

# Modulators of Spike Timing-Dependent Plasticity

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## 6 **Abstract**

7 Spike Timing-Dependent Plasticity (STDP) is a Hebbian form of learning  
8 that captures the temporal relationship between pre- and postsynaptic  
9 spikes. Recent studies have uncovered that the relative concentration of  
10 neuromodulators, in addition to the timing of pre- and postsynaptic spikes,  
11 can affect the polarity of STDP. In this paper we sought to modify existing  
12 STDP rule implementing neural networks to account for experimental  
13 findings and observe the corresponding changes in the synaptic weights.  
14 Initial conclusions support the notion that minimal changes in  $A_P$  or  $A_D$   
15 have little effect on rate of synaptic changes however further work must be  
16 done to verify or refute these findings.

## 17 **1 Introduction**

18 Long-term potentiation (LTP) and long-term depression (LTD) are well known  
19 mechanisms for cortical synaptic plasticity. Traditionally, the given outcome, LTP or LTD,  
20 was determined by varying the presynaptic firing rate or the postsynaptic membrane  
21 potential during conditioning. With the discovery that when a presynaptic spike precedes a  
22 post synaptic spike by several seconds LTP results and when a postsynaptic spike precedes a  
23 postsynaptic spike by several seconds LTD results, the temporal relationship between the  
24 presynaptic and postsynaptic spike became an important factor in the study of synaptic  
25 plasticity.  
26

27 The discovery of spike timing-dependent plasticity [1] [2] has led to many  
28 developments in the study of neural networks. It was found that oscillations greatly facilitate  
29 STDP's ability to detect repeating patterns. Specifically, Masquelier et al.[3] were able to  
30 demonstrate, in simulations, how a single neuron equipped with STDP robustly detects a  
31 pattern of input currents automatically encoded in the phases of a subset of its afferents, and  
32 repeating at random intervals. In addition it was demonstrated that a model network of  
33 cortical spiking neurons modulated with the neuromodulator dopamine (DA) could solve the  
34 distal reward problem [4].

35 Surprisingly, much of the recent progress of STDP involves implementing the STDP  
36 rule into neural networks, and not on the STDP rule. Only recently has significant progress  
37 been made to control the rules of STDP, specifically the given outcome and the time window  
38 for coincidence detection. A recent study found that the relative activation of  
39 neuromodulator receptors coupled to adenylyl cyclase (AC) and phospholipase C (PLC)  
40 signaling cascades can override the order of the timing of the pre- and postsynaptic spikes  
41 [5]. In addition, they found that these modulators could also increase the window of  
42 coincidence detection and effectively "prime" neuron synapses into plastic states.

43 The aim of this project was to attempt to modify existing STDP rule implementing  
 44 neural networks to account for experimental findings on neuromodulators. Since the  
 45 experimental findings were found in an in vitro system, and hence their release and uptake  
 46 were tightly controlled, biologically relevant concentration and kinetics could not be  
 47 obtained. As a result, the focus of this project was to model the possible changes that  
 48 addition of particular neuromodulators can have on the system.  
 49

## 50 2 Methods

### 51 2.2 Leaky Integrate and Fire Neurons

52  
 53 In single neuron or simple neuronal networks, conductance-based models, such as  
 54 the Hodgkin-Huxley (1952) and Morris-Lecar (1981) models, can be useful because such  
 55 models contains variables and parameters that have well-defined biophysical meanings that  
 56 can be measured experimentally [6]. However when neuronal network contain many  
 57 constituents, the number of connections and the vast amount of differential equations that  
 58 must be solved can become computationally taxing. Furthermore, such models, which  
 59 contain detailed accounts of axon potential generation and conductance, often obtain  
 60 parameters from many neurons which are averaged to produce a value.  
 61  
 62

63 For our present purposes, it is thus convenient to use a simple model which  
 64 faithfully reproduces the most important neurcomputational features of the neuron (voltage  
 65 threshold, reset, membrane potential) and omit some of the more taxing features (detailed  
 66 action potential generation and propagation).  
 67

68 Leaky Integrate-and-fire neurons (LIF neuron) are the simplest possible neural  
 69 model. In this model, complicated conductances responsible for action potential generation  
 70 have been omitted and instead the spiking mechanism is replaced by a potential threshold,  
 71  $v_{thres}$  [7]. When  $v(t_1) = v_{thres}$  has been reached, a spike is emitted at  $t_1$  and the potential is  
 72 reset to zero. In this model, the membrane potential follows the differential equation:  
 73  
 74

$$75 \quad C \frac{dv}{dt} = -\frac{v}{R} + I, t > 0$$

76 where  $I = I(t)$  is a stimulation current and  $v(0) = 0$  is an initial condition. Without input  
 77 current and at steady state, the membrane potential is equal to the resting membrane  
 78 potential value of the model, which in this case is zero. If an input current is present, the  
 79 synaptic inputs to a LIF neuron can be simulated as a conductance change:  
 80

$$81 \quad I_{syn}(t) = \sum_{n=1}^{n_{ex}} g_{ex}(t, t_{ex,n})(v - v_{ex}) + \sum_{n=1}^{n_{in}} g_{in}(t - t_{in,n})(v - v_{in})$$

82 Due to their relative simplicity, LIF neurons can be easily expanded to LIF  
 83 networks. LIF neurons in networks operate according to a specific order: (1) check for an  
 84 input spike at the current time,  $t$ , and for networks spikes from previous time,  $t-dt$ ; (2) update  
 85 conductances based on the input spikes and network spikes recorded in (1); (3) update  
 86 potentials, recorded spikes, and return to (1).  
 87

88 Finally, LIF networks can be modeled with the STDP rule resulting in plastic  
 89 network. The network equations for the weight increases and decreases are presented below  
 90 which occur when the LIF network order (see above) reports that cell  $k$  fires in the interval  
 91  $[jdt, (j+1)dt)$ . In order to limit the maximum weight gain, the potentiation of the presynaptic  
 92 weight is hard bounded by  $(W_{max} - W_{k,k_{pre}}^j)$  and the depression of the postsynaptic weights

93 is hard bounded by  $W_{k_{post},k}^j$ .

94

$$W_{k,k_{pre}}^{j+1} = W_{k,k_{pre}}^j + A_P \exp\left(\frac{T_{k_{pre}} - (j+1)dt}{\tau_P}\right) (W_{max} - W_{k,k_{pre}}^j)$$

95

$$W_{k_{post},k}^{j+1} = W_{k_{post},k}^j + A_D \exp\left(\frac{T_{k_{post}} - (j+1)dt}{\tau_D}\right) W_{k_{post},k}^j$$

96

97

## 98 2.2 Neuromodulator sources and kinetics

99

100 In their paper, Seol [5] identified isoproterenol (Iso) and 4-[N-(3-Chlorophenyl)  
101 carbamoyloxy]-2-butynyltrimethylammonium chloride (McN) as the neuromodulators that  
102 (McN) and that induced associative LTP or LTD respectively. Iso is an agonist of  $\beta$ -  
103 adrenergic receptors coupled to the AC cascade and Mcn is a muscarinic cholinergic  
104 receptor M1 agonist coupled to the PLC cascade.

105

106 In order to model their effects, the equations used by Izhikevich (2007) [4] to model  
107 the effects of dopamine were adapted to account for Iso:

108

$$109 \dot{P}_{AC} = -\frac{P_{AC}}{\tau_{P_{AC}}} + STDP(t_{post} - t_{pre})\delta(t - t_{\frac{pre}{post}})$$

110

$$111 \dot{S}_P = P_{AC}A_{Iso}$$

112

$$A_{Iso} = \frac{B_{Iso}}{\tau_{Iso}} + ISO(t)$$

113

114 Where  $P_{AC}$  refers to the activation of an enzyme important for plasticity;  $A_{ISO}$  describes the  
extracellular concentration of Iso;  $\delta(t - t_{\frac{pre}{post}})$  is the Dirac delta function that step-increases

115 the variable  $P_{AC}$ ;  $S_P$  is the potentiating contribution to the synaptic weight;  $\tau_{ISO}$  refers to the  
116 time constant of Iso uptake or its diffusion away from the receptors; and  $ISO(t)$  models the  
117 source of Iso. Likewise, the equations for the effects of McN are below with parameters  
118 exactly as for Iso but adapted for McN.

119

$$120 \dot{D}_{PLC} = -\frac{D_{PLC}}{\tau_{D_{PLC}}} + STDP(t_{post} - t_{pre})\delta(t - t_{\frac{pre}{post}})$$

121

$$122 \dot{S}_D = D_{PLC}B_{McN}$$

123

$$B_{McN} = \frac{B_{McN}}{\tau_{McN}} + MCN(t)$$

124

## 125 2.2 Weight increase and decreases

126

127 The experiments described by Seol [5] were conducted in an in vitro system  
128 consisting of layer IV to layer II/III inputs in the rat visual cortex. Although they modulated  
129 STDP, Iso and McN represent foreign neuromodulators and hence the values for their  
130 concentration at the synapse, their uptake rates, and production rates have not yet been (or  
131 simply cannot be) obtained. The values for dopamine found in Izhikevich's paper could not  
132 be used due to fundamentally different situations.

133

134 In Izhikevich's paper [4], dopamine served as an extracellular reward used to  
identify the correct source of STDP and reinforce the correct pre- post synaptic connection.

135 In our present paper, neuromodulators directly increase or decrease the weights at the  
 136 synapse and do not serve to pick out a particular synaptic connection. As a result, until  
 137 biological relevant neuromodulators are identified in vivo, as well as their kinetics  
 138 measured, we will proceed in our experiment to absorb the values of  $S_P$  into  $A_P$  and the  
 139 values of  $S_D$  into  $A_D$ .  
 140

$$W_{k,k_{pre}}^{j+1} = W_{k,k_{pre}}^j + A_P \exp\left(\frac{T_{k_{pre}} - (j+1)dt}{\tau_P}\right) (W_{max} - W_{k,k_{pre}}^j)$$

141  
 142

$$W_{k_{post},k}^{j+1} = W_{k_{post},k}^j + A_D \exp\left(\frac{T_{k_{post}} - (j+1)dt}{\tau_D}\right) W_{k_{post},k}^j$$

143

## 144 **3 Results**

145

### 146 **3.1 Establishment of controls**

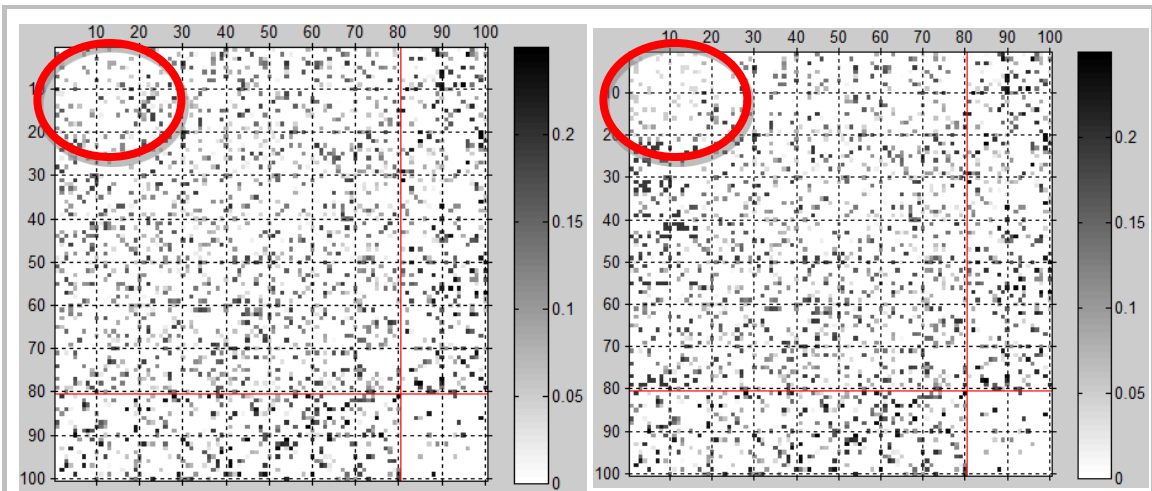
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148 A neural network (with the STDP rule) [8] of 100 neurons, 80 excitatory cells and  
 149 20 inhibitory cells, was used to establish a baseline value (the code is found in [8]). The  
 150 matrix of weights between excitatory-excitatory, excitatory-inhibitory, and inhibitory-  
 151 excitatory cells was set at 25% connectivity whereas the matrix of weights of inhibitory-  
 152 inhibitory cells was set at 5%. A simultaneous input was delivered to the excitatory  
 153 conductance's of the first 16 excitatory cells causing them to fire synchronously just before  
 154 the simulation.

155

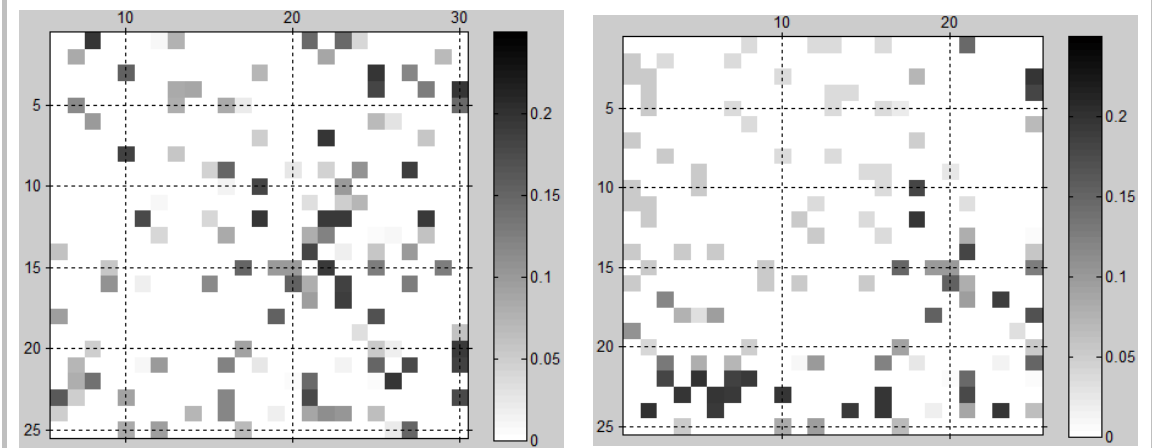
156 A simulation with  $A_P = 0.1$ ,  $A_D = 0.3$ , with a time course of two seconds was run  
 157 (Fig 1). A plot of synaptic weights at the beginning of the simulation (Fig 1A, 1C) and after  
 158 (Fig 1B, 1D) clearly show that following the simulation, the synaptic weights of the first 16  
 159 excitatory neurons show decreased strength, evident in the brighter plots found in Fig 1B,  
 160 1D compared to Fig 1A, 1C. As the first 16 neurons were simultaneously driven by an input  
 161 stimulus, the synaptic connections between neighboring neurons were expected to degrade  
 162 over time as the external input would override the minimal contribution of the synaptic  
 163 weights.  
 164

165 The STDP rule in the neural network also causes synaptic weights to either increase  
 166 to a maximal value (in this case, 0.2) or decrease to zero over time (Fig 1E). The value of  $A_D$   
 167 = 0.3 is represented by the top blue dots; the value of  $A_P = 0.1$  is represented by the bottom  
 168 blue dots. The greater value for  $A_D$  compared to  $A_P$  results in synaptic weights decreasing  
 169 much quicker to 0 then increasing to 0.2 Fig 1F illustrates the initial random distribution of  
 170 possible synaptic weights before the simulation has begun (red trace) and after the  
 171 simulation has completed (green trace); as time passes, the fraction of neurons at synaptic  
 172 values of 0 or 0.2 increases.  
 173



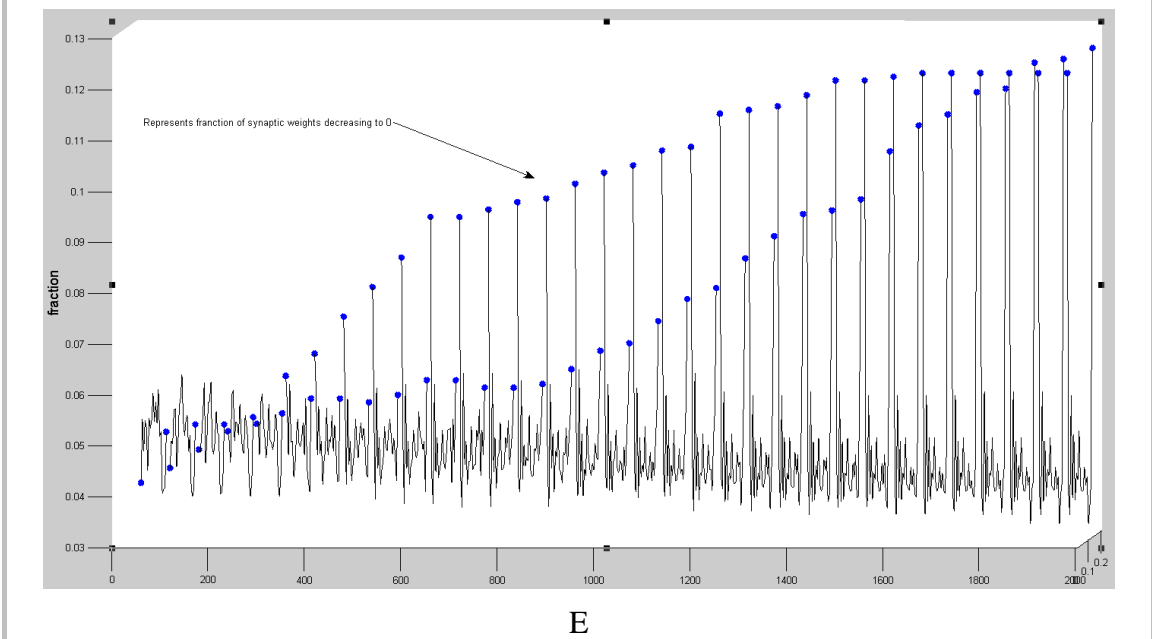
A

B

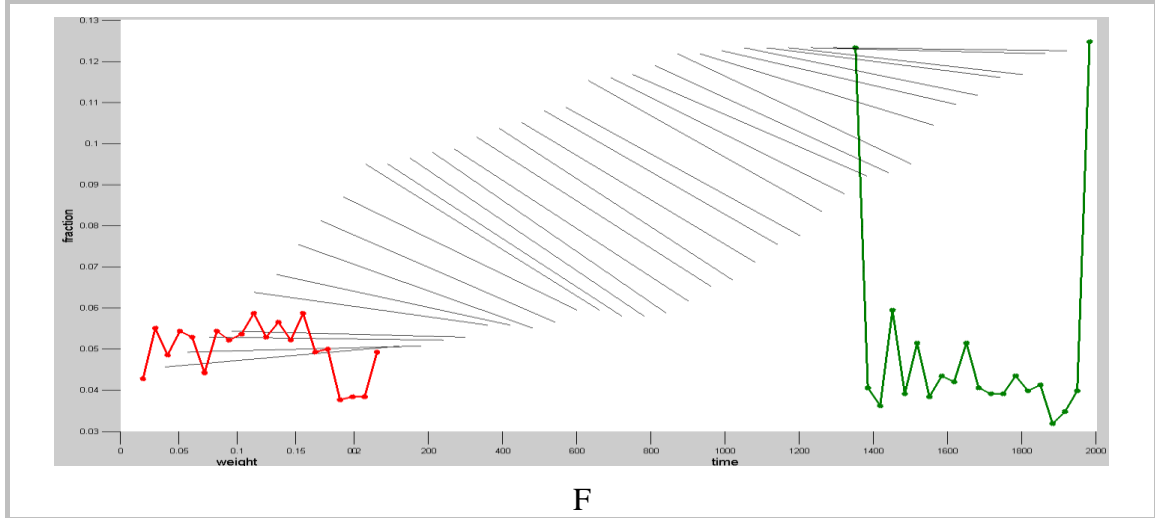


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E

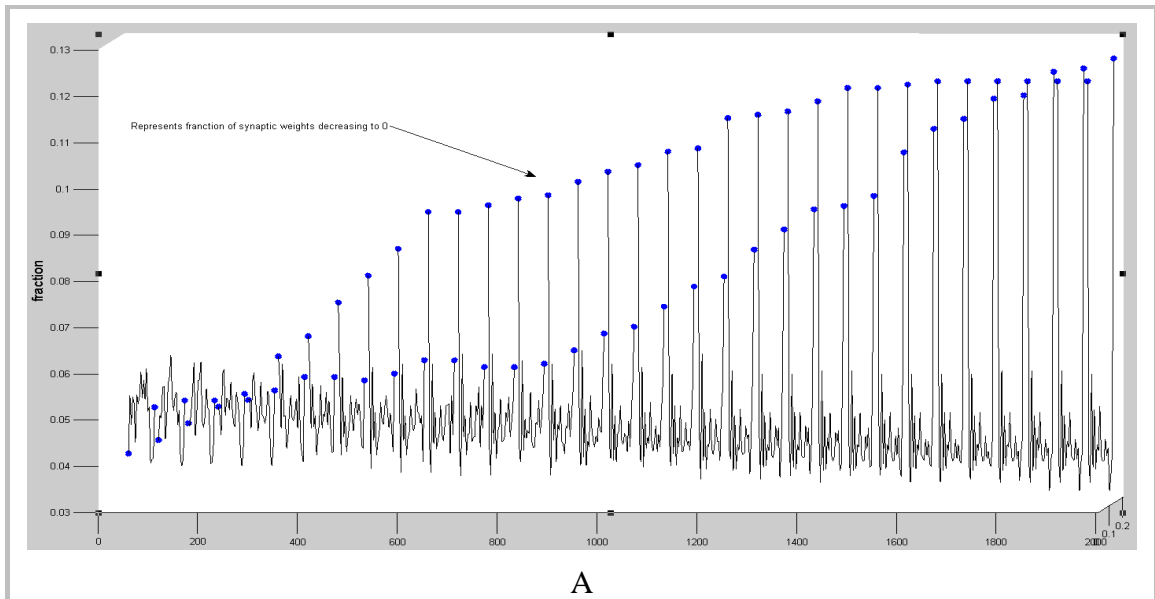


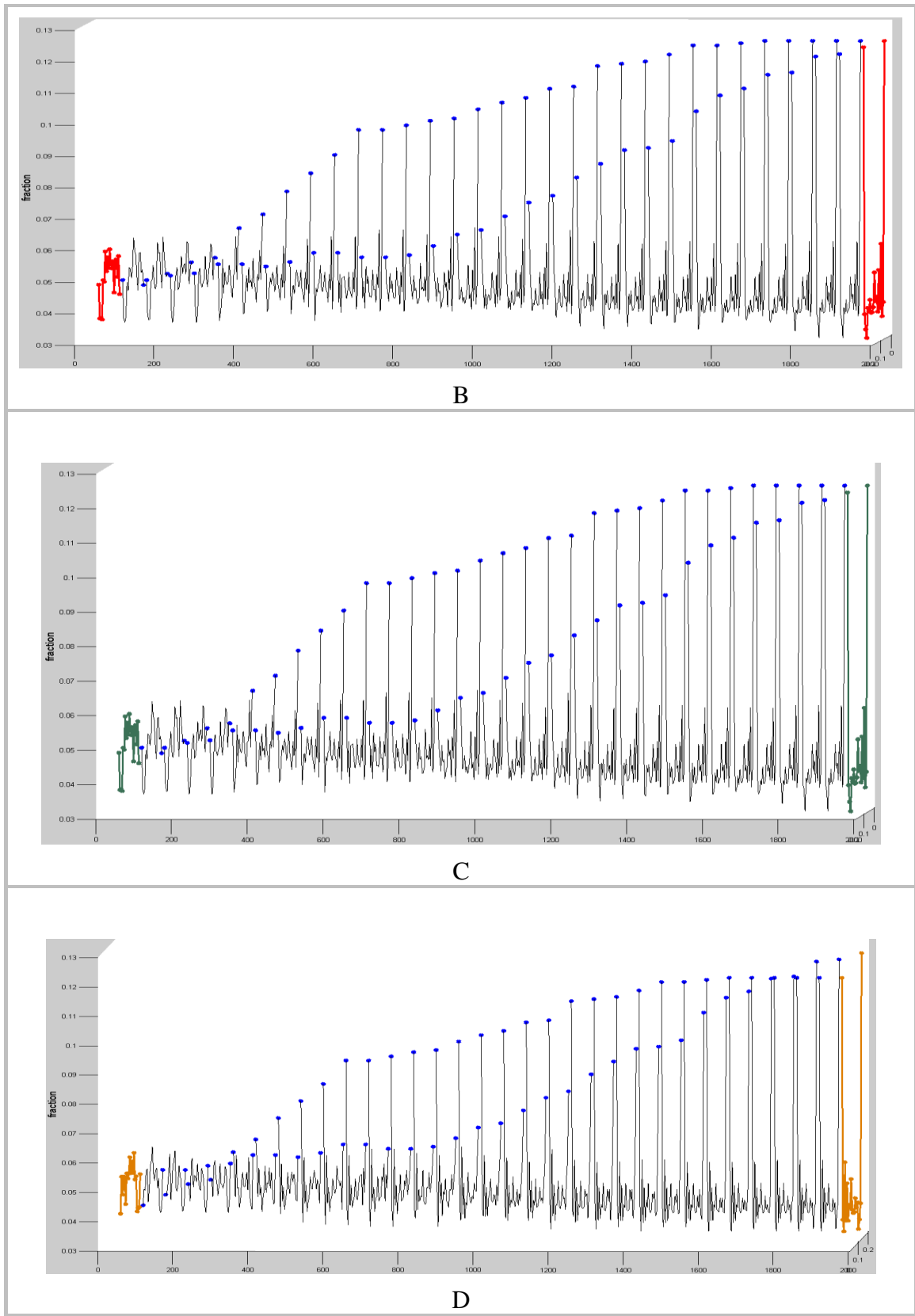
174 Figure 1: Baseline values. (A) A plot of synaptic weights before the simulation;  $A_P = 0.1$ ,  $A_D = 0.3$ , total time = 2000msec; x and y axis denote neuron number; red lines differentiate the  
 175 weight matrices; the red circle encloses the first 16 excitatory cells (B) A plot of synaptic  
 176 weights after the simulation. (C) Blow up of the 16 excitatory cells in (A) before the  
 177 simulation. (D) Blow up of the 16 excitatory cells in (B) following the simulation. (E) Plot  
 178 of the fraction of synaptic weights over time (in msec); top blue dots represents fraction of  
 179 synaptic weights at 0; bottom blue dots represents fraction of synaptic weights at 0.2 (F) The  
 180 red trace is distribution of synaptic weights at the beginning of the simulation. The green  
 181 trace is the distribution of synaptic weights at the end of the simulation.  
 182

183  
 184

### 3.2 Neuromodulators and increase in synaptic weights

185 In order to replicate the effects of Iso neuromodulator addition, the values for  $A_P$   
 186 were increased in steps of 0.1 while the value of  $A_D$  remained constant at 0.3 (Fig 2A-D).  
 187 Surprisingly all changes in  $A_P$  resulted in identical plots which would either suggest that  
 188 small variations in  $A_P$  would have no effect on the rate of synaptic weight changes or the  
 189 code is unresponsive to the changes.  
 190





191 Figure 2: Variations in  $A_p$ ; unless noted, top blue dots represent fraction of synaptic weights

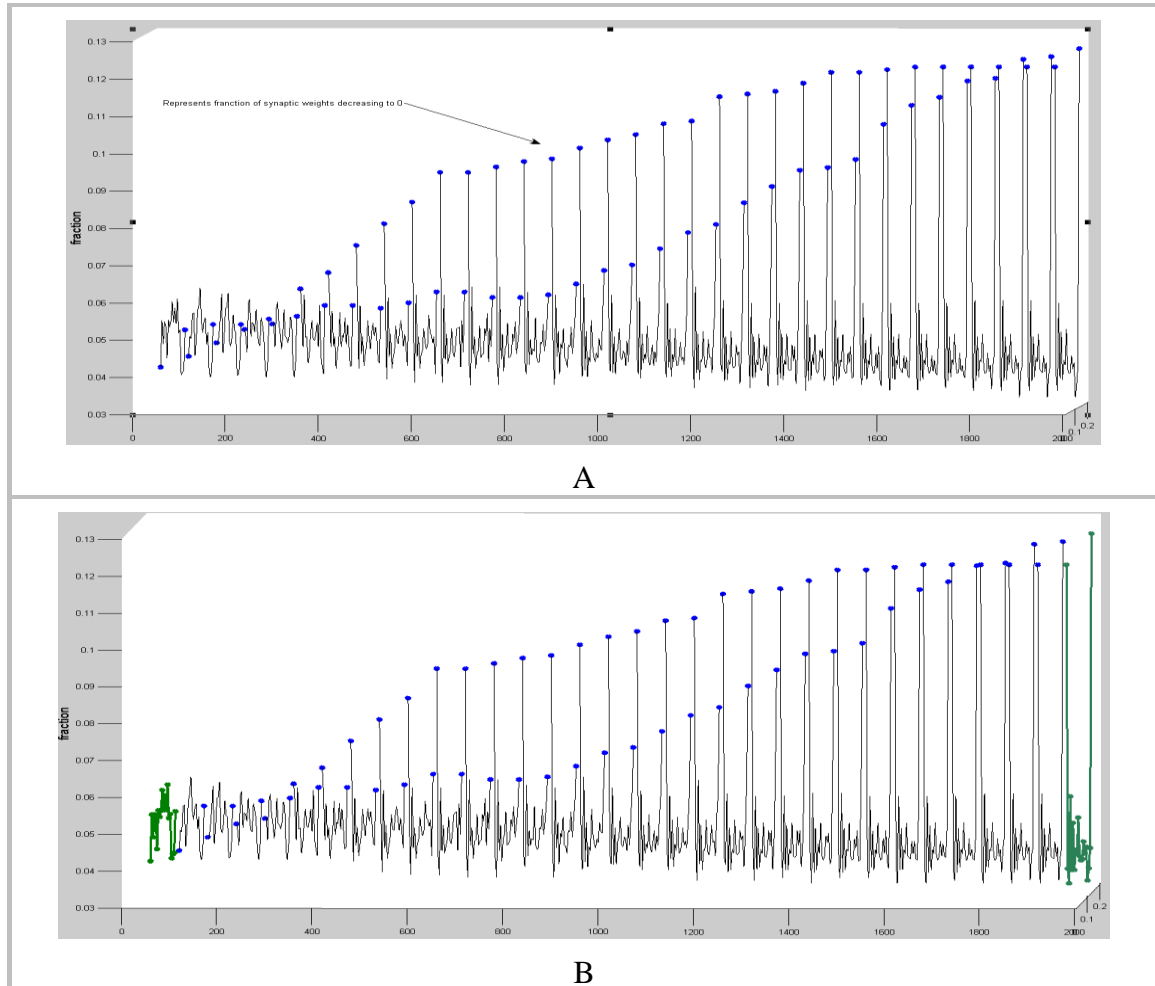
192 at 0 (A)  $A_P = 0.1$ ,  $A_D = 0.3$ , total time = 2000msec (B)  $A_P = 0.2$ ,  $A_D = 0.3$ , total time =  
193 2000msec (C)  $A_P = 0.3$ ,  $A_D = 0.3$ , total time = 2000msec (D)  $A_P = 0.4$ ,  $A_D = 0.3$ , total time =  
194 2000msec

195  
196

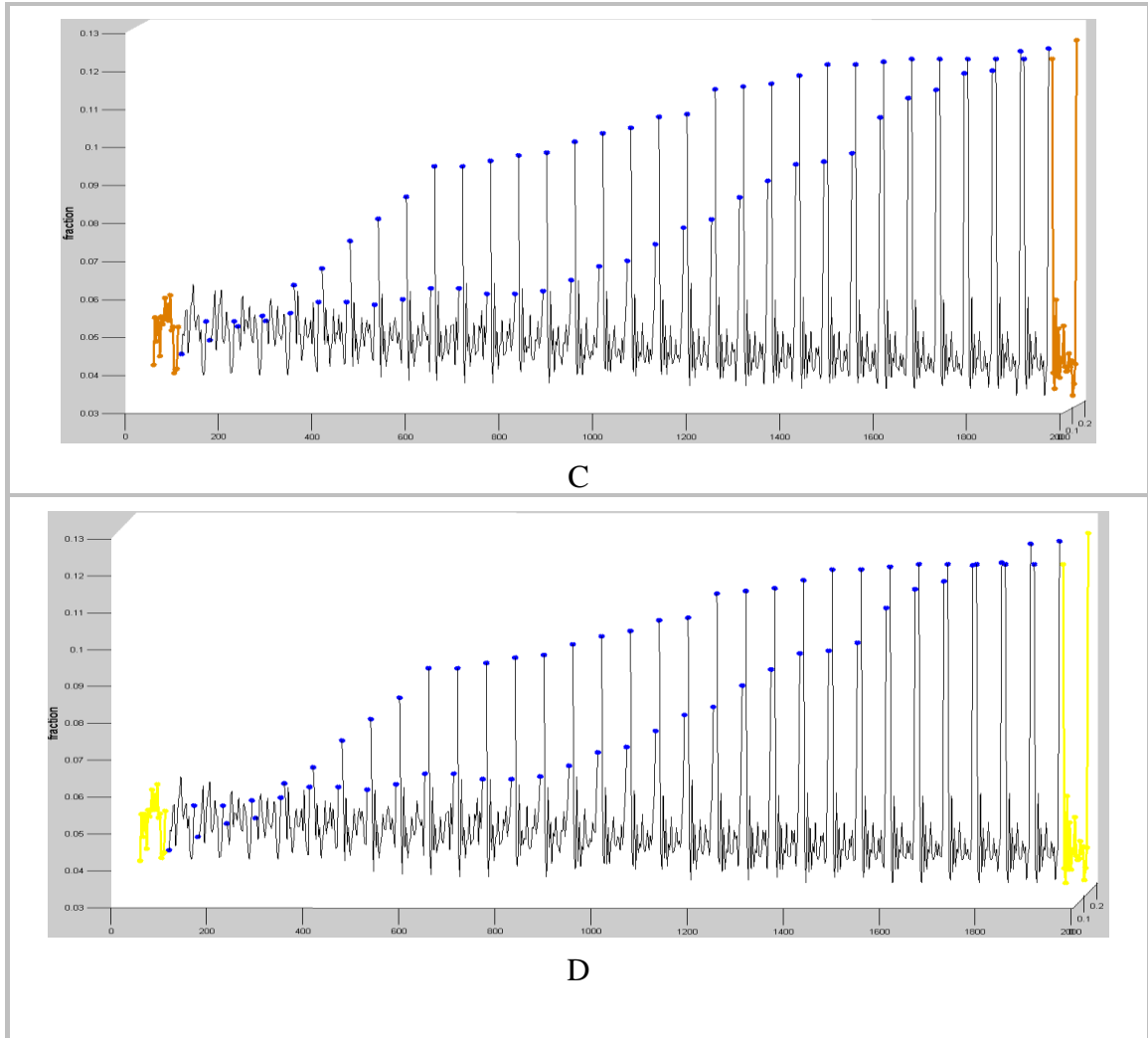
### 3.3 Neuromodulators that decrease in synaptic weights

197 In order to replicate the effects of Iso neuromodulator addition, the values for  $A_D$   
198 were increased in steps of 0.1 while the value of  $A_P$  remained constant at 0.1 (Fig 3A-D).  
199 Interestingly the results found in changing  $A_P$  while holding  $A_D$  fixed appeared identical to  
200 the ones obtained by changing  $A_D$  while holding  $A_P$  fixed.

201







202 Figure 3: Variations in  $A_D$  (A)  $A_P = 0.1$ ,  $A_D = 0.3$ , total time = 2000msec (B)  $A_P = 0.1$ ,  $A_D =$   
 203  $0.4$ , total time = 2000msec (C)  $A_P = 0.1$ ,  $A_D = 0.1$ , total time = 2000msec (D)  $A_P = 0.1$ ,  $A_D =$   
 204  $0.5$ , total time = 2000msec

205

## 206 4 Conclusion

207 The similarity in the results found in modifying  $A_P$  while holding  $A_D$  fixed and  
 208 reverse situation would lead to two different conclusions: either the network is insensitive to  
 209 minimal changes of  $A_P$  or  $A_D$  or simply the code was hardcoded to account for only a  
 210 particular value of  $A_P$  or  $A_D$ . Although the results would agree with the latter, extensive  
 211 review of the code would lead to no indication that the values of  $A_P$  or  $A_D$  are hard coded in  
 212 the system.

213 Recent studies also support the conclusion that neural networks supporting the  
 214 STDP rule are incredibly efficient at learning. Masquelier et al.[3] have found that in the  
 215 context of learning with phase-of-firing coding (PoFC); learning was possible even when  
 216 only ~10% of afferents exhibited PoFC. Further testing on the code by the owners[8] have  
 217 shown that the STDP rule applied to the random networks provides it the ability of pattern  
 218 completion.

219

220 Ultimately, additional testing in other models [9] is needed to determine whether  
 221 STDP neural networks are insensitive to small changes in  $A_P$  or  $A_D$ . Also, consultation of the

222 literature for larger values of  $A_P$  or  $A_D$  would be informative though on a first pass, no such  
223 values were discovered.

224

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226 Thanks to Jeff Bush for his computational input and feedback as well as support and also to  
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229 providing an accompanying Matlab code.

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