Dynamic behavior for a network of Izhikevich neurons with recurrent synaptic input

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Abstract

Many different neuron models have been devised in order to explain the functioning of neurons. The most common being the Integrate and Fire and the Hodkin-Huxley models. The integrate and fire neuron model offers fast simulation with simple description of the system. While Hodkin-Huxley model offers a detailed description of the system with slow simulation time. In order to overcome the shortcomings that either model offers, this paper uses the Izhikevich neuron model to simulate large networks ($N > 10000$) of excitatory and inhibitory neurons. In addition Synaptic weights between the different types of neurons, time dependent random synaptic input, and random connectivity in the network are included in the model to achieve simulations close to actual observations. Simulation results in output that is analogous to many local field potential measurements for the awake cortex.

1 Introduction

The past couple decades have shown promise in expanding the boundaries of our knowledge on the brain. A significant amount of effort has been directed towards the simulation of large neural networks using different neuron models. The common model has been the integrate and fire neuron, which allows for fast simulation but lacks the detail of other models. Recent studies by Amit and Brunel [1] and [2] along with Fourcaud and Brunel [8], Brunel and Ledoux [6], and Brunel and Hakim [4] have analyzed the dynamics for neural networks of integrate and fire neurons. A model such as the Hodkin-Huxley model would be preferred due to its complexity, however, realistically the time it takes to simulate large networks of these neurons is undesirable. A simple, descriptive model was derived using bifurcation theory by Izhikevich [9]. By including the Izhikevich model along with excitatory neurons, inhibitory neurons, and synaptic weights, different simulations with large neural networks can be produced in a reasonable amount of time.

2 Methods

2.1 The basic model

The neuron model proposed by Izhikevich [9], resulting from bifurcation theory, is given by the set of differential equations:

$$\frac{dV}{dt} = 0.04V(t)^2 + 5V(t) + 140 - u + I(t) \tag{1}$$

$$\frac{du}{dt} = a(bV(t) - u) \tag{2}$$

where

- V is the membrane potential
- u is the parameter that describes the Na^+ and K^+ ion channel gating
- I is the current between synapses
- a is the time scale for the parameter u to recover from a spike
- b determines the strength of correlation between the membrane potential V and the gating variable u
- c is the membrane reset potential
- d is the post-spike reset for the gating variable u and is equivalent to the slow recovery of the Na^+ and K^+ concentrations

Typical values for the parameters a, b, c, and d for excitatory and inhibitory neurons can be found in Table 1. The parameters were chosen in a similar fashion to Izhikevich [9] with the exhibitory neurons being a regular spiking neuron (normal firing rate) and the inhibitory neurons as a fast spiking neuron (fast firing rate).

2.2 Randomization of the Input Current

The total current going to the neuron is given by the external current along with an additional recurrent current, which is determined by pre-synaptic and post-synaptic potential. The net current is simply given by

$$I(t) = I_{ext}(t) + I_{recurrent}(t) \tag{3}$$

In order to randomize the current that goes to each neuron we first must assert three assumptions made by Amit and Brunel [1] about the properties of the neuron:

- Every neuron spikes many times over its integration period
- Depolarization is small compared to threshold
- Spike timing of different neurons are independent, the rates, however, can still be dependent

These assumptions allow for the external current of the neuron to be approximated by a Gaussian diffusion process. The external current that activates each neuron can be rewritten as

$$I_{ext}(t) = J_{\alpha E} N(t) \tag{4}$$

where $\alpha \epsilon E, I$ and $J_{\alpha E}$ are the synaptic weights between the different types of neurons that are connected at a synapse (excitatory to excitatory (EE), excitatory to inhibitory (EI), and inhibitory to inhibitory (II)). The Gaussian process is given by the Poisson distribution $N(t)$ with the mean of the distribution at $\epsilon N_E \upsilon_E dt$ with N_E being the number of excitatory neurons and υ_E the firing rate of the neurons. The random and fixed connectivity of the neurons to other neurons in the network is represented by ϵ.

2.3 Creating a recurrent network

The recurrent term is defined by a difference between the sum of the excitatory neurons that spiked and the sum of the inhibitory neurons that spiked. Both terms are scaled by different parameters as follows

$$I_{recurrent} = \sum_{j \epsilon E} \rho_j(t) J_{ij} \tau_j S_j(t - \tau_{ij}) - \sum_{j \epsilon I} \rho_j(t) J_{ij} \tau_j S_j(t - \tau_{ij}) \tag{5}$$

the first parameter $\rho_j(t)$ is the probability (r) that a spike in one neuron will activate a spike in a neighboring neuron, indicated by $\rho_j(t) = 1$. The strength of the synaptic connections are represented by the J_{ij} terms. The time delay between receiving input and generating a spike is τ_j and $S_j(t - \tau_{ij}) = 0, 1$ represents if a spike is activated on this time interval.

Table 1 lists all of the above parameters and their associated values for this simulation (units are given where appropriate). The ratio of excitatory to inhibitory is taken from observation to be $4:1$.

Table 1: Parameters used for simulation

Parameter	**Value**
a_E, a_I	$0.02, 0.06 \pm 0.04$
b_E, b_I	$0.2, 0.25$
c_I, c_I	$-65, -65$
d_I, d_I	$8 \pm 6, 2$
J_{EE}, J_{EI}	0.2mV, $0.6mV$
J_{IE}, J_{II}	$0.4mV$, $0.6mV$
N_E	8000
N_I	2000
ϵ	0.2
r	0.2
τ_E, τ_I	$10ms$, $5ms$

3 Results

Figure 1 represents the averaged sum of the potentials in the network of $N = 10000$ neurons over a period of $1000ms$. Due to the network's large size this output signal could easily be the measured local field potential of a region on the cortex. Simulations over smaller networks would produce similar results except for less spiking.

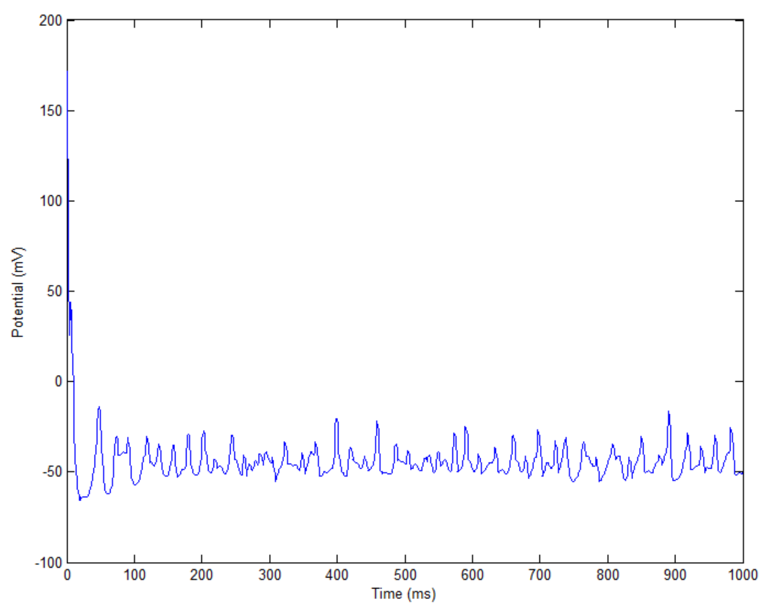

Figure 1: Recurrent Neural Network Potential of 10000 neurons over 1 second of simulation

A raster plot of 100 randomly chosen neurons is shown in Figure 2, where neuron index from 1 to 80 corresponds to excitatory neurons and 81 to 100 are inhibitory neurons. The firing rate observed in

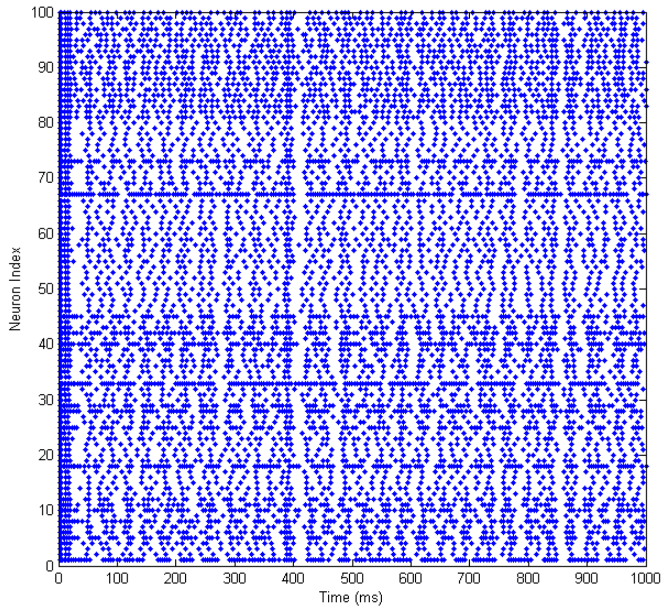

Figure 2: Raster for Recurrent Neural Network of 100 random neurons over 1 second of simulation

the raster turn out to be much higher than would be expected for this network for both excitatory and inhibitory neurons. A breakdown of the firing rate distributions over the total network are broken down by excitatory and inhibitory neurons in Figures 3 and 4 respectively. Neither is a probability distribution, but rather just the firing rate for each neuron in the network. With the average firing

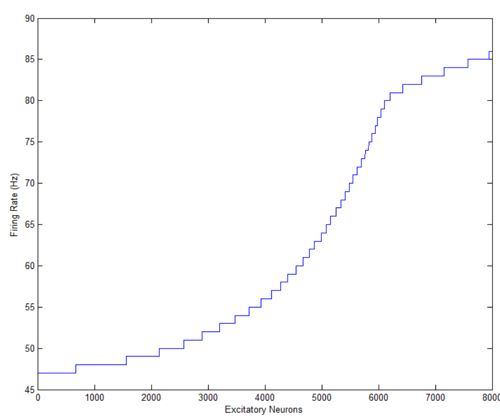

Figure 3: Firing rates for each excitatory neuron in the network

rate being 70 Hz for excitatory neurons the majority of neurons fire with a rate equal to or below this rate, while around 0.2N fire at higher rates.

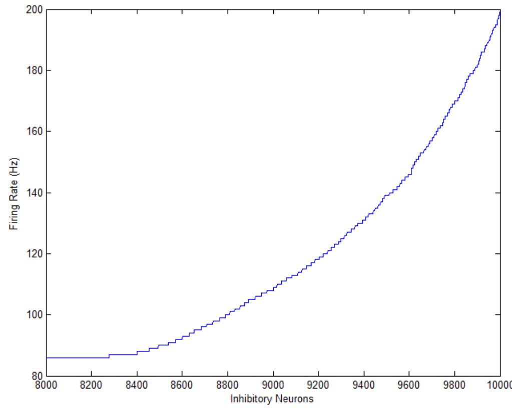

Figure 4: Firing rates for each inhibitory neuron in the network

The average firing rate is > 100 Hz for inhibitory neurons with the majority of neurons firing with a rate equal to or greater than this rate which would indicate serious problems with the model or parameters used.

4 Conclusions

Even with the firing rates are not all within reasonable range, the overall voltage output from the network still has similarities to local field potential measurements. Reasonable firing rates found by Amit and Brunel [1] are up to 60Hz for excitatory neurons and up to 80Hz for inhibitory neurons. Possible ways to correct for this error would be to change the Izhikevich parameters or the mean of the Poisson Distribution for both neuron types.

While the Izhikevich does allow for fast simulation of large networks, one must also take into account software limitations. When trying to run the network size at $N = 100,000$ MATLAB runs out of memory for the relevant variables. Further work would include simulation on a program which could handle the demands for such large networks. In addition it would be informative to plot the firing rate as a a function of each parameter in the Izhikevich model.

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