

# UCSD Department of Bioengineering

## BENG 221 – Mathematical Methods in Bioengineering

Fall 2016

**Class:** Lectures: Tue & Thu, 11:00am –12:20pm, PFBH 161  
Problem Solving Sessions: Fri, 11:00am-11:50am, PFBH 161

**Instructor:** Professor Gert Cauwenberghs  
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**URL:** <http://isn.ucsd.edu/courses/beng221>

**Reading:** See *Reading Materials* on the class web page

<b>Grading:</b>	Homework	40%
	Midterm	20%
	Problem Solving Project	40%

## Homework and Exam Policies

ALL parts of the homework must be turned-in and will be graded. All parts of exams must be answered unless otherwise stated. Both the exams and homework must be legible, show good use of English, and be organized. Points will be given or deducted for these qualities accordingly. Late homeworks will not be accepted. Any requests for re-grading must be made in writing to the TAs and will result in re-grading of the entire homework or exam, not just a specific problem, first by the TAs themselves and if following this a dispute remains, by the course instructor. Such re-grading could result in a higher or lower overall grade.

The Department of Bioengineering adheres to the UCSD Policy on Integrity of Scholarship. An excerpt of this Policy states that "Students are expected to complete the course in compliance with the instructor's standards. No student shall engage in any activity that involves attempting to receive a grade by means other than honest effort...." In other words, cheating of any kind whatsoever WILL NOT BE TOLERATED in any form in this course and will be punishable to the maximum extent possible as per university rules and policies. The full policy is available in the general catalog (e.g., at <http://www.ucsd.edu/catalog/AcadRegu.htm>). Group study and discussion of homework assignments is allowed, but the contributions of others should be clearly documented in writing in the homework. Students are allowed to discuss approaches to problem-solving and design, but are not allowed to obtain or provide written solutions. In cases of suspected academic dishonesty, the case will be referred to the appropriate Dean. Generally, the penalty for academic dishonesty is a failing grade at minimum, with other more severe academic penalties possible.

## **Problem Solving and Presentations**

The problem solving sessions offer an exercise in bioengineering problem formulation and solving, demonstrating creative thinking, effective teamwork, and clear presentation skills. In groups of two students you will:

- a. Formulate a problem in bioengineering in quantitative mathematical terms;
- b. Simplify the problem to arrive at an analytical solution, and interpret the solution in terms of the dependence on the parameters;
- c. Use numerical methods to arrive at an approximate solution to the full problem, and interpret any discrepancies between the numerical solution and the simplified analytical solution;
- d. Present the problem and solutions during one of the presentation sessions, and answer questions from peer students;
- e. Write and submit a report, presenting and documenting the problem and solutions.

Group assignments, and the scheduling of presentations, are made during the first week of classes. Each group presents once, for a total of two presentations each meeting time. Time your presentations not to exceed a total of 20 minutes, not including questions. Use the whiteboard for your presentations. Computer use is allowed for presentation of numerical results, but not for analytical results. The report is due one week after your presentation, and should include title, presenters' names, problem statement, simplified solution, numerical validation, conclusion, and references/credits. Use diagrams and graphs to document the problem and the solutions. The report should be submitted by e-mail to the TA in charge, in PDF format. Reports will be posted on the class web page as a service to future students taking the class searching for ideas and inspiration.

You may consult anyone on your problem statement and methods, but you must do the problem solving yourselves, working with your group companion. You must also give credit to all sources where these guided your problem statement or solutions. Where these sources are in the open literature, you must cite the publications in print or on-line.

Success, and enjoy!

**BENG 221**  
**Mathematical Methods in Bioengineering**  
**Fall 2016**

**CLASS SCHEDULE**

**START OF CLASSES**

- September 22 Introduction. Ordinary differential equations (ODEs), and initial and boundary conditions. Solution of homogeneous and inhomogeneous ODEs. Eigenvalue and eigenvector analysis, and multimode analysis. Linear time-invariant systems, impulse response, and transfer function.
- September 23 Tutorial and problem solving session. Introduction to Matlab for linear systems, ODEs and PDEs. Analytical and numerical techniques. Solution to example problems, using paper and pencil, and verified by numerical simulation.

**WEEK 1**

- September 27 Introduction to PDEs. One-dimensional heat equation, and its equivalents in electrical and chemical transport with applications to biomedical engineering. Flux through membranes. One-dimensional wave equation in an electrical transmission line, with open and short circuit termination. Finite difference PDE approximations.
- September 29 Solutions to PDEs over bounded and unbounded domains. Separation of variables. Boundary value problem and solution of the  $x$  dependent equation. Product solution of the PDEs with specified boundary conditions. Solutions over infinite domains using Fourier transforms.
- September 30 Problem solving session—student presentations.

**WEEK 2**

- October 4 Heat equation. Temperature, thermal energy, and flux. Diffusion of thermal energy, and boundary conditions on temperature and flux. Thermal equilibrium.
- October 6 Analytical solution to the inhomogeneous heat equation with space varying source and boundary conditions. Decomposition of the solution into a particular steady-state solution, and Fourier series eigenmodes of the homogeneous solution. Fourier series expansions of initial conditions revisited.
- October 7 Problem solving session—student presentations.

### **WEEK 3**

- October 11 Analytical solution to inhomogeneous PDEs using Green's functions. Relationship to impulse response of linear time and space invariant systems. Green's solution to the inhomogeneous heat equation with time-varying and space-varying heat source.
- October 13 Extended Green's solution to the inhomogeneous heat equation with time-varying value and flux boundary conditions. Solutions on infinite domains using Laplace and Fourier transforms.
- October 14 Problem solving session—student presentations.

### **WEEK 4**

- October 18 Heat and diffusion equation in space and time. Separation of variables for cartesian separable boundary conditions. Bounded, infinite, and semi-infinite domains.
- October 20 Review and sample midterm
- October 21 Problem solving session—student presentations.

### **WEEK 5**

- October 25 Brownian motion, and diffusion. Theory for one-dimensional displacement. Scaling of diffusion in space and time. Viscous flow, and Reynolds number.
- October 27 Midterm.
- October 28 Problem solving session—student presentations.

### **WEEK 6**

- November 1 Review of vector calculus. Gradients, divergence, curl, and Laplacian. Transformation between Cartesian, cylindrical, and radial coordinates. Fields and potentials. Divergence theorem, and Stokes theorem.
- November 3 Diffusion in polar and cylindrical coordinates. Analytical solution using Bessel functions. Value and flux boundary conditions in terms of roots and extrema of Bessel functions. Fourier-Bessel series expansion of initial conditions.

November 4 Problem solving session—student presentations.

### **WEEK 7**

November 8 Gradient descent optimization. First-order and higher-order methods for null-finding and function minimization. Introduction to linear and nonlinear control systems in bioengineering.

November 10 Numerical solution to PDEs using finite element methods. Orthogonal, non-orthogonal, and triangular elements. Practical applications in bioengineering.

November 11 Veterans Day

### **WEEK 8**

November 15 Electrostatics. Coulomb's law. Electric field and potential. Work and moving charge. Equivalence of surface/field product and enclosed charge. Gauss's law, and Poisson's and Laplace's equation. Electric field near and in conductors. Dielectric phenomena. Capacitance.

November 17 Introduction to electromagnetism using Maxwell's equations. Wave propagation in homogeneous and inhomogeneous media. Far and near field. RF telemetry and power delivery. Tissue absorption.

November 18 Problem solving session—student presentations.

### **WEEK 9**

November 22 The one dimensional wave equation. The vibrating string as a boundary value problem. Vibrating string clamped at both ends. Standing waves and summation of traveling waves.

November 24 Thanksgiving

### **WEEK 10**

November 29 Sound. Transmission of waves in gases. Pressure variation in a sound wave.

December 1 Course review.

December 2 Problem solving session—student presentations. Final project reports due.