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BENG221 project

Disinfecting Mad-Cow Diseased Beef: A Heat Transfer Approach

Background

Bovine spongiform encephalopathy (BSE), also known as mad-cow disease, is a progressive neurological disorder that causes the animal to lose physical coordination. The disease leads to the spongy degeneration of the brain and the spinal cord, and has an incubation period of approximately 4 years. As neurological tissue degrades, signal propagation is detrimentally affected, leading to fatal consequences. The source of this disease is believed to be an infectious protein known as a prion. A prion (**pr**oteinaceous **i**nfectious particle) is a misfolded protein, which is extremely stable. Aggregation of the particles in infected tissue forms plaques known as amyloids. Amyloids are responsible for the disruption of the tissue structure, which results in the creation of vacuoles in neurons. Because of their stability, prions are highly resistant to denaturization through chemical and physical agents. Unfortunately, BSE is transmissible to the human population and Creutzfeldt-Jakob disease (CJD) is the human equivalent of BSE. Effective prion decontamination depends on protein hydrolysis or reduction using bleach and strong acidic detergents. Prions are able to retain protein conformation even at boiling temperatures of 100 degrees C. Literature suggests that autoclave temperatures of 134 degrees C for 18 minutes may be able to denature the infectious protein agents. For our project, we are interested in solving for the temperature profile of the beef in respect to time and thickness. This solution is important because the United States consume significant amounts of beef on an annual basis. Knowing the how long to cook a piece of steak with specified thickness will allow the general public to disinfect CJD prior to consumption, or at least know if such a solution to BSE is possible.

Setup

Assumptions

- Assume that beef is approximately 70% fluid, such as that of the human body. This should be fairly accurate.
- Assume that fluid evaporation is minimal even though it is not entirely accurate. This is necessary, as evaporation will cause changes in the density of tissue within the steak.
- Assume that beef slab is sandwiched between two heating plates at temperature of 250 degree C. We can say this assumption if fairly accurate, as many people do cook their meats in George Foreman Grills.
- Assume beef is a slab and this is fairly accurate a piece of steak is a slice of meat after all.
- Assume uniform distribution of PRION throughout the slab of meat, even though the accuracy is questionable. We must do this in order to maintain continuity.

- Assume uniform heat capacity throughout the steak, which is very important because the model will not function at all without this.
- Assume that there is no radial temperature change, or in this case only 1-D diffusion. This is very important for our model, but inaccurate in the real world since heat transfers in all directions.
- Assume constant boundary condition at both ends, meaning the grill is flat and in perfect contact with the steak. In reality, steak is not perfectly flat and in complete contact with the grill since the topography is rather bumpy.

Equations

Applicable equations to this problem is the Heat Equation, which describes the rate of change of temperature at a point over time in terms of the heat constant and second spatial derivatives (thermal conductions) of temperature.

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

In order to solve for the temperature profile, we will first use separation of variables, then apply initial and boundary conditions. The boundary conditions the temperature of the steak at boundary d and –d, it is held at the temperature of the grill T_0 (250 degrees Celsius). For the initial condition, the initial temperature of the steak is T internal T_i.

Problem Solving

Start out using the heat equation.

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} = \lambda^2 \; or \; 0$$

where λ^2 is a constant.

Use separation of variables to solve for the temperature profile:

$$T(x,t) = \psi(x)\phi(t)$$

$$\frac{\partial \psi(x)\phi(t)}{\partial t} = \alpha \frac{\partial^2 \psi(x)\phi(t)}{\partial x^2}$$

$$\frac{1}{\phi(t)}\frac{\partial\psi(x)}{\partial t} = \alpha\frac{1}{\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2} = +\lambda^2, -\lambda^2\ or\ 0$$

$+\lambda^2$ yield exponential growth function → not applicable in our scenario

Solving for the remaining 2 scenario where heat equation = 0 or $-\lambda^2$

$$\frac{1}{\phi(t)}\frac{\partial\psi(x)}{\partial t} = \alpha\frac{1}{\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2} = -\lambda^2$$

$$\frac{1}{\alpha}\frac{1}{\phi(t)}\frac{\partial\psi(x)}{\partial t} = -\lambda^2$$

$$\psi(x) = e^{-\lambda^2\alpha t}$$

$$\frac{1}{\alpha}\frac{1}{\phi(t)}\frac{\partial\psi(x)}{\partial t} = 0$$

$$\psi(x) = \psi_0$$

$$\frac{1}{\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2} = -\lambda^2$$

$$r^2 - \lambda^2 = 0$$

$$r = +i\lambda\ or - i\lambda$$

$$\psi(x) = (A\cos\lambda x + B\sin\lambda x)$$

Recall: $T(x,t) = \psi(x)\phi(t)$

$$T = e^{-\lambda^2\alpha t}(A\cos\lambda x + B\sin\lambda x)$$

Initial condition for homogenous solution:

$T(x,0) = T_0 - T_i$ (homogenous solution= final solution- steady state solution)

$$T(-d,0) = 0$$

$$T(d,0) = 0$$

Substituting initial into equation to solve for the constants

$$T(d,0) = 0 = e^{-\lambda^2\alpha 0}(A\cos\lambda d + B\sin\lambda d)$$

$$T(-d,0) = 0 = e^{-\lambda^2 \alpha 0}(A\cos\lambda d - B\sin\lambda d)$$

$$0 = B$$

$$0 = 2A\cos\lambda d$$

$$0 = \cos\frac{(2n-1)}{2}\pi$$

$$\lambda d = \frac{2n-1}{2}\pi$$

$$\lambda = \frac{2n-1}{2d}\pi \;\; for\ n\ is\ an\ interger > 0$$

$$T = \sum e^{-\left(\frac{2n-1}{2d}\pi\right)^2 \alpha t}\left(A\cos\frac{2n-1}{2d}\pi x\right)$$

$$\int_{-d}^{d} T dx = \int_{-d}^{d}\sum e^{-\left(\frac{2n-1}{2d}\pi\right)^2 \alpha t}\left(A\cos\frac{2n-1}{2d}\pi x\right)dx$$

For simplicity, substitute $\lambda_n$back into the equation

$$\lambda_n = \frac{2n-1}{2d}\pi \;\; for\ n\ is\ an\ interger > 0$$

$$\int_{-d}^{d} T dx = \int_{-d}^{d}\sum e^{-(\lambda_n)^2 \alpha t}(A\cos\lambda_n d)\,dx$$

Multiplying both side by $A\cos\lambda_m x$

$$\int_{-d}^{d} T(A\cos\lambda_m x)dx = \int_{-d}^{d}\sum e^{-(\lambda_n)^2 \alpha t}(A\cos\lambda_n x)\,(A\cos\lambda_m x)dx$$

Basic trigonometry to remember:

$$\int_{-\pi}^{\pi}\sin nx\sin mx\,dx = \int_{-\pi}^{\pi}\cos nx\cos mx dx = 0$$

$$when\ m \neq n$$

$$\int_{-\pi}^{\pi}\sin^2 nx\,dx = \int_{-\pi}^{\pi}\cos^2 nx\,dx = \pi$$

$$\int_{-d}^{d} T(\cos\lambda_m x)dx = \int_{-d}^{d}\sum e^{-(\lambda_n)^2\alpha t}(A\cos\lambda_n x)\,(\cos\lambda_m x)dx$$

In another word, only when $\lambda_n = \lambda_m$ one would yield a non-zero answer

$$\int_{-d}^{d} T(\cos\lambda_m x)dx = \int_{-d}^{d}\sum e^{-(\lambda_n)^2\alpha t}(A\cos^2\lambda_n d)\,dx$$

$$\int_{-d}^{d} T(x,0) * (\cos\lambda_n x)dx = \sum e^{-(\lambda_n)^2\alpha 0}\;\; A\int_{-d}^{d}(\cos^2\lambda_n x)dx$$

Applying initial condition to solve for A:

$$\int_{-d}^{d}(T_0 - T_i) * (\cos\lambda_n x)dx = \sum e^{-(\lambda_n)^2\alpha 0}\;\; A\int_{-d}^{d}(\cos^2\lambda_n x)dx$$

$$\int_{-d}^{d}(T_0 - T_i) * (\cos\lambda_n d)dx = A\frac{2d}{2}$$

$$\lambda = \frac{2n-1}{2d}\pi \;\; for\ n\ is\ an\ interger > 0$$

$$\frac{(T_0 - T_i)}{(2n-1)\pi}2d\left(\sin\frac{2n-1}{2d}\pi x\right) = A\frac{2d}{2}$$

Choosing n to be odd to eliminate the sine term:

$$\frac{(T_0 - T_i)}{(2n-1)\pi}2d\left(\sin\frac{1}{2d}\pi x\right) = Ad$$

$$\frac{(T_0 - T_i)}{(2n-1)\pi}2d(1 - -1) = Ad$$

$$4\frac{(T_0 - T_i)}{(2n-1)\pi} = A$$

$$T_h = (T_0 - T_i)\sum e^{-\left(\frac{2n-1}{2d}\pi\right)^2\alpha t}\left(\frac{4}{(2n-1)\pi}\cos\frac{2n-1}{2d}\pi x\right)$$

Initial condition for steady state solution:

$$T(x,0) = T_0$$

$$\frac{1}{\phi(t)}\frac{\partial\psi(x)}{\partial t} = \alpha\frac{1}{\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2} = 0$$

$$T = Ax + b$$

$$T_{ss} = T_i$$

Final solution:

$$T = T_{ss} + T_h$$

$$T = T_{i+}(T_0 - T_i)\sum e^{-\left(\frac{2n-1}{2d}\pi\right)^2 \alpha t}\left(\frac{4}{(2n-1)\pi}\cos\frac{2n-1}{2d}\pi x\right)$$

Numerical Analysis with Matlab

We are showing four cases of cooking a steak with Matlab's pde solver.
Case I: George Forman Style (double-side heating or oven-like heating). Thickness of the steak is 2cm. Heater's temperature is 250℃

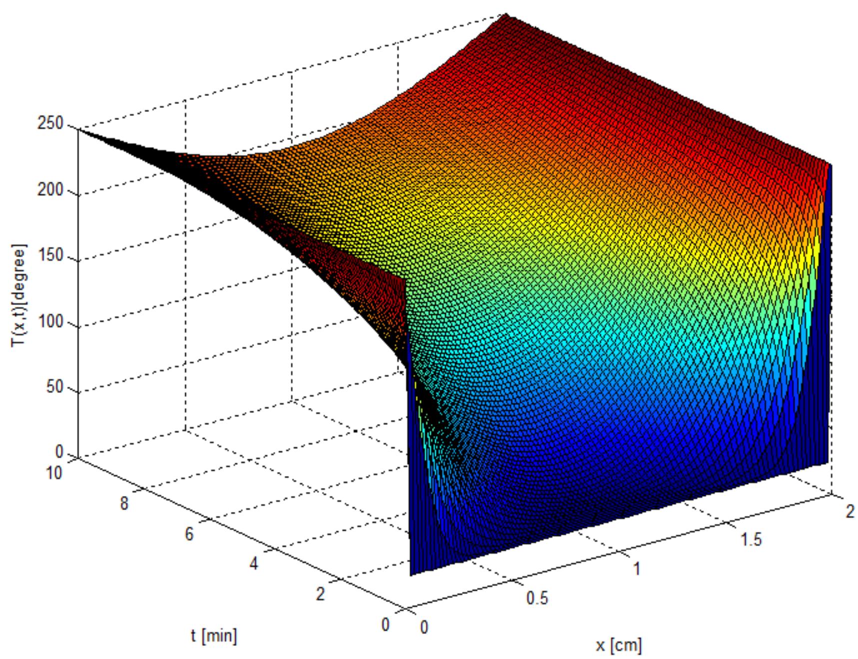
Case II: Grill case (one-side heating). Thickness of the steak is 2cm. Grill's temperature is 800℃, and there is heat loss through air on the other side. (same for case III and IV)

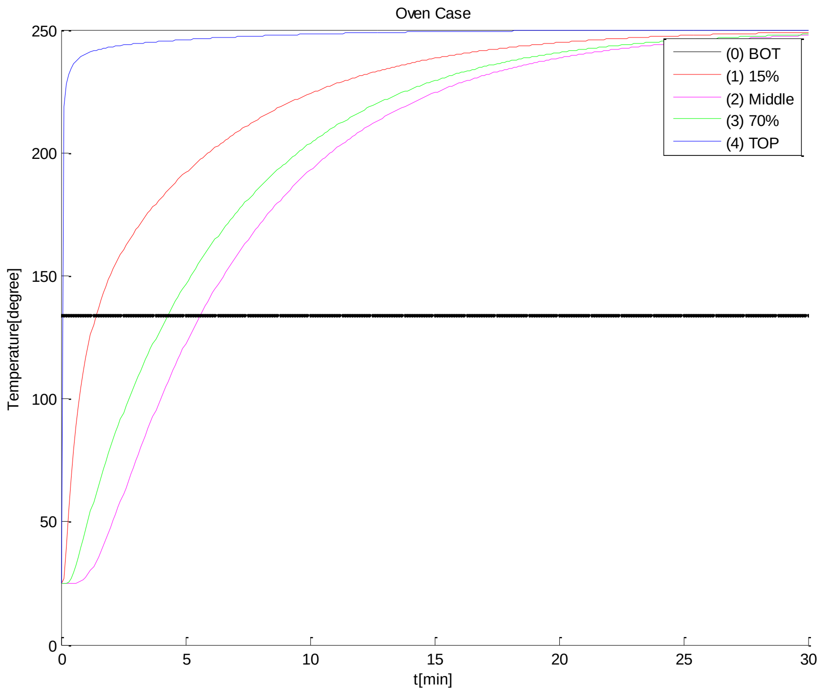
Case III: Grill case (one-side heating). Thickness of the steak is 1cm.

Case IV: Grill case with one flipping (one-side heating). Thickness of the steak is 2cm.

Criterion for denaturing all Prions: All Prions within the steak must be exposed to 134℃ for 18 minutes to denature protein

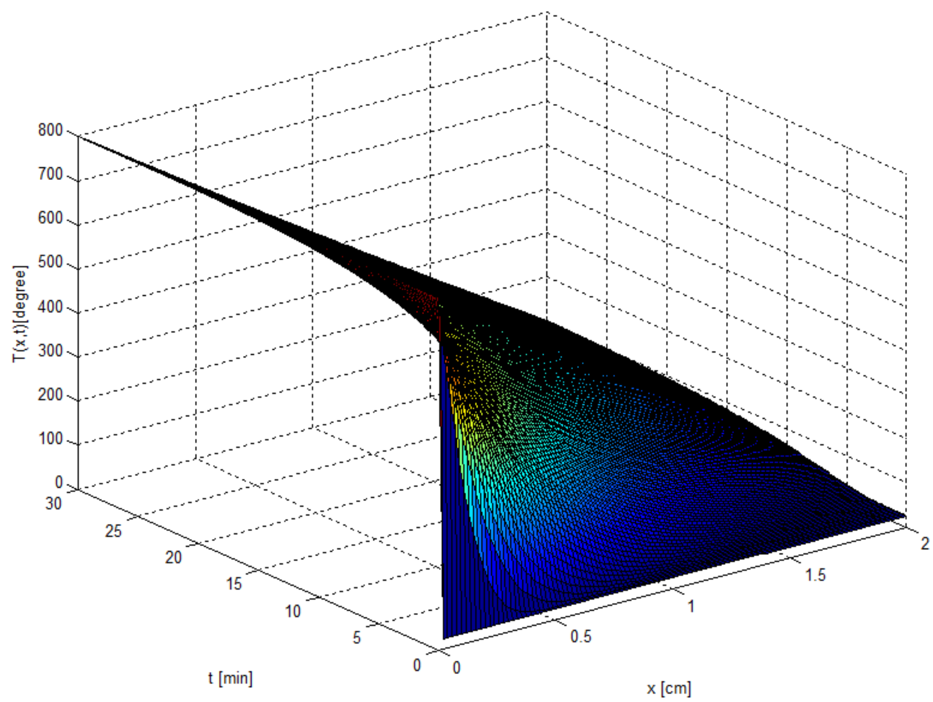
Case I: George Forman Style and 2cm-thick steak.

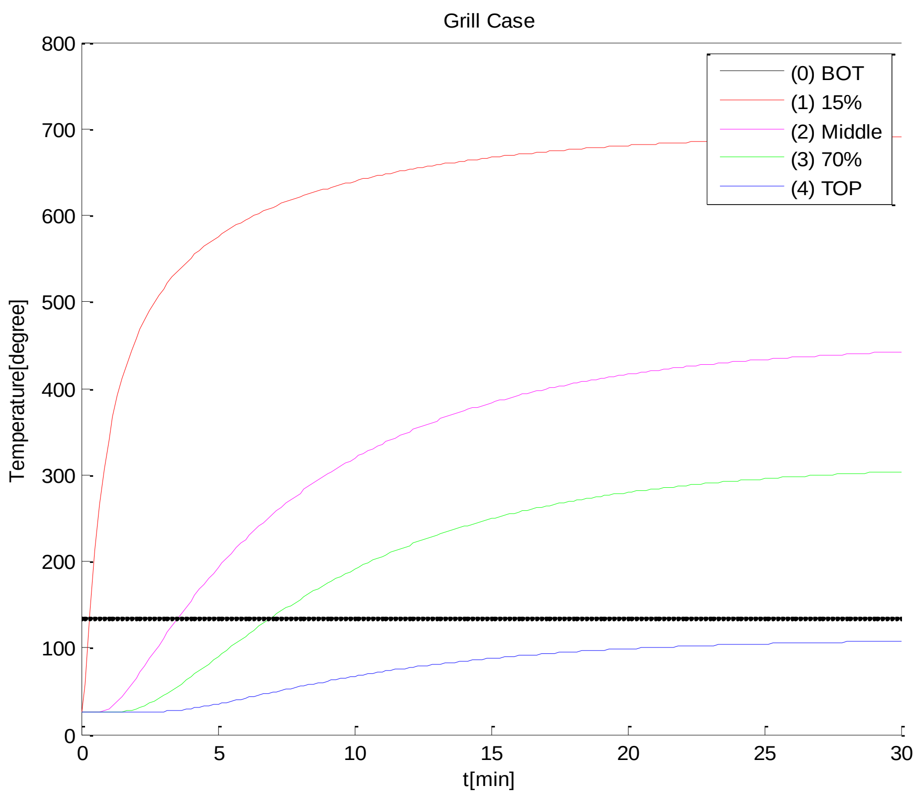




As shown in the 3D plot above, both sides are fixed with 250℃and the middle in the steak has lowest temperature. Because heat comes from both side, the center easily reach over 134℃at 6 minutes. Thus, we can denature all Prions within 24 minutes.

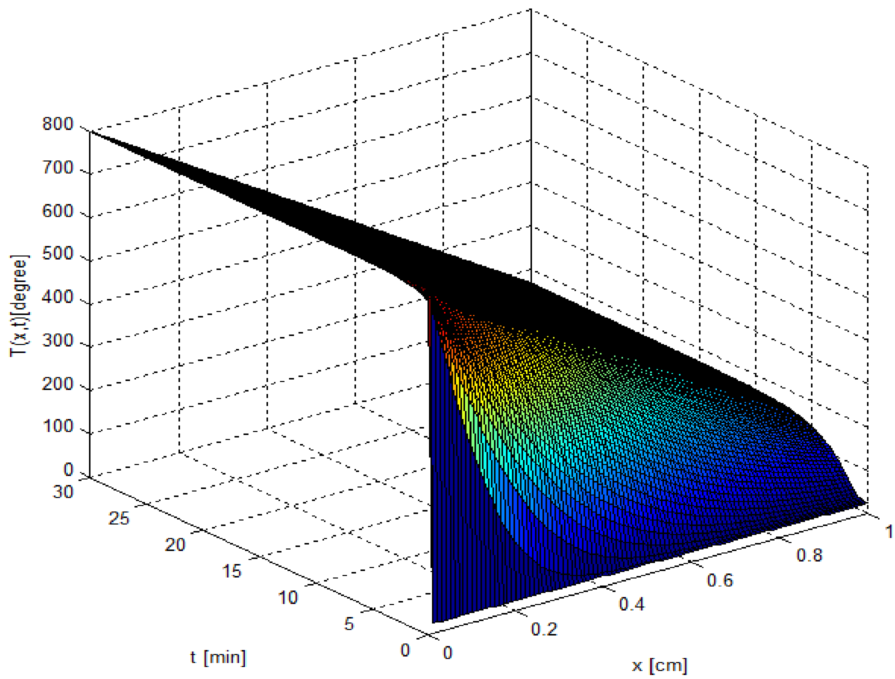
Case II: One side grilling of a 2cm-thick steak.

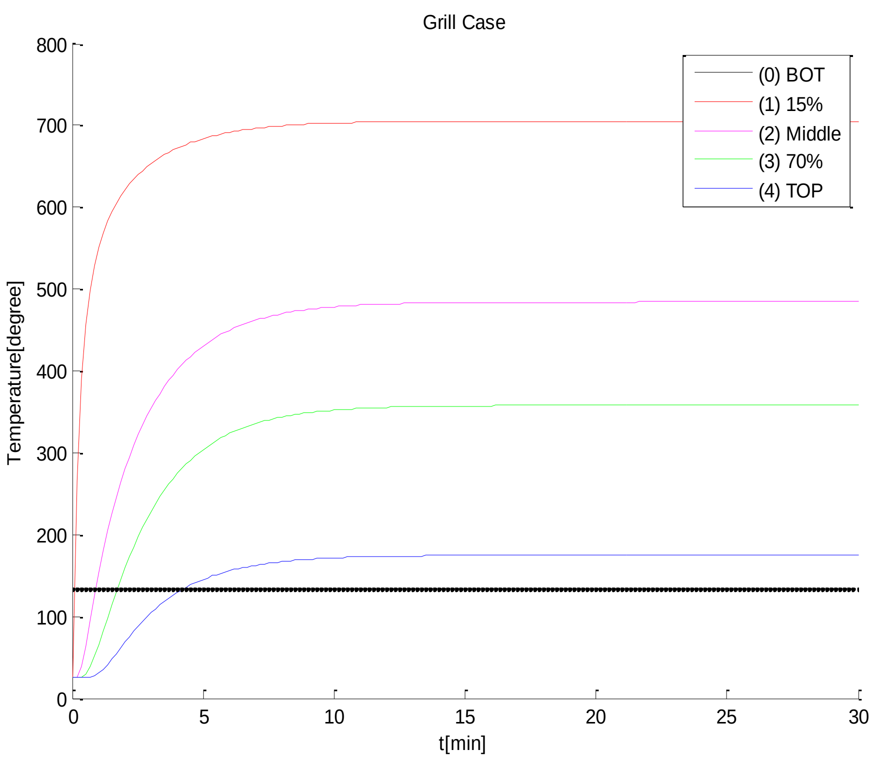




In this case, we heat the bottom of the steak with grill at 800℃and assume that there is heat loss from the top. The 2D graph shows the temperature of the top cannot reach to 134℃ because of heat loss.

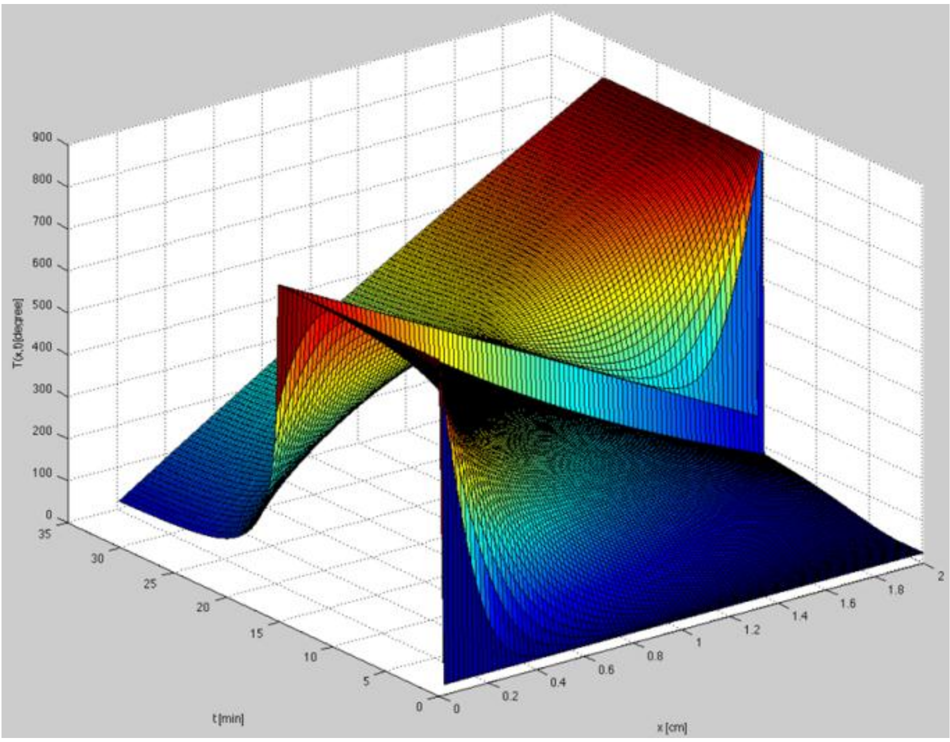
Case III: one side grilling of a 1cm-thick steak.

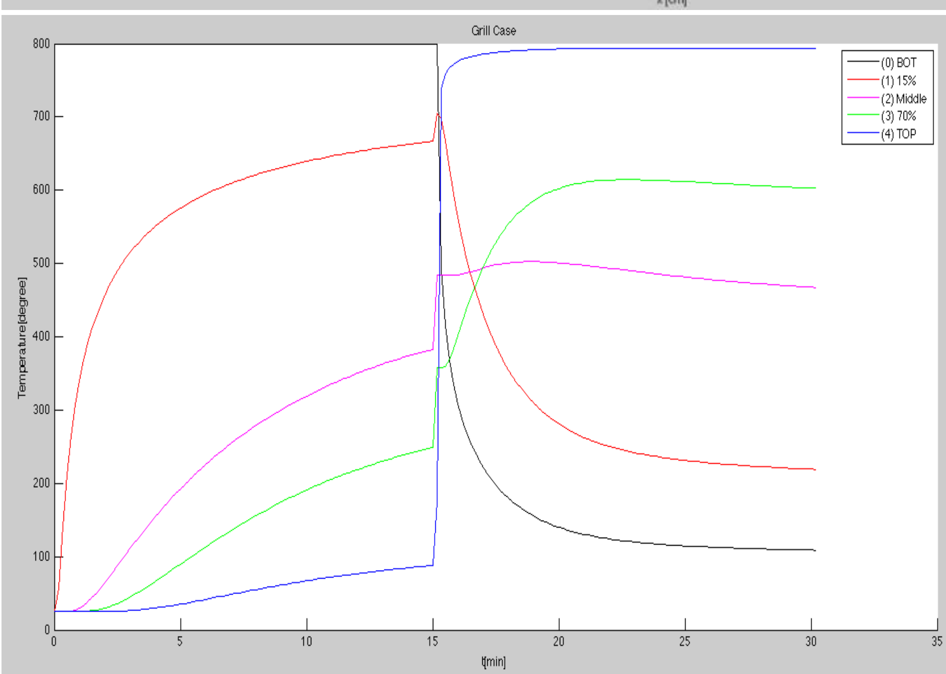




In order to denature all Prions and cook the steak well, we halved the thickness of the steak. As a result, the temperature of the top can reach over 134℃in 4 minutes hence the steak can be prion-free in 22 minutes.

Case IV: 1 side grilling of a 2cm-thick steak and flipping at 15 minutes.





We decided that a 2-cm steak gives more texture, so we will alter our cooking method by flipping the steak every 15 minutes. As illustrated in the temperature profile, temperature at each depth will reach 134°Ceventually. Hence, this 2-cm steak can be prion-free in 33 minutes.

Conclusions

Our results suggest that it is possible to completely denature the prions in a steak, however the necessary cooking time is too large to yield an edible product. Starting with the assumption that holding the meat at 134 degrees Celsius for 18 minutes will denature all the prions, our model suggests that we would have to cook the steak for approximately 24 minutes with a George Forman Grill (2cm thickness), and 22 minutes with a traditional fry pan (1 cm thickness). This amount of time at such a high temperature would burn all of the sugars in the muscle tissue, giving you a thin, dry, and tasteless slab of meat. This point is true regardless of the boundary conditions we set and the grill set-up we used. This result makes an important point about the safety of meat consumption. Since BSE can be transferred between animals and humans simply by digesting prions, it is possible to get the disease from infected meat even when it is thoroughly cooked.

Future Work

While our model firmly established the amount of time it would take to denature prions by grilling a steak, we could explore cooking techniques that used lower temperatures for longer periods of time. Since the temperature needs to be held at 134 degrees for a relatively long time, perhaps slow cooking the meat would denature prions without rendering the meat inedible. We could model this by submerging the slab into a liquid medium of uniform heat distribution and setting the boundary values to a constant temperature slightly greater than 134 degrees. While the cooking time would be much longer, it would provide a meaningful result because the meat would still be edible.

We could also try modeling the system with an inhomogeneous heat equation. The inhomogeneous term could come from a number of sources, including the changing properties of the meat as it cooks, or the heat that dissipates as air flows over the steak on the grill. This would require more knowledge on the the chemistry of meat as temperature changes, although considering the amount of water that dissipates would account for most of this difference.

APPENDIX

(1) Code for Oven Case

```
function steak_burn_oven_case
% Oven Case
dt = 6;
t = 0:dt:dt*300;
L = 0.02;
dx = L/100;
x = 0:dx:L;

N = length(t);
X = length(x);

U = zeros(N,X);

% Calculations with PDE solver
solution = pdepe(0,@pdex,@pdexic,@pdexbc,x,t);
U = solution(:,:,1);

% 3D Plot
figure(1);
surf(x*100, t/60, U)
xlabel 'x [cm]'; ylabel 't [min]'; zlabel 'T(x,t)[degree]';

% 2D Plot
figure(2);
hold on;
plot(t/60,U(:,1),'black'); % 0m
plot(t/60,U(:,16),'r'); % 10m
plot(t/60,U(:,51),'m'); % 50m
plot(t/60,U(:,71),'g'); %70m
plot(t/60,U(:,100),'b'); % 100m
plot(t/60,134,'--k');
hold off;
legend('(0) BOT','(1) 15%', '(2) Middle' , '(3) 70%','(4) TOP');
title('Oven Case');
xlabel('t[min]')
ylabel('Temperature[degree]')

function [c,f,s] = pdex(x,t,u,DuDx)
k = 1.09e-7; %thermal diffusivity of beef [m^2/s]
c = 1;
f = k*DuDx;
s = 0;

function u0 = pdexic(x)
% initial condition at x = 25
```

```
u0 = 25;


function [pl,ql,pr,qr] = pdexbc(xl,ul,xr,ur,t)
pl = ul-250; % temperature at the top is 250
ql = 0;
pr = ur-250; % temperature at the bottom is 250
qr = 0;
```

(2) Code for Grill case

```
function steak_burn_grill_case
% Grill Case
dt = 10;
t = 0:dt:dt*180;
L = 0.01;
dx = L/100;
x = 0:dx:L;

N = length(t);
X = length(x);

U = zeros(N,X);

% Calculations by PDE solver
solution = pdepe(0,@pdex,@pdexic,@pdexbc,x,t);
U = solution(:,:,1);

% 3D Plot
figure(1);
surf(x*100, t/60, U)
xlabel 'x [cm]'; ylabel 't [min]'; zlabel 'T(x,t)[degree]';

% 2D Plot as time goes by
figure(2);
hold on;
plot(t/60,U(:,1),'black'); % 0m
plot(t/60,U(:,16),'r'); % 10m
plot(t/60,U(:,51),'m'); % 50m
plot(t/60,U(:,71),'g'); %70m
plot(t/60,U(:,100),'b'); % 100m
plot(t/60,134,'--k');
hold off;
legend('(0) BOT','(1) 15%', '(2) Middle' , '(3) 70%','(4) TOP');
title('Grill Case');
xlabel('t[min]')
ylabel('Temperature[degree]')

function [c,f,s] = pdex(x,t,u,DuDx)
k = 1.09e-7; %thermal diffusivity of beef [m^2/s]
```

```
c = 1;
f = k*DuDx;
s = 0;

function u0 = pdexic(x)
% initial condition at x = 25
u0 = 25;

function [pl,ql,pr,qr] = pdexbc(xl,ul,xr,ur,t)
k_air = 0.024;  % %thermal conductivity of air [m^2/s]
%k_air = 1.9e-5;  % %thermal diffusivity of air [m^2/s]
pl = ul-800; % Grill temperature
ql = 0;
pr = 0.002*k_air*(ur-25);% heat loss via air
qr = 1;
```

(3) Code for Grill Flipping case

```
function steak_grill_flip
% Fliping case
dt = 10;
t = 0:dt:dt*90;
L = 0.02;
dx = L/100;
x = 0:dx:L;

N = length(t);
X = length(x);
U = zeros(N,X);

% Calculations
solution = pdepe(0,@pdex,@pdexic,@pdexbc,x,t);
U = solution(:,:,1);

figure(1);
surf(x*100, t/60, U)
xlabel 'x [cm]'; ylabel 't [min]'; zlabel 'T(x,t)[degree]';

figure(2);
hold on;
plot(t/60,U(:,1),'black'); % 0m
plot(t/60,U(:,16),'r'); % 10m
plot(t/60,U(:,51),'m'); % 50m
plot(t/60,U(:,71),'g'); %70m
plot(t/60,U(:,100),'b'); % 100m
plot(t/60,134,'--k');
hold off;
legend('(0) BOT','(1) 15%', '(2) Middle' , '(3) 70%','(4) TOP');
title('Grill Case');
xlabel('t[min]')
```

```
ylabel('Temperature[degree]')

U2 = zeros(N,X);

% Calculations
solution = pdepe(0,@pdex,@pdexic2,@pdexbc,x,t);
U2 = solution(:,:,1);

figure(3);
surf(x*100, t/60, U2)
xlabel 'x [cm]'; ylabel 't [min]'; zlabel 'T(x,t)[degree]';

t2 = 0:dt:dt*180+dt;
size(t2)
size(t)
UT = zeros(2*N,X);
UT(1:N,:) = U;
UT(N+1:2*N,X:-1:1) = U2;

size(UT)

figure(4);
hold on;
plot(t2/60,UT(:,1),'black'); % 0m
plot(t2/60,UT(:,16),'r'); % 10m
plot(t2/60,UT(:,51),'m'); % 50m
plot(t2/60,UT(:,71),'g'); %70m
plot(t2/60,UT(:,100),'b'); % 100m
plot(t2/60,134,'--k');
hold off;
legend('(0) BOT','(1) 15%', '(2) Middle' , '(3) 70%','(4) TOP');
title('Grill Case');
xlabel('t[min]')
ylabel('Temperature[degree]')

figure(5);
surf(x*100, t2/60, UT)
xlabel 'x [cm]'; ylabel 't [min]'; zlabel 'T(x,t)[degree]';


function [c,f,s] = pdex(x,t,u,DuDx)
k = 1.09e-7; %thermal diffusivity of beef [m^2/s]
c = 1;
f = k*DuDx;
s = 0;


function u0 = pdexic(x)
% initial condition at x = 25
```

```
u0 =  25;

function u0 = pdexic2(x)
% initial condition for the case after flipping
a = [    800.0000  793.6654   787.3307  780.9961  774.6616  768.3270
761.9926  755.6582  749.3239  742.9897  736.6557  730.3217  723.9879
717.6543  711.3208  704.9875 … … 168.0525];
u0 = a( round(101-x*50*100));
function [pl,ql,pr,qr] = pdexbc(xl,ul,xr,ur,t)
k_air = 0.024;  % %thermal conductivity of air [m^2/s]
%k_air = 1.9e-5;  % %thermal diffusivity of air [m^2/s]
pl = ul-800; % Grill temperature
ql = 0;
pr = 0.002*k_air*(ur-25);% heat loss via air
qr = 1;
```