

Determination of the Correlation between Conductance and Architecture of Disordered β -amyloid Channels

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Abstract

It has been hypothesized that the insertion of cation preferential channels into neuronal cell membranes disrupts homeostasis and function leading to Alzheimer's disease. These channels have been shown to be composed of variable numbers of subunits. The nonhomogenous configuration of these subunits in the channels has introduced difficulties in defining set values of ion conductance. Here we model β -amyloid ($A\beta$) channels as a cylinder spanning the cell membrane of variable radius dependent on subunit configuration. Various subunit motifs were examined and the expected conductance for each configuration determined. The theoretically determined conductance was then compared to experimentally observed values and a new mechanism for pore growth through successive subunit addition is proposed. This work will allow for the introduction of these channels into electrodynamic models of neurons to illuminate the effects of channel insertion into the cell membrane and expand our understanding of neurodegenerative diseases.

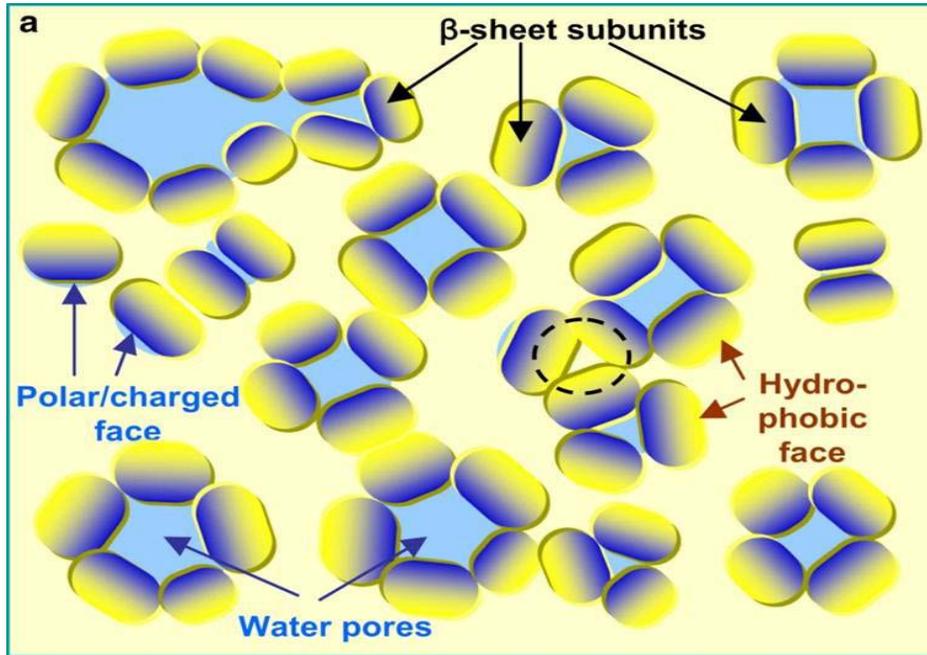
1 Introduction

Alzheimer's disease (AD) is a class of protein misfolding disease resulting in neurotoxicity and neurodegeneration. AD is characterized by the formation of fibrils by β -amyloid ($A\beta$) in the brain, the 1-42 fragment of the Amyloid Precursor Protein. Fibril formation was initially thought to lead to acute cell death, but mounting evidence suggests that these fibrils do not contribute to toxicity[1, 2]. The basic mechanism of physiological pathology is a loss of ionic homeostasis facilitated by oligomers of $A\beta$, inducing oxidative stress on the cells. Oxidative stress apoptosis results from persistent cellular destabilization by the oligomers[3].

The mechanism of oligomeric $A\beta$ toxicity is poorly understood, with receptor binding, lipid fluidity and thinning, and activation of signaling factors all leading to the accumulation of oxidative reagents, being suggested as possible causes. Recent evidence suggests that $A\beta$ can form channels that are cation preferential[4, 5]. This results in a direct path for loss of ionic homeostasis as ions can enter neurons through these pores. Reconstitution of $A\beta$ monomers into bilayers results in ionic conductance shortly after the bilayer is deposited over a pored[4]. Due to the disordered nature of the proteins resulting in varying oligomer lengths, the conductance observed varies widely, this is thought to be primarily due to the inconsistent diameter size created by having pores composed of varying numbers of monomers and subunits (Figure 1). AFM images of reconstituted $A\beta$ show the presence channel like structures with varied subunit configurations ranging from 3 to 6 subunits[5]. Molecular dynamics simulations also indicate that the oligomers can form stable pore structures of different sizes[6, 7]. However, little is known about the mechanism of channel formation and the dynamics of conformation change.

42 This work examines how the conductances observed experimentally are related pore structure.
 43 This is accomplished through modeling the structure of protein pores with different numbers of
 44 oligomer subunits and also varying the number of A β monomers in each subunit. Pore diameters
 45 and conductances were subsequently calculated for each conformation. The selectivity of the pores
 46 also widely varies in electrical recording experiments and can result in non-specific ionic flux. To
 47 better understand ion selectivity of the pores, the electric fields for different pore geometries were
 48 calculated. This work provides better understanding of how A β channels form and the properties
 49 associated with them. This will be important for future models of neuronal activity simulating the
 50 conditions present in AD and how it impacts cellular dynamics and connectivity.

51



52
 53 **Figure 1:** Molecular dynamics (MD) simulations suggest that A β channels are dynamic in nature
 54 and non-uniform in size and shape. Modified from Jang et al. (2009) [6]

55

56 2 Methods

57 2.1 Model geometry

58 The complex, dynamic, and undefined geometry of A β pores present a challenge for effective
 59 modeling. The system was thus modeled under a simplified geometry to allow for comparison
 60 between configurations. Each pore subunit was assumed to be a cylinder composed of equal
 61 numbers of A β monomers. The diameter of each subunit was assumed to be equal to the width of
 62 the monomers within and the length equal to that of a cell membrane, 5 nm. Each monomer was
 63 assumed to be a cylinder with a diameter of 1 nm based on the findings of Stine et al. [8]. The pore
 64 was formed by arranging the subunits such that the subunit centers were positioned at the vertices
 65 of a regular polygon. The conducting pore radius could thus be calculated from the difference in
 66 the area of the polygon and the bound subunit fraction based on the equation:

67

$$A_{pore} = \frac{n^2 N}{4 \tan\left(\frac{\pi}{N}\right)} - \left(\frac{N-2}{2}\right) \frac{\pi n^2}{4}$$

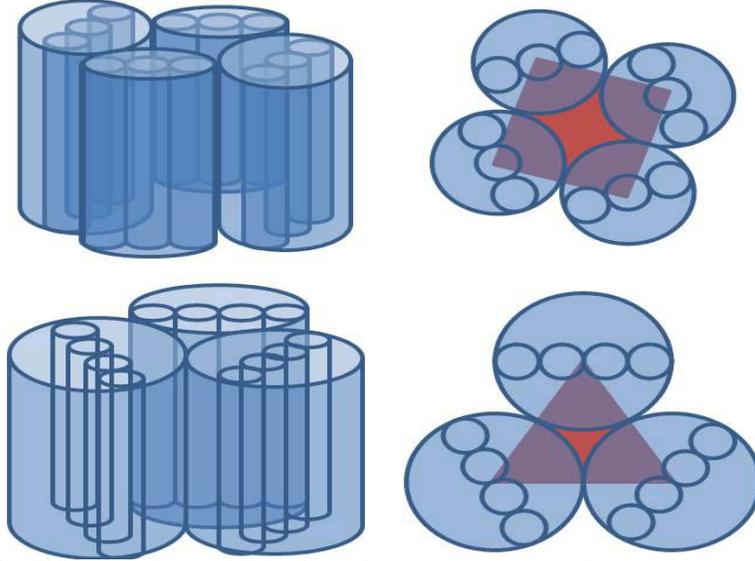
68

$$R_{pore} = \sqrt{\frac{A_{pore}}{\pi}}$$

69

70 where N is the number of subunits and n is the number of monomers per subunit.

71



72

73 **Figure 1:** Simplified geometry for various β -amyloid pore configurations. Left: Pores were
 74 formed by the aggregation of cylindrical subunits comprised of equal numbers of subunits. Right:
 75 The conducting area of the pore (red) is calculated from the difference in area between the
 76 polygon and bound subunit fraction (overlap of blue and red).

77

78 2.2 Conductance modeling

79 The equations derived by Tagliazucchi et al. [9] for charged conducting pores was modified to fit
 80 the parameters of the $A\beta$ model. Two pore scenarios were modeled: 1) Charge residing solely
 81 within the pore, 2) Charge extending onto the surrounding membrane.

82 Scenario I: Charge residing exclusively within the pore

83 The resistivity of the pore is calculated by the equation:

84

$$\Omega = \frac{1}{2R_{pore}\sigma_s} + \frac{1}{\pi} \frac{C_{salt}L}{\rho_g\sigma_s R_{pore}}$$

85

86 where C_{salt} is the solution concentration, σ_s the solution resistivity, ρ_g the surface charge density,
 87 and L the pore length.

88 Scenario II: Charge extending onto surrounding lipid bilayer

89

$$\Omega = \frac{1}{\pi\sigma_s} \left[\frac{2C_{salt}}{\rho_g} \left(\ln \left(R_{pore} + \frac{1}{2} \frac{\rho_g}{C_{salt}} \right) - \ln(R_{pore}) \right) \right] + \frac{1}{\pi} \frac{C_{salt}L}{\rho_g\sigma_s R_{pore}} + \left(\frac{1}{2} - \frac{1}{\pi} \right) \frac{1}{\sigma_s R_{pore}}$$

90

91 For both scenarios, the system was modeled in 150 mM KCl solution with a solution resistivity of
 92 1.5 S/m [10]. Based on the primary structure of A β , each monomer has a charge of -3 at
 93 physiological pH. The surface charge density of the pore is thus calculated as:

94

$$\rho_g = \frac{3Nn}{2\pi R_{pore}L}$$

95

96 The conductance of the pore, calculated as the inverse of resistivity, for each scenario was
 97 determined for three to ten subunit pores composed of one to six monomers per subunit.

98

99 2.3 Electric field modeling

100 In addition to modeling the conductance through the pore, the electric field within the pore was
 101 modeled. Point charges, corresponding to the overall charge of each subunit, were spatially
 102 distributed around the pore at the vertices of the bounding polygon. The resultant electric field at
 103 every point within the pore, resulting from the superposition of the contribution from each
 104 individual point charge, was calculated via Coulomb's law:

105

$$E = \sum_{i=1}^{n_Q} \frac{1}{4\pi\epsilon\epsilon_0} \frac{Q_i}{r_i^2} \hat{r}_i$$

106

107 3 Results

108 3.1 Pore Radius

109 Using the model discussed in section 2.1, varying numbers of subunits and monomers were
 110 analyzed and the radius of each geometry was calculated in Matlab (Table 1). Increasing the
 111 number of subunits and monomers results in increased pore radius.

112

113 **Table 1: Radius (nm) of Different Pore Geometries.** Cells highlighted in blue correspond with
 114 those in Table 2 (to be discussed later)

Radius		Number of Monomers					
		1	2	3	4	5	6
Number of Subunits	3	0.11	0.23	0.34	0.45	0.57	0.68
	4	0.26	0.52	0.78	1.05	1.31	1.57
	5	0.42	0.83	1.25	1.66	2.08	2.49
	6	0.57	1.14	1.72	2.29	2.86	3.43
	7	0.73	1.46	2.19	2.92	3.65	4.38
	8	0.89	1.77	2.66	3.55	4.44	5.32
	9	1.05	2.09	3.14	4.18	5.23	6.27
	10	1.20	2.41	3.61	4.82	6.02	7.22

115

116 **3.2 Pore Conductances**

117 The conductances calculated for scenario I (Table 2) show that larger pores are more conducting
 118 than smaller pores, as expected. The blue highlighted cells in Table 2 demonstrate the difficulty in
 119 calculating pore diameter from observed conductances (Figure 4) as multiple subunit
 120 conformational motifs (different number of monomers and subunits) result in the same
 121 conductance value. From examination of Table 1, this is not strictly driven by radius as the radii
 122 of these pores are not the same, indicating that the net charge, which is determined by the total
 123 number of monomers, also contributes to the conductance.

124

125 **Table 2: Conductance (nS) of Aβ pore for Scenario I.** Cells highlighted in blue demonstrate the
 126 difficulty in correlating an experimentally derived conductance with a pore diameter – different
 127 subunit motifs produce identical conductance.

Scenario I		Number of Monomers					
		1	2	3	4	5	6
Number of Subunits	3	0.31	0.61	0.92	1.22	1.53	1.83
	4	0.66	1.31	1.97	2.62	3.28	3.93
	5	1.00	1.99	2.99	3.99	4.99	5.98
	6	1.33	2.67	4.00	5.33	6.66	8.00
	7	1.67	3.33	5.00	6.66	8.33	9.99
	8	2.00	3.99	5.99	7.98	9.98	11.97
	9	2.32	4.65	6.97	9.29	11.62	13.94
	10	2.65	5.30	7.95	10.60	13.25	15.90

128

129 Introducing charge on the surrounding membrane, which replicates the effect of having charged
 130 lipids, results in the reduction of the conductance dynamic range for Scenario II (Table 3) (0.68-
 131 10.36 nS) as compared to Scenario I (Table 2) (0.31-15.90 nS). The charged lipids are negatively
 132 charged, which draws cations towards the surface. When the pore is small, this increases the
 133 proximity of cations around the pore, effectively drawing cations toward the pore and increasing
 134 the likelihood of transport across the membrane. The Tagliazucchi model indicates that there are
 135 two important regions that dictate conductance through a charged pore: 1) A hemisphere with the
 136 same diameter as the pore directly above the conducting region, 2) The bulk fluid and surface
 137 surrounding the pore. When the surface is uncharged, the hemisphere dominates the conductance –
 138 for a large pore there is a large electric field within the hemisphere, which draws cations toward
 139 the pore. However, when the membrane is charged, the radius of the charged hemisphere increases
 140 and draws cations toward a region larger than the pore itself. Whereas the cations in the uncharged
 141 scenario were funneled toward a region of the same size as the pore, the charged membrane brings
 142 ions to a region with a larger footprint than the pore. This effectively shields the pore by
 143 diminishing the probability of cation interaction with the pore, as compared to the uncharged
 144 membrane scenario. In other words, cations get trapped on the lipid membrane near the pore
 145 surface, reducing the ionic flux. Since transport is a probabilistic event, a charged membrane
 146 increases the chances of ionic transport in small pores while decreasing the probability in a large
 147 pore. Since ionic transport is equivalent to conductance, small pores will see greater conductance
 148 while large pores will have lower conductance for Scenario II (Table 3) as compared to Scenario I
 149 (Table 2).

150

151
152

Table 3: Conductance (nS) of A β pores for Scenario II.

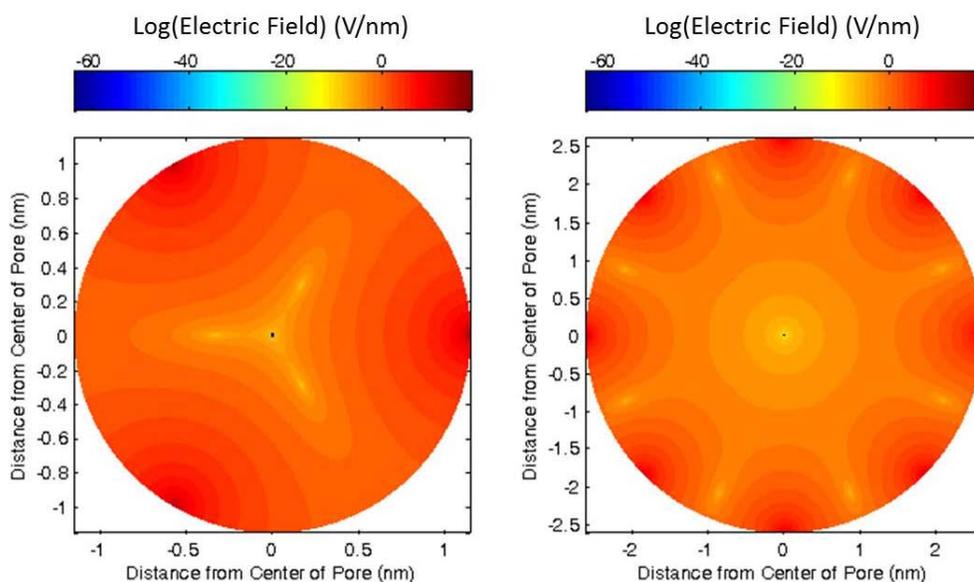
Scenario	Number of Monomers						
	1	2	3	4	5	6	
Number of Subunits	3	0.68	1.32	1.91	2.46	2.98	3.46
	4	1.30	2.43	3.42	4.29	5.06	5.75
	5	1.83	3.33	4.57	5.62	6.50	7.25
	6	2.30	4.10	5.51	6.65	7.56	8.30
	7	2.75	4.77	6.30	7.47	8.37	9.07
	8	3.16	5.38	6.97	8.13	8.99	9.63
	9	3.55	5.91	7.54	8.68	9.48	10.05
	10	3.91	6.40	8.03	9.12	9.86	10.36

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154

155 3.2 Electric Field in Pores

156 The effect of electric field on the cation selectivity of A β pores was examined. Figure 3 shows the
157 electric field in two different pores, one with a small pore radius (left) and the other with a large
158 pore radius (right). The octomer (right) exhibits more diffuse low field strength regions than the
159 trimer (right). Additionally, the low field regions in the octomer extend further from the pore
160 center than in the trimer. This suggests that electrostatics play less of a role in ion conductance
161 through large pores than small pores and large pores are thus less cation selective.

162



163

164 **Figure 3: A β Pores become less cation selective as the diameter increases.** The resultant
165 electric field within pores is shown as a heat map with areas of high field strength appearing in red
166 and low field strength appearing yellow. The plot on the left is for a trimer of dimers (three

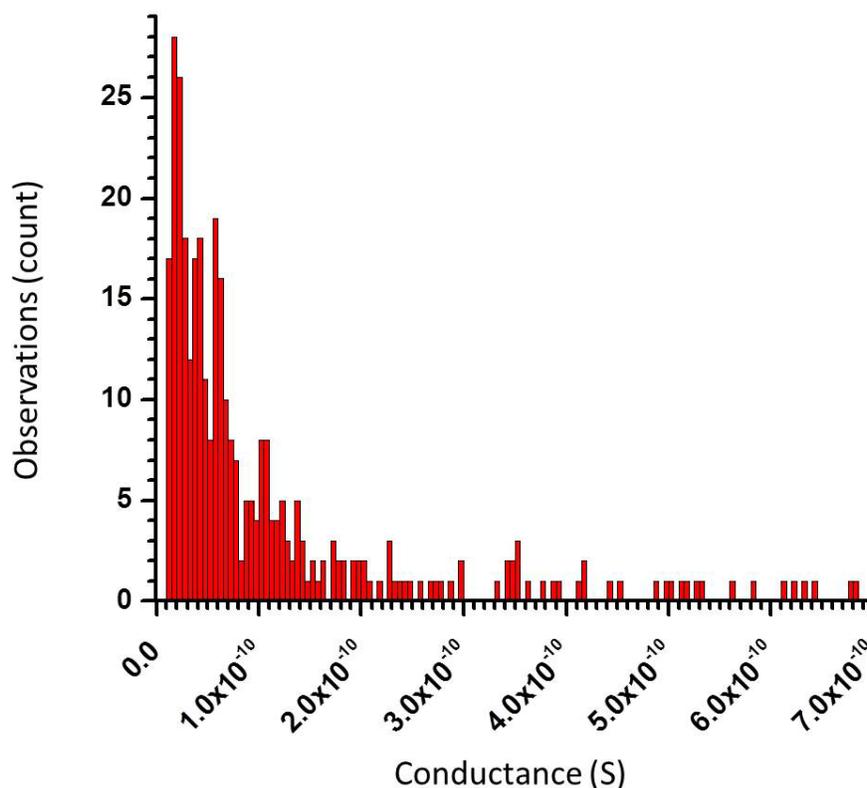
167 subunits with two monomers per subunit) while the right shows an octomer of dimers (eight
168 subunits with two monomers each).

169

170 4 Conclusion

171 Comparing experimental (Figure 4) data to the theoretical results presented here indicates that
172 most pores are of a small diameter. This agrees with current understanding of A β pore formation
173 that suggests that smaller pores are more stable than large pores, as increasing the pore radius
174 would result in a structure that collapses. This is supported by the experimental results indicating
175 that most conductance are less than 0.7 nS, which correlates to pores of 3-4 subunits based on
176 theoretical estimates of pore conductances (Table 1 and 2).

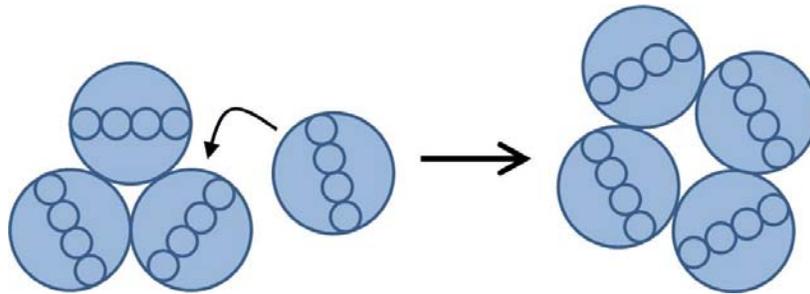
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178

179 **Figure 4: Experimentally observed conductance of A β pores.** Electrical recording of lipid
180 bilayers was carried out with A β added to the media. The integrity of the membrane was ensured
181 via lack of conductance prior to protein addition. Step changes in the conductance were recorded.
182 The histogram shows that extremely low conductance (pS scale) steps dominate the recording.
183 This suggests growth of small pores (3-4 subunits) through successive subunit addition rather than
184 spontaneous assembly of large pores with high (nS scale) conductance.

185 The data contained in Figure 4 also shows conductances that occur as a result of conformational
186 change (i.e. the conductance is measured at one value and then a step increase occurs resulting in a
187 higher conductance value and the change in conductance is reported). This suggests that the pore
188 has increased in diameter by a small amount resulting in slight increase in conductance, as the
189 change in conductance observed is less than any of the conductances predicted by the models.
190 This change in conductance indicates that a new pore has not formed in the bilayer, but a small
191 pore has become larger. Large pores are unlikely to form via spontaneous aggregation of subunits
192 within the cellular membrane, but rather through successive growth of small pores through subunit
193 addition (Figure 5), which is supported by our analysis. The larger pore will exhibit a small step
194 increase in conductance as the size grows.



195

196 **Figure 5: Proposed model for successive subunit addition and pore expansion.** A pore of 3
 197 subunits is expanded to a pore consisting of 4 subunits via subunit addition.

198 Future work in this area will focus on testing new geometric configurations to resolve differences
 199 between the values obtained via the model and experimental data. The current model, while
 200 geometrically convenient does not necessarily represent the proper configuration of A β pores in
 201 biology. The true structure of the subunits is likely more elongated and elliptical than the idealized
 202 cylinders used for the present analysis. Future simulations will focus on elliptical geometries and
 203 varying numbers of monomers within each subunit in the simulation – in this manner, native
 204 configurations can be more closely simulated. Additionally, the effects of the modeled A β
 205 conductance on spiking neurons will be modeled to simulate the pathophysiology of amyloid
 206 diseases such as Alzheimer’s. Furthermore, the effects of A β pore toxicity, whether in a single cell
 207 or in multiple cells in small networks of cells, will be examined to help determine how cell
 208 impairment affects single cell dynamics and neural connectivity.

209

210 **Acknowledgments**

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 212 experimental data

213 **References**

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