Glossal Immobilization to Metallic Beam at Hyperborean Temperatures

"I triple dog dare you"

Jean Wang and Zachary King

BENG 221

October 28, 2011

Table of Contents

Background and Motivation

Glossal immobilization to a metallic beam was a common thematic element in blockbuster comedies through the 1980's and 1990's, and it remains an important comedic tool for screenwriters in Hollywood. Here we examine two landmark films in the history of glossal immobilization to a metallic beam: 1983's *A Christmas Story* and 1994's *Dumb and Dumber*.

A Christmas Story depicts 9-year-old Ralphie Parker's Christmastime adventures, from his encounter with the school bully to his family's Christmas Eve feast at a local Chinese restaurant—the dogs got a hold of the Christmas turkey—to Ralphie's inevitable encounter with the a real Red Ryder BB gun. One of the most memorable moments in the movie is a scene in which one of Ralphie's friends convinces another friend to stick his tongue to the metal flagpole in the middle of winter. (The scene can be viewed here: http://youtu.be/ZLZj3zOUZNs.) Within a few seconds, the unfortunate boy's tongue has become frozen to the metal pole, and he screams and wales until the fire department shows up to free him.

Dumb and Dumber contains a similar, though admittedly less realistic, depiction of glossal immobilization. In this scene, Harry Dunne has driven from Rhode Island to Aspen, CO, with best friend Lloyd Christmas to return a suitcase full of money, and now he sits on a ski lift with Mary Swanson, the woman with whom both men have fallen madly in love. In a moment of pathetic abandon, Harry notices frost on the metal pole of the ski lift chair, and he sticks his tongue to it. (The scene can be viewed here: http://youtu.be/-whpvv2KpsQ. It's pretty hilarious.) Again, the freezing process takes only seconds.

As children, we have often found ourselves wondering if these depictions of glossal immobilization to a metallic beam were realistic. Lacking the vacuous courage to try such a thing, we waited until graduate school to investigate the matter further. The goal of this project is to evaluate these two scenes by creating a simplified, mathematical model for glossal immobilization. In this report, we define a 1-dimensional model of the tongue in contact with a metallic beam below the freezing temperature. We present analytical solutions and numerical approximations performed with MATLAB, and we ultimately show that our model demonstrates the mechanics of the glossal immobilization process but does not match the depictions of the process seen in *A Christmas Story* and *Dumb and Dumber*.

Model and Assumptions

To approximate the mechanics of a tongue in contact with a metal bar, a simple, 1-dimensional model was created, as shown in Figure 1.

Figure 1. A 1-dimensional model for glossal immobilization to a metallic beam.
(Image courtesy of Warner Bros. Studios)

Table 1 shows the constants selected for both the analytical solutions and numerical approximations. Thermal diffusivity, D, and length of the tongue, L, were found in literature. Critical freezing distance, $L_{critical}$, was chosen to be the depth of a lingual papilla. The rational for this distance is that the saliva at the tip of the tongue must freeze completely before the tongue becomes immobilized, and this saliva should be present at the tip of the tongue to about the depth of the lingual papillae. Once frozen, the saliva forms to the contour of the tip of the tongue, creating a physical immobilization to the metallic beam.

The temperature of the beam was chosen based on our experience with Midwest wintertime conditions, and the human body temperature has been established to be 37 °C.

Table 1. Constants selected for glossal immobilization model.

Thermal diffusivity	D	$3.7 \bullet 10^4$ cm^2/s [1]
Length of the tongue	L	10 cm [2]
Critical freezing distance	$L_{critical}$	0.75 mm [3]
Temperature of the beam	T_0	-10 °C
Body temperature	T_L	37 °C

Model 1: 1-D Homogeneous Model

Analytical Solution

In our 1-dimensional case, heat diffusion follows the differential equation

$$\text{PDE: } \frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2},$$

with boundary conditions

$$\text{BC: } \begin{aligned} T(0,t) &= T_0 \\ T(L,t) &= T_L, \end{aligned}$$

and initial condition

$$\text{IC: } T(x,0) = T_L.$$

The full analytical solution can be found in Appendix A. Using the “Extracting the Poison Tooth” method, we determine the closed form solution

$$T(x,t) = \frac{T_L - T_0}{L}x + T_0 + \sum_{n=1}^{\infty}\frac{2(T_L - T_0)}{n\pi}\cdot \sin(\frac{n\pi}{L}x)\cdot e^{-D(\frac{n\pi}{L})^2 t}.$$

Figure 2 shows the analytical solution plotted as a surface plots for n = 50 terms. This plot shows how temperature decays near the surface of the tongue.

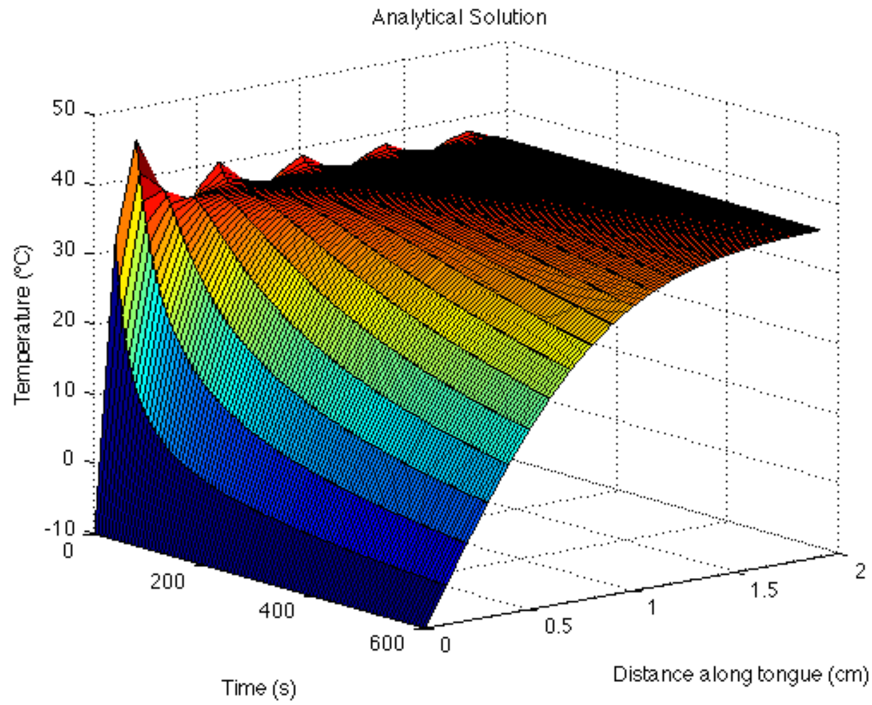

Figure 2. Surface plot of analytical solution of homogeneous equation. Calculated using n = 50.

Numerical Analysis

Using the MATLAB pdepe function (code is in Appendix B), the differential equation was numerically plotted over the same range. Figure 3 shows the surface plot, which matches the analytical solution nicely.

To investigate the problem at hand—determining how long freezing to the pole will take—we present this data and incremental time-points over a narrower distance near the tip of the tongue. This is shown for our pdepe data in Figure 4. The purple line is the steady-state solution. In this homogeneous system, temperature will approach this distribution as time goes to infinity.

Our critical freeze length for this model is x = 0.75 mm, and here we see that freezing to this critical limit will take around 96 seconds.

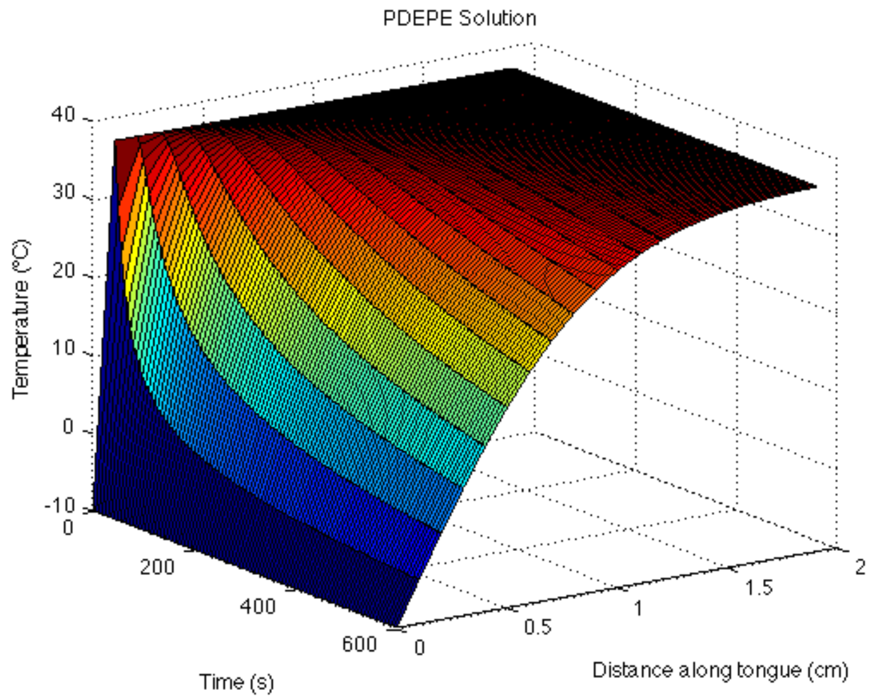

Figure 3. Surface plot of PDEPE approximation for homogenous equation.

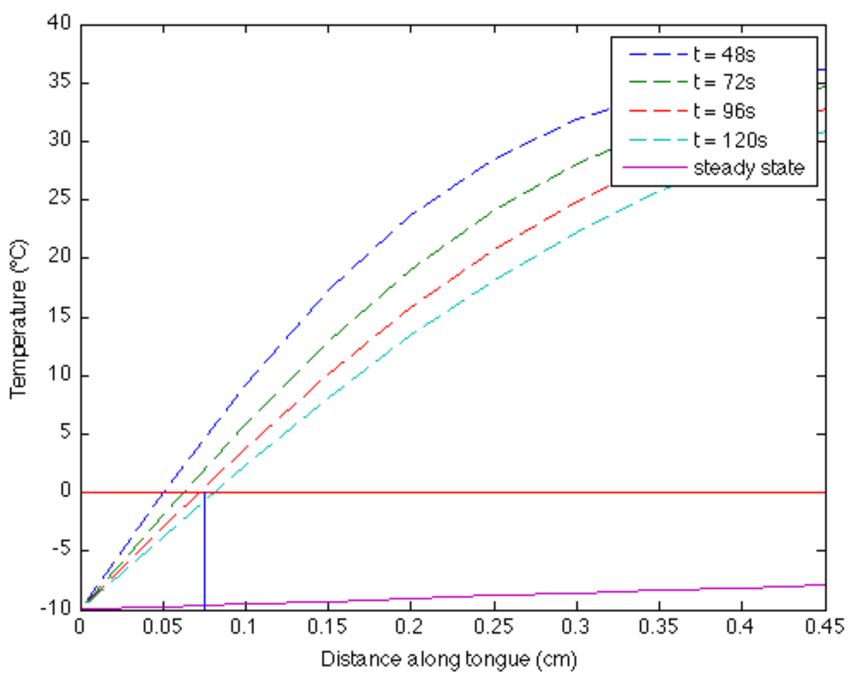

Figure 4. Temperature vs. distance plots for the PDEPE approximation of the homogenous equation. Time points from 48 to 120 seconds are shown along with the steady-state solution.

Model 2: Nonhomogeneous Model with a Heat-loss Term

The assumptions made to simplify our analytical analysis present a number of limitations. For example, this model:

- Assumes a 1 dimensional tongue
- Assumes no heat loss from the sides of the tongue
- Ignores blood flow in the tongue
- Ignores complex geometries of the tip of the tongue
- Ignores exothermic freezing reaction

To make our analysis more rigorous, we chose to address what we believe is the biggest factor of these limitations: Heat loss from the sides of the tongue. Based on the literature, we were able to calculate a constant heat loss of 68 °C/hour based on the surface area and mass of an average human tongue [4,5]. This was added to our differential equation to yield

$$\text{PDE: } \frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2} + Q_0$$

$$\text{BC: } \begin{matrix} T(0,t) = T_0 \\ T(L,t) = T_L \end{matrix}$$

$$\text{IC: } T(x,0) = T_L$$

where Q_0 = -68 °C/hour. Green's function was used to solve this analytically. (See Appendix A for the complete solution). Our solution as a function of time and distance along the tongue is

$$T(x,t) = \sum_{n=1}^{\infty} \frac{2T_L}{n\pi}(1-(-1)^n)\cdot \sin(\frac{n\pi}{L}x)\cdot e^{-D(\frac{n\pi}{L})^2 t} + \sum_{n=1}^{\infty}\left[\frac{2Q_0L^2}{Dn^3\pi^3}\left(1-(-1)^n\right) + \frac{2}{n\pi}\left(T_0 - T_L(-1)^n\right)\right]\cdot \sin(\frac{n\pi}{L}x)\cdot\left(1 - e^{-D(\frac{n\pi}{L})^2 t}\right)$$

Figure 5 shows the analytical solution plotted with n = 50 terms, and Figure 6 shows the corresponding numerical approximation. These surface plots have similar shapes, but the analytical solution trends toward T = 0 at both boundaries. This points to a possible error in our analysis.

Figure 7 shows temperature plotted against distance for time points between 48 and 120 seconds. Here, we see that the time for critical freezing is still approximately 96 seconds, not much different from our original solution without a heat loss term. The reason for this is because at short time frames, the heat loss term does not have enough time to affect the solution. When plotting to much longer time frames (e.g. 1200 seconds, plot not shown) the difference between the homogenous PDE and the nonhomogenous PDE with heat loss Q_0 becomes much more apparent.

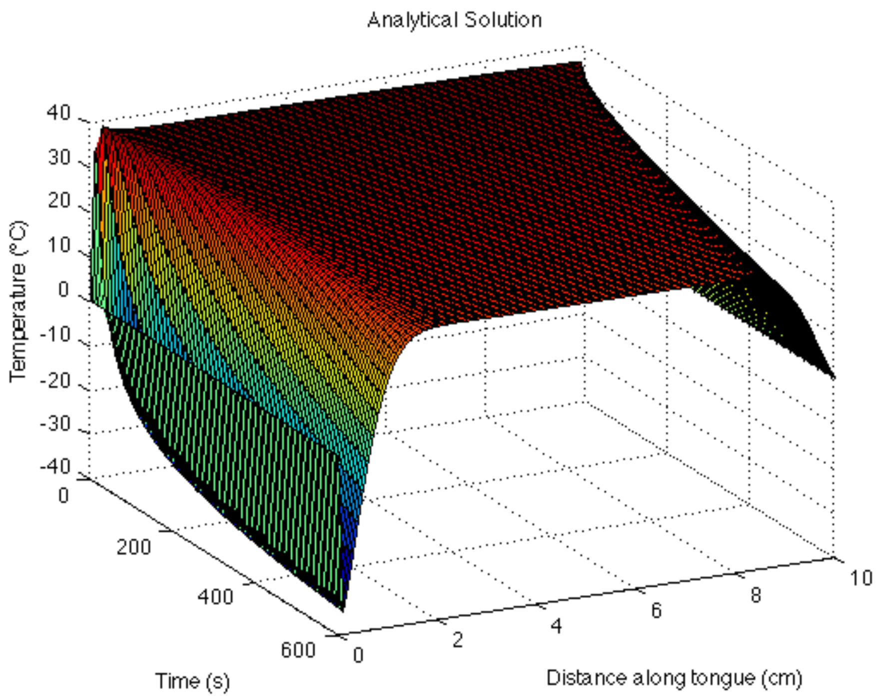

Figure 5. Surface plot of analytical solution of nonhomogeneous equation with heat-loss term. Calculated using n = 50.

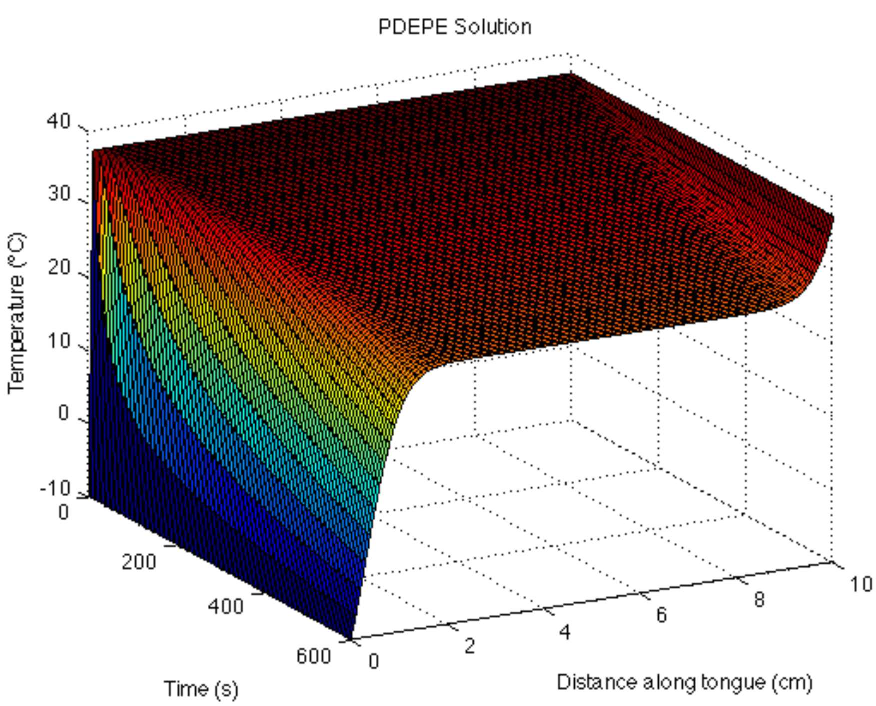

Figure 6. Surface plot of PDEPE approximation for nonhomogenous equation with a heat-loss term.

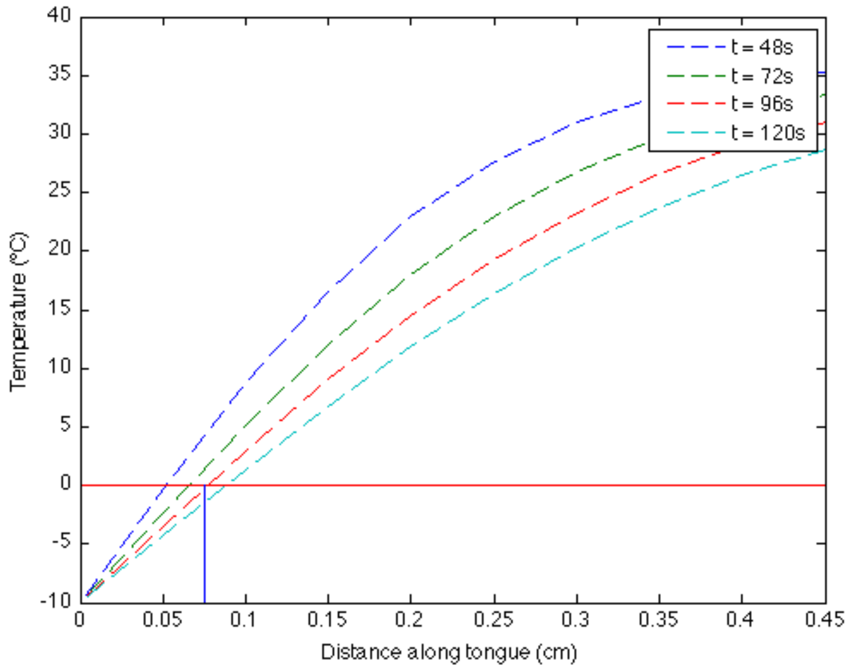

Figure 7. Temperature vs. distance plots for the PDEPE approximation of the nonhomogenous equation with a heat-loss term Q_0. Time points from 48 to 120 seconds are shown.

Furthermore, modeling heat loss through the sides of the tongue with a constant Q term is not the ideal solution for this problem. As the evaporative heat loss is a flux that certainly depends on the temperature difference between the tongue and the outside air. This system would be better modeled in two or three dimensions, where heat loss from the tongue can be modeled as a gradient along the length of the tongue. We chose to use the Q term as a work-around to solve this analytically in 1-D.

Conclusion

These models demonstrated the mechanism by which a tongue can freeze to a metallic beam at subfreezing temperatures. For the first, 1-dimensional model, a closed-form solution was analytically determined, and a numerical analysis was performed using MATLAB's pdepe function. This project was focused on the question of how long it might actually take for a tongue to freeze to a metallic beam. According to our first model, this process would take approximately a minute and a half.

Many simplifications were made in this first model, so we added another term to encompass what we thought was the most significant omission in the first model: evaporative heat loss from the sides of the tongue. A constant heat loss, Q, was added to the differential equation, and analytical and numerical analyses were performed again. Unfortunately, this attempt to improve our model did not decrease the predictive time for tongue freezing noticeably.

It is important to clarify that these models cannot *disprove* any hypothesis regarding the freezing of a tongue to a metallic beam. An empirical investigation would be necessary to determine whether tongue can freeze to a metallic beam as quickly as the movies would have us believe. However, modeling the process remains important for understanding the heat loss process, for determining which terms play the biggest role—it seems that evaporative heat loss is relatively unimportant at shorter timeframes —and for making predictions when similar situations are encountered. That is the real value of a model like this: It can guide and predict as we use freezing in a medical or industrial setting.

Impact and Future Work

A number of therapies and procedures rely on cooling or freezing tissues. Modeling the cooling process could be very valuable for further development of these technologies.

One of the most common is cryotherapy for removing skin abnormalities such as warts or moles [6]. Generally, liquid nitrogen is applied for a short period of time (less than 1 minute) to the skin. Liquid nitrogen cools the tissue enough to kill cells in the undesired region, and the body naturally discards and replaces this tissue.

Another, rather less established, therapy is known as *coolsculpting*. Here, a cooling treatment is applied to areas of the body with undesired fat. This procedure is intended to have cosmetic benefits for the user. It recently gained FDA approval, and one imagines that the folks at *coolsculpting* did some mathematical modeling of their cooling device to demonstrate its safety and effectiveness.

The models that we have developed here could be expanded and improved to help model therapies such as cryotherapy and coolsculpting. As the models become more complicated (multi-dimensional, non-homogeneous), leveraging computational approaches such as the pdepe function in MATLAB becomes extremely valuable.

References

1. Werner, U., Giese, K., Sennhenn, B., Plamann, K., & KÃ¶lmel, K. (1992). Measurement of the thermal diffusivity of human epidermis by studying thermal wave propagation. Physics in medicine and biology, 37(1), 21-35. Retrieved from http://www.ncbi.nlm.nih.gov/pubmed/1741425

2. Robin Kerrod (1997). MacMillan's Encyclopedia of Science. 6. Macmillan Publishing Company, Inc.

3. Segovia, C., Hutchinson, I., Laing, D. G., & Jinks, A. L. (2002). A quantitative study of fungiform papillae and taste pore density in adults and children. Brain research. Developmental brain research, 138(2), 135-46. Retrieved from http://www.ncbi.nlm.nih.gov/pubmed/12354641

4. Cooney, David O. 1976. Biomedical Engineering Principles: An Introduction to Fluid, Heat and Mass Transport Processes. New York: Marcel Dekker, Inc.

5. Collins, L. M. C., & Dawes, C. (1987). The Surface Area of the Adult Human Mouth and Thickness of the Salivary Film Covering the Teeth and Oral Mucosa. Journal of Dental Research, 66(8), 1300-1302. doi:10.1177/00220345870660080201

6. http://www.webmd.com/skin-problems-and-treatments/cryotherapy-for-warts

7. http://www.coolsculpting.com/

Appendix A: Analytical solutions

Homogenous model

Partial differential equations that describe the system, with boundary conditions and initial conditions

PDE: $\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2}$

BC: $T(0,t) = T_0$
$T(L,t) = T_L$

IC: $T(x,0) = T_L$

First find the steady state solution $T_E(x)$

- If $\frac{\partial T}{\partial t} = 0$, then $T_E(x) = ax + b$
- Plugging in the boundary conditions

$$T_E(x) = \frac{T_L - T_0}{L}x + T_0$$

Let $T(x,t) = T_E(x) + T_H(x,t)$, where $T_H(x,t)$ has a homogeneous PDE and boundary conditions, which are the following:

- We know that $\frac{\partial T}{\partial t} = \frac{\partial T_H}{\partial t} + \frac{\partial T_E}{\partial t} = \frac{\partial T_H}{\partial t}$ and $\frac{\partial^2 T}{\partial^2 x} = \frac{\partial^2 T_H}{\partial^2 x} + \frac{\partial^2 T_E}{\partial^2 x} = \frac{\partial^2 T_H}{\partial^2 x}$, so

$$\frac{\partial T_H}{\partial t} = D\frac{\partial^2 T_H}{\partial x^2}$$

- For the initial conditions, $T_H(x,0) = T(x,0) - T_E(x)$

IC: $T_H(x,0) = (T_L - T_0)(1 - \frac{x}{L})$

BC: $T_H(0,t) = 0$
$T_H(L,t) = 0$

Solving for $T_H(x,t)$ by separation of variables

$$T_H = \Phi(x)G(t),\ \frac{d^2\Phi}{dx^2} + \Phi\lambda = 0,\ \frac{dG}{dt} = -\lambda G$$

- To find λ, plug boundary conditions into $\Phi(x)$
  - If $\lambda > 0$, $\Phi(x) = A_n \cos(\sqrt{\lambda_n}x) + B_n \sin(\sqrt{\lambda_n}x)$, after plugging in BC, we find that

$$A_n = 0\text{, and } \lambda_n = \frac{n\pi}{L}$$

- For the λ = 0 and λ < 0 cases, we find the trivial solution Φ(x) = 0, after plugging in BC

- By the principle and superposition

$$T_H(x,t) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{L}x\right)\cdot e^{-D(\frac{n\pi}{L})^2 t}$$

- To find C_n, we plug in the initial conditions

$$T_H(x,0) = (T_L - T_0)\cdot\left(1-\frac{x}{L}\right)\sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{L}x\right)$$

$$C_n\left(\frac{L}{2}\right) = \int_0^L (T_L - T_0)\cdot\left(1-\frac{x}{L}\right)\cdot \sin\left(\frac{n\pi}{L}x\right)dx$$

- After integration and some algebra

$$C_n = \frac{2(T_L - T_0)}{n\pi}$$

Combining everything, we get the solution to our original PDE to be

$$T(x,t) = \frac{T_L - T_0}{L}x + T_0 + \sum_{n=1}^{\infty} \frac{2(T_L - T_0)}{n\pi}\cdot \sin\left(\frac{n\pi}{L}x\right)\cdot e^{-D(\frac{n\pi}{L})^2 t}$$

Non-homogeneous model

With the addition of heat loss, the equation that describes our model becomes

$$\text{PDE: } \frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2} + Q_0$$

$$\text{BC: } \begin{array}{l} T(0,t) = T_0 \\ T(L,t) = T_L \end{array}$$

$$\text{IC: } T(x,0) = T_L$$

Solving this using Green's Function

- For value-value boundary conditions:

Green's function: $G(x,t;x_0,t_0) = \sum_{n=1}^{\infty} \frac{2}{L}\cdot \sin\left(\frac{n\pi}{L}x_0\right)\cdot \sin\left(\frac{n\pi}{L}x\right)\cdot e^{-D(\frac{n\pi}{L})^2(t-t_0)}$

$$\frac{\partial}{\partial x_0}G(x,t;x_0,t_0)=\sum_{n=1}^{\infty}\frac{2n\pi}{L^2}\cdot\cos(\frac{n\pi}{L}x_0)\cdot\sin(\frac{n\pi}{L}x)\cdot e^{-D(\frac{n\pi}{L})^2(t-t_0)}$$

Solution:

$$T(x,t)=\int_0^L g(x_0)G(x,t;x_0,t_0)dx_0+\int_0^L\int_0^t Q(x_0,t_0)G(x,t;x_0,t_0)dt_0dx_0+\ldots$$
$$\int_0^t T_0(x_0)k\frac{\partial}{\partial x_0}G(x,t;x_0,t_0)dt_0-\int_0^t T_L(x_0)k\frac{\partial}{\partial x_0}G(x,t;x_0,t_0)dt_0$$

- Plugging in everything

$$T(x,t)=\int_0^L\sum_{n=1}^{\infty}\frac{2T_L}{L}\cdot\sin(\frac{n\pi}{L}x_0)\cdot\sin(\frac{n\pi}{L}x)\cdot e^{-D(\frac{n\pi}{L})^2t}dx_0+\int_0^L\int_0^t\sum_{n=1}^{\infty}\frac{2Q_0}{L}\cdot\sin(\frac{n\pi}{L}x_0)\cdot\sin(\frac{n\pi}{L}x)\cdot e^{-D(\frac{n\pi}{L})^2(t-t_0)}dt_0dx_0\ldots$$
$$\int_0^t\sum_{n=1}^{\infty}\frac{2T_0Dn\pi}{L^2}\cdot\sin(\frac{n\pi}{L}x)\cdot e^{-D(\frac{n\pi}{L})^2(t-t_0)}dt_0-\int_0^t\sum_{n=1}^{\infty}\frac{2T_LDn\pi(-1)^n}{L^2}\cdot\sin(\frac{n\pi}{L}x)\cdot e^{-D(\frac{n\pi}{L})^2(t-t_0)}dt_0$$

- After integration and algebra, the solution to the model with heat loss term Q is

$$T(x,t)=\sum_{n=1}^{\infty}\frac{2T_L}{n\pi}(1-(-1)^n)\cdot\sin(\frac{n\pi}{L}x)\cdot e^{-D(\frac{n\pi}{L})^2t}+\sum_{n=1}^{\infty}\left[\frac{2Q_0L^2}{Dn^3\pi^3}\left(1-(-1)^n\right)+\frac{2}{n\pi}\left(T_0-T_L(-1)^n\right)\right]\cdot\sin(\frac{n\pi}{L}x)\cdot\left(1-e^{-D(\frac{n\pi}{L})^2t}\right)$$

Appendix B: MATLAB code

Homogeneous model

```
function problemsolving

    global D L T1 T2

    D = 3.7E-4; % diffusion of heat in the tongue, cm^2/s
    L = 10; % length of tongue, cm
    T1 = 37;    % body temperature, Celcius
    T2 = -10;   % temperature of metal pole, Celcius
    trange = 120; %s
    x = linspace(0,L,101);
    t = linspace(0,trange,101);    % simulation time, seconds

    figure(1);
    sol = pdepe(0,@pdefun,@icfun,@bcfun,x,t);
    sol = sol';
    surf(t,x(1:20),sol(1:20,:));
    title('PDEPE Solution','fontweight','b')
    ylabel('Distance along tongue (cm)')
    xlabel('Time (s)')
    zlabel('Temperature (°C)')
    % ----

    initializer = ones(length(x),length(t));

    TB = T2; TH = T1;

    nterms = 50;
    sol_an = TB .* initializer;

    Hexp = initializer; Hsin = initializer;

    for n = 1:nterms
        for d = 1:length(x)
            for e = 1:length(t)
                if n==1
                    sol_an(d,e) = sol_an(d,e) + (TH-TB)/L*x(d);
                end
                Hexp(d,e) = exp(-D*(n*pi/L)^2 * t(e));
                Hsin(d,e) = sin(n*pi/L * x(d));
            end
        end

        Hconst = initializer .* ( 2*(TH-TB)/(n*pi) );

        sol_an = sol_an + Hconst .* Hexp .* Hsin;
    end

    figure(2);
    surf(t,x(1:20),sol_an(1:20,:));
    title('Analytical Solution','fontweight','b')
    ylabel('Distance along tongue (cm)')
    xlabel('Time (s)')
```

```
    zlabel('Temperature (∫C)')

    figure(3);

    TSS = (TH-TB)/L .* x + TB .* ones(size(x));

    plot(x(1:30),sol(1:30,40),'--', ...
        x(1:30),sol(1:30,60),'--',...
        x(1:30),sol(1:30,80),'--',...
        x(1:30),sol(1:30,101),'--', ...
        x(1:30),TSS(1:30),'-');
    line([0 3],[0 0],'Color','red');
    legend('t = 48s','t = 72s','t = 96s','t = 120s','steady state');
    xlabel('Distance along tongue (cm)')
    ylabel('Temperature (∫C)')

end

function [c,f,s] = pdefun(x,t,u_pdepe,DuDx)
global D
c = 1;
f = D*DuDx;
s = 0;
end
% -----
function u0 = icfun(x)
global T1
u0 = T1;
end
% -----
function [pl,ql,pr,qr] = bcfun(xl,ul,xr,ur,t)
global T1 T2
pl = ul - T2;
ql = 0;
pr = ur - T1;
qr = 0;
end
```

Non-homogeneous model

```
function BENG221Proj
close all
global D L T0 TL Q

D = 3.7E-4; % diffusion of heat in the tongue, cm^2/s
L = 10; % length of tongue, cm
T0 = -10;    % body temperature, Celcius
TL = 37;   % temperature of metal pole, Celcius
Q = -68/3600;
x = linspace(0,L,101);
t = linspace(0,600,101);    % simulation time, seconds

figure(1)
sol = pdepe(0,@pdefun,@icfun,@bcfun,x,t);
u_pdepe = sol(:,:,1);
surf(t,x,sol')
title('PDEPE Solution','fontweight','b')
```

```
ylabel('Distance along tongue (cm)')
xlabel('Time (s)')
zlabel('Temperature (∞C)')

figure(2)
ana = analytical(x,t,200);
surf(t,x,ana)
title('Analytical Solution','fontweight','b')
ylabel('Distance along tongue (cm)')
xlabel('Time (s)')
zlabel('Temperature (∞C)')

end

% -----
function [fxn_sum] = analytical(x,t,series)
global T0 TL L D Q
C = zeros(1, series);
fxn = zeros(length(x),length(t), series);
fxn_sum = zeros(length(x),length(t));
for j = 1:length(x)
    for k = 1:length(t)
        for n = 1:series
            lambda = n*pi/L;
            fxn(j,k,n) = 2*TL/(n*pi)*(1-(-1)^n)*sin(lambda*x(j))*exp(-
D*lambda^2*t(k)) + ...
                (2*Q*L^2/(D*n^3*pi^3)*(1-(-1)^n) + 2/(n*pi)*(T0-TL*(-
1)^2))...
                *sin(lambda*x(j))*(1-exp(-D*lambda^2*t(k)));
            fxn_sum(j,k) = fxn_sum(j,k) + fxn(j,k,n);
        end
    end
end
end
% -----
function [c,f,s] = pdefun(x,t,u_pdepe,DuDx)
global D Q
c = 1;
f = D*DuDx;
s = Q;
end
% -----
function u0 = icfun(x)
global TL
u0 = TL;
end
% -----
function [pl,ql,pr,qr] = bcfun(xl,ul,xr,ur,t)
global T0 TL
pl = ul - T0;
ql = 0;
pr = ur - TL;
qr = 0;
end
```