepe methods as shown in the code below.

Student A

Diffusion Equation:	Initial Conditions:	Boundary Conditions:
$\frac{\partial C}{\partial t} = D \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial C}{\partial r} \right) \right]$	$C(r < 6 \text{ cm}, t = 0) = 0.05$ $C(r > 6 \text{ cm}, t = 0) = 0$	$\frac{\partial C}{\partial r}(r = 0, t) = 0$ $C(r = 6.5 \text{ cm}, t) = 0$

Diffusion Equation:	Initial Conditions:	Boundary Conditions:
$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}$	$C(r < 6 \text{ cm}, t = 0) = 0.05$ $C(r > 6 \text{ cm}, t = 0) = 0$	$\frac{\partial C}{\partial r}(r = 0, t) = 0$ $C(r = 6.5 \text{ cm}, t) = 0$

Student B

Diffusion Equation:	Initial Conditions:	Boundary Conditions:
$\frac{\partial C}{\partial t} = D \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial C}{\partial r} \right) \right] + Q$ $Q(r < 6 \text{ cm}) = 0.0187$ $Q(r > 6 \text{ cm}) = 0$	$C(r < 6 \text{ cm}, t = 0) = 0$ $C(r > 6 \text{ cm}, t = 0) = 0$	$\frac{\partial C}{\partial r}(r = 0, t) = 0$ $C(r = 6.5 \text{ cm}, t) = 0$

Student C

Diffusion Equation:	Initial Conditions:	Boundary Conditions:
$\frac{\partial C}{\partial t} = D \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial C}{\partial r} \right) \right] + Q$ $Q(r < 6 \text{ cm}) = 0.01 \sin^2(\pi t)$ $Q(r > 6 \text{ cm}) = 0$	$C(r < 6 \text{ cm}, t = 0) = 0.05$ $C(r > 6 \text{ cm}, t = 0) = 0$	$\frac{\partial C}{\partial r}(r = 0, t) = 0$ $C(r = 6.5 \text{ cm}, t) = 0$

Student A (Cartesian) MATLAB pdepe Code

```
% STUDENT A CARTESIAN

function pdexfunc
m = 0;
x = linspace(0,6.5,100);
% Thickness of stomach lining is 0.5 cm
t = linspace(0,6,100);
% Time is 6 hours

sol = pdepe(m,@pdex,@pdexic,@pdexbc,x,t);
u=sol(:,:,1);

figure(1);
plot(t,u(:,1), t,u(:,96), t,u(:,99))
xlabel('Time (hours)')
ylabel('Concentration (ABV)')
title('Concentration Profile (Student A - Cartesian)','FontSize',16);
legend('x=0cm', 'x=6.25cm', 'x=6.5cm')

figure(2);
plot(x,u(51,:))
xlabel('Position (cm)')
ylabel('Concentration (ABV)')
title('Concentration Profile at t = 3 hours','FontSize',16);

function [c, f, s] = pdex(x, t, u, DuDx)
if x < 6
    D = 10^15; % This forces the concentration to be 0.05 from 0 to 6 cm
else
    D = 10^-4*3600; % cm^2 / hour
end
c = 1;
f = D * DuDx;
s = 0;

function u0 = pdexic(x)
if x < 6
    u0 = .05;
else
    u0 = 0;
end

function [p1, q1, pf, qf] = pdexbc(x1,u1,xf,uf,t)
p1 = 0 ;
q1 = 1;
pf = uf;
qf = 0;
```

Student A (Spherical) MATLAB pdepe Code

```
% STUDENT A SPHERICAL

function pdexfunc
m = 2;
x = linspace(0,6.5,100);
% Thickness of stomach lining is 0.5 cm
t = linspace(0,6,100);
% Time is 6 hours

sol = pdepe(m,@pdex,@pdexic,@pdexbc,x,t);
u=sol(:,:,1);

figure(1);
plot(t,u(:,1), t,u(:,96), t,u(:,99))
xlabel('Time (hours)')
ylabel('Concentration (ABV)')
title('Concentration Profile (Student A - Spherical) ','FontSize',16);
legend('x=0cm','x=6.25cm','x=6.5cm')

figure(2);
plot(x,u(51,:))
xlabel('Position (cm)')
ylabel('Concentration (ABV)')
title('Concentration Profile at t = 3 hours ','FontSize',16);

function [c, f, s] = pdex(x, t, u, DuDx)
if x < 6
    D = 10^15; % This forces the concentration to be 0.05 from 0 to 6 cm
else
    D = 10^-4*3600; % cm^2 / hour
end
c = 1;
f = D * DuDx;
s = 0;

function u0 = pdexic(x)
if x < 6
    u0 = .05;
else
    u0 = 0;
end

function [p1, q1, pf, qf] = pdexbc(x1,u1,xf,uf,t)
p1 = 0 ;
q1 = 1;
pf = uf;
qf = 0;
```

Student B (Spherical) MATLAB pdepe Code

```
% STUDENT B SPHERICAL

function pdexfunc
m = 2;
x = linspace(0,6.5,100);
% Thickness of stomach lining is 0.5 cm
t = linspace(0,60,100);
% Time is 6 hours

sol = pdepe(m,@pdex,@pdexic,@pdexbc,x,t);
u=sol(:,:,1);

figure(5);
plot(t,u(:,1), t,u(:,96), t,u(:,99))
xlabel('Time (hours)')
ylabel('Concentration (ABV)')
title('Concentration Profile (Student B - Spherical) ','FontSize',16);
legend('x=0cm','x=6.25cm','x=6.5cm')

figure(6);
plot(x,u(51,:))
xlabel('Position (cm)')
ylabel('Concentration (ABV)')
title('Concentration Profile at t = 30 hours ','FontSize',16);

function [c, f, s] = pdex(x, t, u, DuDx)
if x < 6
    D = 10^15; % This forces the concentration to be 0.05 from 0 to 6 cm
else
    D = 10^-4*3600; % cm^2 / hour
end
c = 1;
f = D * DuDx;
if x < 6    % v is the driving force
    v = .0187;
else
    v = 0;
end
s = v;

function u0 = pdexic(x)
u0 = 0;

function [p1, q1, pf, qf] = pdexbc(x1,u1,xf,uf,t)
p1 = 0;
q1 = 1;
pf = uf;
qf = 0;
```


Student C (Spherical) MATLAB pdepe Code

```
% STUDENT C SPHERICAL

function pdexfunc
m = 2;
x = linspace(0,6.5,100);
% Thickness of stomach lining is 0.5 cm
t = linspace(0,6,100);
% Time is 6 hours

sol = pdepe(m,@pdex,@pdexic,@pdexbc,x,t);
u=sol(:,:,1);

figure(7);
plot(t,u(:,1), t,u(:,96), t,u(:,99))
xlabel('Time (hours)')
ylabel('Concentration (ABV)')
title('Concentration Profile (Student C - Spherical )','FontSize',16);
legend('x=0cm','x=6.25cm','x=6.5cm')

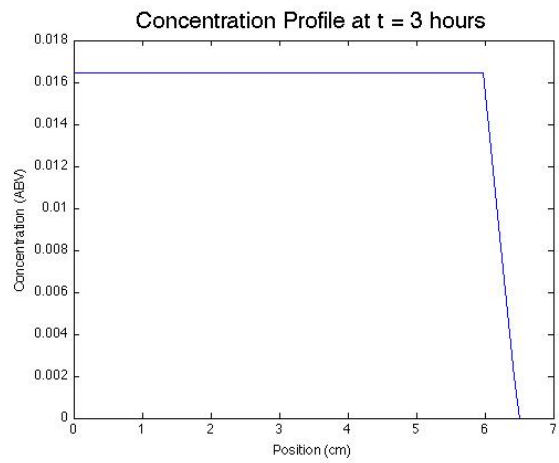
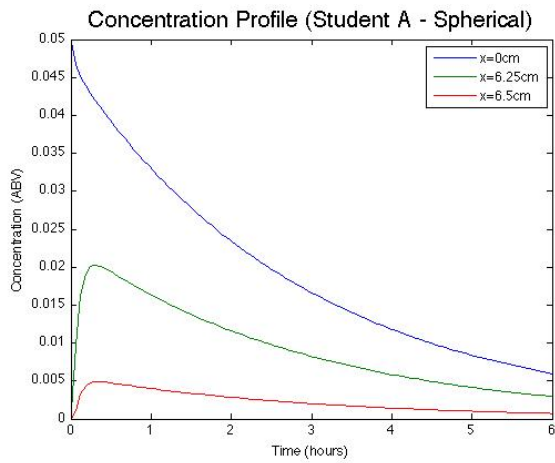
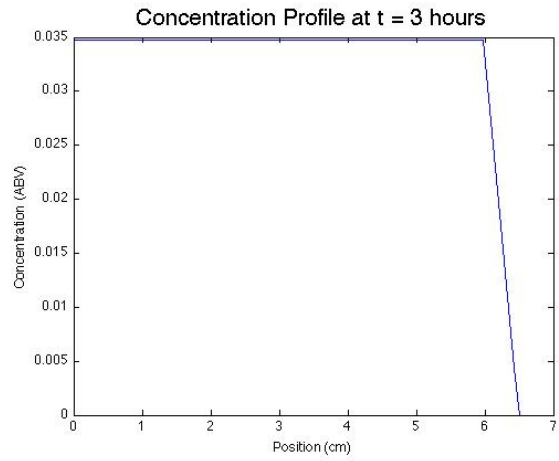
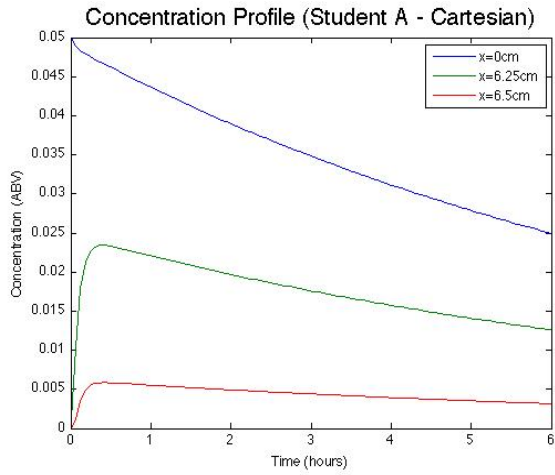
figure(8);
plot(x,u(51,:))
xlabel('Position (cm)')
ylabel('Concentration (ABV)')
title('Concentration Profile at t = 3 hours','FontSize',16);

function [c, f, s] = pdex(x, t, u, DuDx)
if x < 6
    D = 10^15; % This forces the concentration to be 0.05 from 0 to 6 cm
else
    D = 10^-4*3600; % cm^2 / hour
end
c = 1;
f = D * DuDx;
if x < 6
    v = .01*sin(pi*t)^2;
else
    v = 0;
end
s = v;

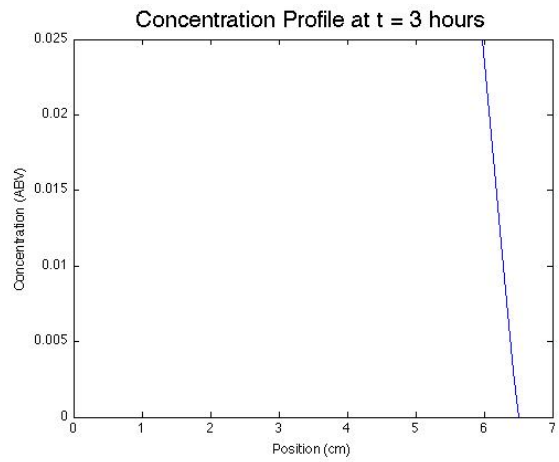
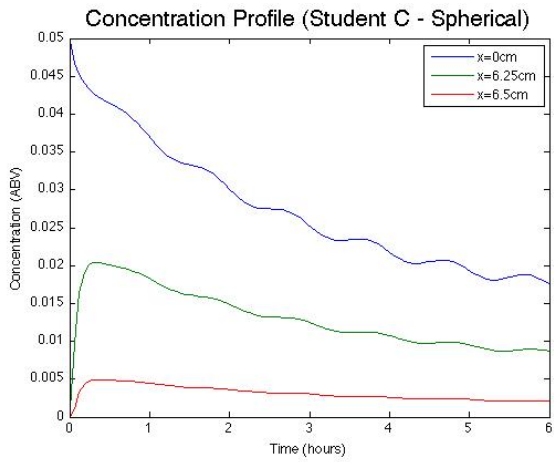
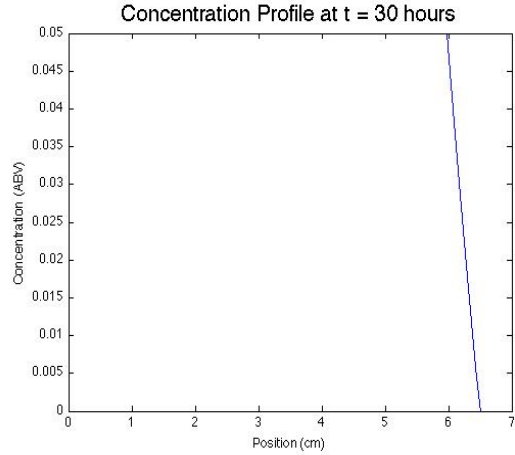
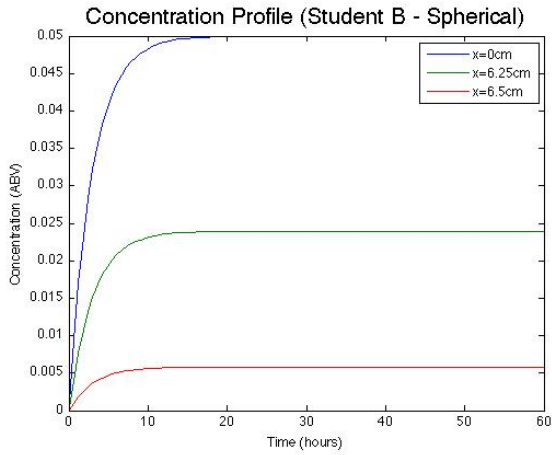
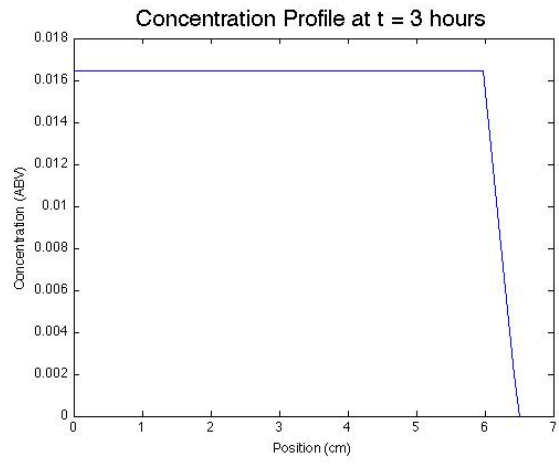
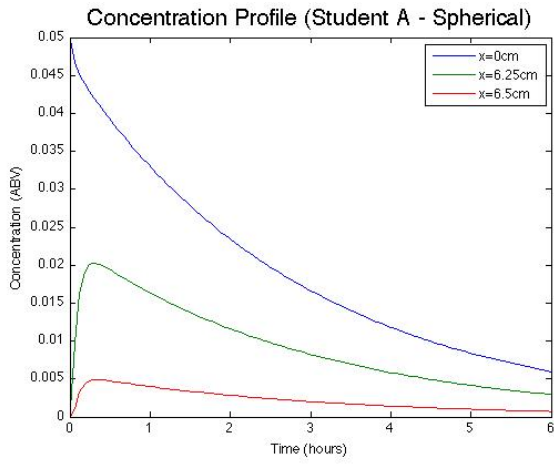
function u0 = pdexic(x)
if x < 6
    u0 = 0.05;
else
    u0 = 0;
end

function [p1, q1, pf, qf] = pdexbc(x1,u1,xf,uf,t)
p1 = 0;
q1 = 1;
pf = uf;
qf = 0;
```

Results: Cartesian Coordinates VS Spherical Coordinates (Student A)



Results: Spherical Coordinates (Student A, B, and C)



Conclusion

For Student A, the concentration inside the stomach is initially 0.05 and the concentration in stomach lining is initially 0. The concentration inside the stomach decreases over time. The rate at which the concentration decreases does not depend on position within the stomach volume inside the inner wall. However, this is not the case for the stomach lining. The concentration in the stomach wall initially increases, and then eventually decreases over time. The concentration depends on the position within the stomach lining. The solution makes physical sense, as the system begins with a fixed concentration of ethanol, and no additional ethanol is added to the system. Eventually, all of the ethanol should diffuse out the stomach into the bloodstream.

The plots for Student A's stomach also show a difference between the two coordinate systems. The concentration in the spherical stomach decreases much more rapidly than the concentration in the Cartesian system. This is expected because the sphere has a greater surface area to diffuse through. The concentration will approach zero much sooner in the spherical system as opposed to the Cartesian system. This highlights the importance of being able to model the spherical coordinate system.

For Student B, the concentration is initially 0 everywhere. A constant driving force is applied to the system in order to simulate Student B's constant drinking. The concentration of the stomach increases everywhere and eventually levels off when it reaches steady-state. The steady-state concentration is dependent on the position within the stomach lining.

Student C is similar to Student A. With Student C, multiple impulses are applied to the system in order to simulate his periodic drinking behavior or drinking pattern. This is modeled by using a driving force that is a squared sine function. The plots are very similar between Students A and C except that concentration profile for student C oscillates due to the sine term. Student C's oscillating drinking pattern causes the concentration to slightly increase at each point in time, then decrease. This makes sense physically, as the student is repeatedly drinking beer, then diluting the concentration of alcohol in his stomach with water. Furthermore, the overall trend of the concentration profiles of Student A and C are similar; they both gradually decrease and level off. However, it takes longer for the concentration of alcohol in Student C to reach 0, because he is still consuming more alcohol after time 0.

Future Work

Future work could be conducted to make fewer assumptions. One assumption was made in the initial condition that the concentration of the stomach is 0.05 ABV. This assumes that person has only consumed beer with 0.05 ABV, and thus, the person's stomach has only beer in it. We have neglected the presence of food as well as various stomach fluids. Our model could also take into account the other foods and beverages that the individual has consumed in recent hours. An equation would have to be included for the initial condition. This equation would include variables for the amount of food and non-alcoholic beverages consumed.

Another assumption that could be removed would be the shape of the stomach. This problem assumes a stomach with a spherical shape. This is not true in actuality. However, it would be difficult to model a different shape in order to approximate a solution.

A more accurate model would include gastric emptying into the small intestine. A negative driving force term could be used to simulate liquid moving from stomach to the small intestine. A differential equation modeling absorption from the small intestine could be coupled to this model

of the stomach in order to determine overall alcohol absorption into the bloodstream. This would allow for blood alcohol content (BAC) to be determined. Then this problem could be expanded to determine issues such as at what time the students would be safe to drive, or would be considered no longer intoxicated.

References

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