520.492 Mixed-Signal VLSI Systems

Week 7

Linear Filters

References

1. Geiger, Allen and Strader, Sections 8.4 and 8.5 (pp 673-728).
2. C.A. Mead, *Analog VLSI and Neural Systems* (Addison Wesley, 1992), pp 127-192.
3. Franca and Tsividis, Chapters 6 and 7.

LINEAR FILTERS

Geiger et al.: chapters 8.4 & 8.5 (pp 673–728)
Carver Mead, Analog VLSI and Neural Systems (pp 127–192)
Franca & Tsividis, Chapter 6 & 7

CONTINUOUS-TIME FILTERS

→ Laplace: $s = \frac{d}{dt}$ $\qquad F(s) = \int_0^{\infty} f(t)\, e^{-st}\, dt$

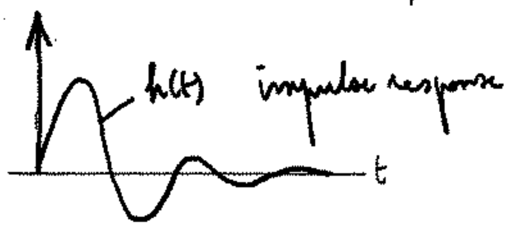
→ Fourier: $j\omega = \frac{d}{dt}$ $\qquad F'(\omega) = \int_{-\infty}^{+\infty} f(t)\, e^{-j\omega t}\, dt$

$$x(t) \longrightarrow \boxed{\begin{matrix} h(t) \\ H(s) \end{matrix}} \longrightarrow y(t)$$

$X(s) \qquad\qquad Y(s)$

$$y(t) = \int_{-\infty}^{t} h(t-\vartheta)\, x(\vartheta)\, d\vartheta$$

$$Y(s) = H(s) \cdot X(s)$$



→ CONTINUOUS-TIME ANALOG

DISCRETE-TIME (SAMPLED) FILTERS

→ z-transform: $z = [t \to t + \frac{1}{f_s}]$ $\qquad F''(z) = \sum_{i=0}^{+\infty} f_i \cdot z^{-i}$

$$x_i \longrightarrow \boxed{\begin{matrix} \{h_j\} \\ H''(z) \end{matrix}} \longrightarrow y_i$$

$X''(z) \qquad\qquad Y''(z)$

$$y_i = \sum_{j=-\infty}^{i} h_{i-j} \cdot x_j$$

$$Y''(z) = H''(z) \cdot X''(z)$$

finite impulse response (FIR): $h_i \equiv 0$ for $i > i_{max}$, $i < 0$

infinite impulse response (IIR): $h_i \equiv 0$ only for $i < 0$

→ SWITCHED-CAPACITOR FILTERS

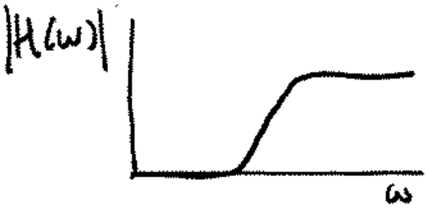
→ DIGITAL FILTERS

FILTER CATEGORIES (in frequency domain)

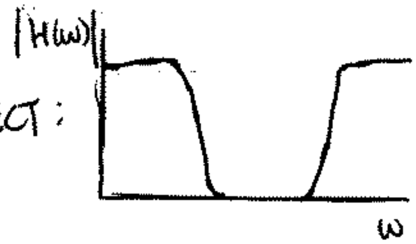
LOWPASS:

BANDPASS:

HIGHPASS:

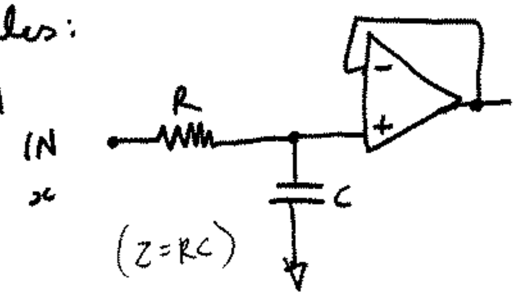


BANDREJECT:



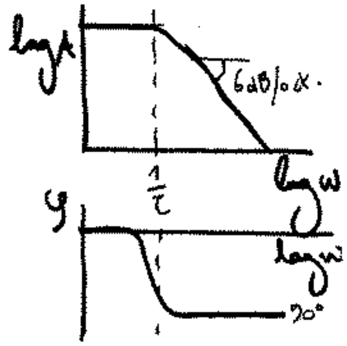
examples:

1)

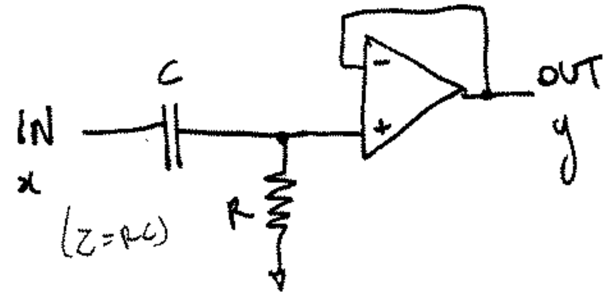


$$Y(s) = \frac{1}{1+\tau s}$$

$$y(t) \approx \frac{1}{\tau}\int_{-\infty}^{t} x(\theta)\,d\theta$$

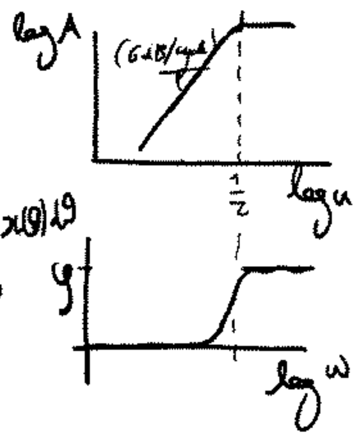


2)

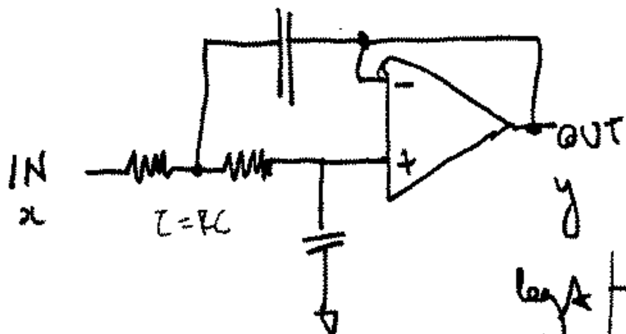


$$Y(s) = \frac{\tau s}{1+\tau s}$$

$$y(t) \approx x(t) - \frac{1}{\tau}\int_{-\infty}^{t} x(\theta)\,d\theta$$

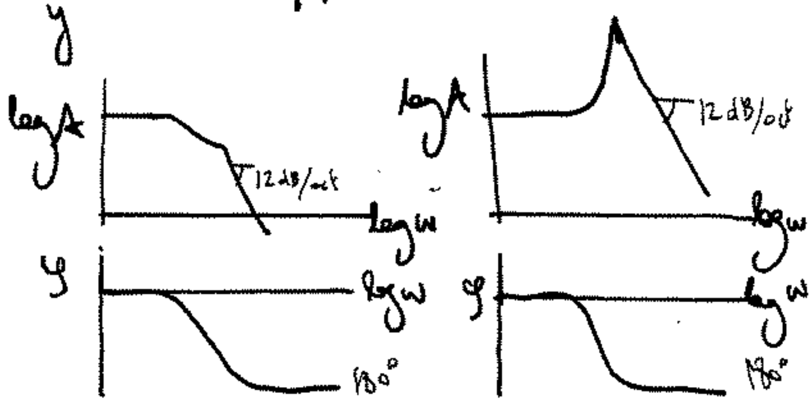


3)



$$Y(s) = \frac{1}{1+a\tau s+b(\tau s)^2}$$

(Sallen and Key)



FILTER SYNTHESIS

ANALOG

Continuous-time: $x(t) \to \boxed{H(s)} \to y(t)$

$$H(s) = \frac{\sum_{j=0} b_j s^j}{\sum_{i=0} a_i s^i}$$

- canonical decomposition:

$$s\cdot\vec{z} = \bar{\bar{A}}\cdot\vec{z} + \vec{B}\cdot x$$
$$y = \vec{C}\cdot\vec{z}$$

→ integrators

- cascade decomposition:

$$H(s) = A\cdot\underbrace{\frac{1+b_1' s + b_2' s^2}{1+a_1' s + a_2' s^2}}_{\text{biquad}}\cdot\frac{1+..}{1+..}$$

discrete-time: $x^i \to \boxed{H'(z)} \to y^i$

$$H'(z) = \frac{\sum_{j=0} b_j' z^j}{1 - \sum_{i=1} a_i' z^{-i}}$$

....

$$z\cdot\vec{z} = \bar{\bar{A}}'\cdot\vec{z} + \vec{B}'\cdot x$$
$$y = \vec{C}'\cdot\vec{z}$$

→ accumulators

....

$$H'(z) = A'\cdot\underbrace{\frac{1+b_1' z^{-1} + b_2' z^{-2}}{1-a_1' z^{-1} - a_2' z^{-2}}}_{\text{biquad}}\cdots$$

DIGITAL

- Tsividis (2004)

discrete time (direct synthesis)

- FIR: $H'(z) = \sum_{j=0}^{N} b_j' z^{-j}$ $(a_i' \equiv 0)$

$$\to y^i = \sum_{j=0}^{N} b_j' x^{i-j}$$

- IIR: $y^i = \sum_{j=1}^{M} a_j' y^{i-j} + \sum_{j=0}^{N} b_j' x^{i-j}$

CONTINUOUS

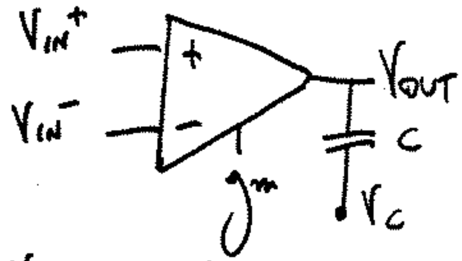
SAMPLED

ANALOG CONTINUOUS-TIME IMPLEMENTATION

- **Transconductance mode:**

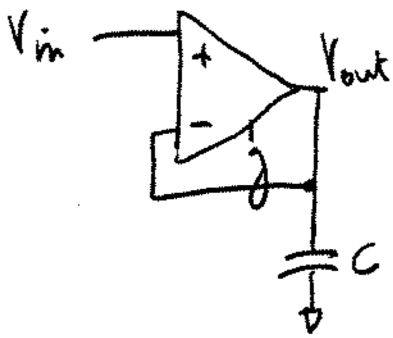
INTEGRATOR:

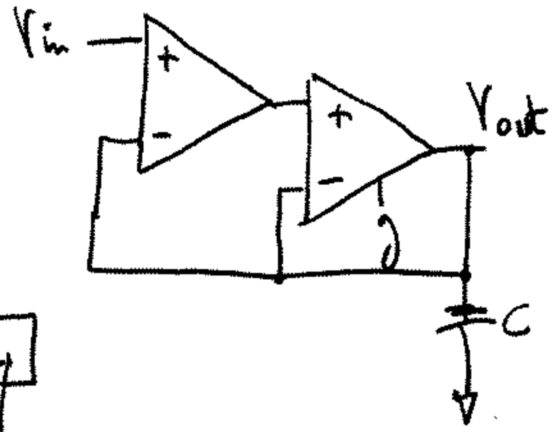
- g_m: { voltage in (HI-Z) differential; current out (HI-Z) }
- C: integrating current to voltage conversion

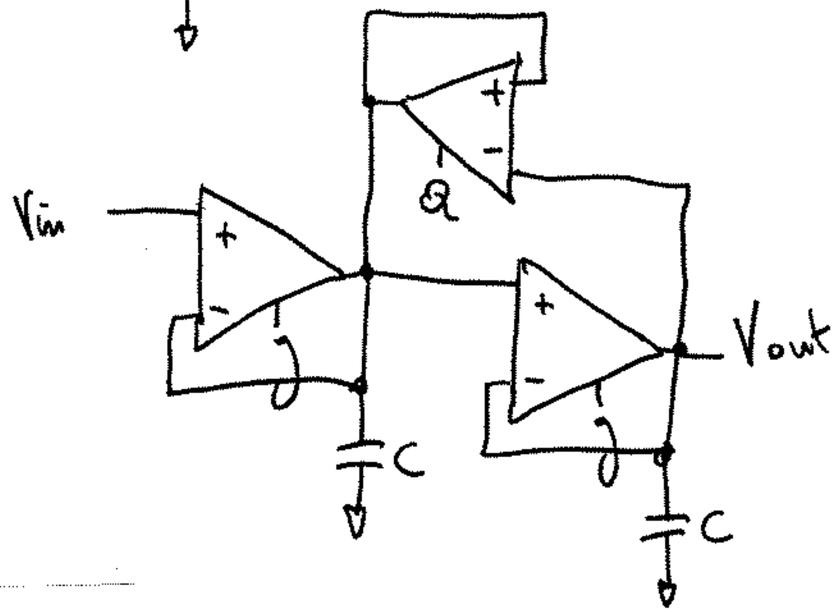


$$C\frac{dV_{out}}{dt} = C\frac{dV_c}{dt} + (V_{in}^+ - V_{in}^-)\,g_m$$

examples:





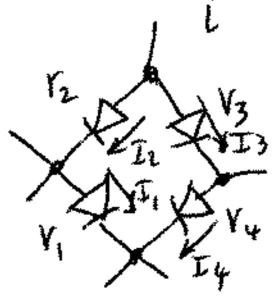


- <u>Current mode</u> : LOG-DOMAIN FILTERS

  - Static translinear principle:

$$I \propto e^{V/V_{th}} \text{ then } \quad \sum_i V_i = \sum_j V_j \Rightarrow \prod_i I_i = \prod_j I_j$$

$\left(V_{th} = \frac{kT}{q} = 25\,mV \ @ \ T_{300K} \right)$

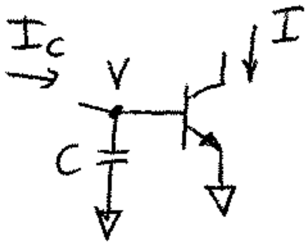


$V_1 + V_2 = V_3 + V_4 \Rightarrow I_1 I_2 = I_3 I_4$

also: BJTs, or MOS in subthreshold

  - Dynamic translinear principle:

$$I \propto e^{V/V_{th}} \text{ then } \quad \frac{d}{dt} I = \frac{1}{V_{th}} \frac{d}{dt} V \cdot I$$

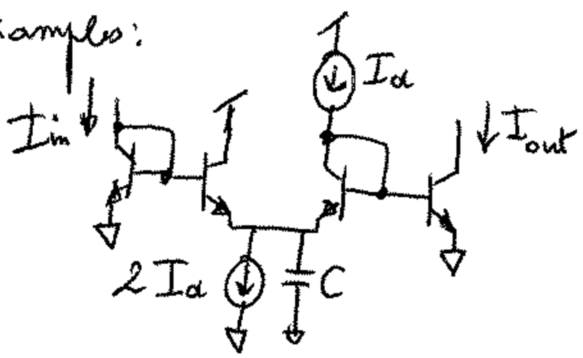


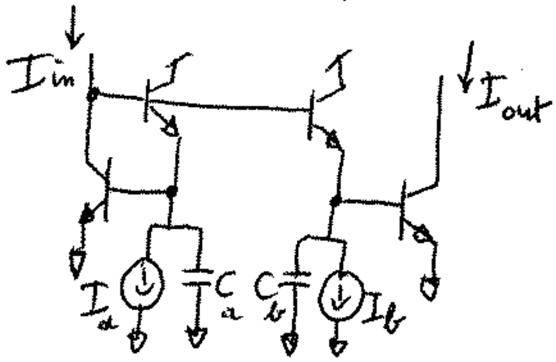
$$I_C = C \frac{dV}{dt} \Rightarrow \frac{dI}{dt} = \frac{I \cdot I_C}{C V_{th}}$$

$(\beta_F = \infty)$

Product $I \cdot I_C$ implemented with STATIC translinear principle

Examples:



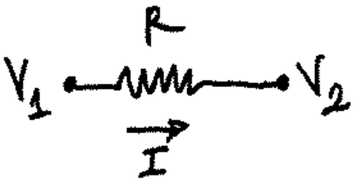


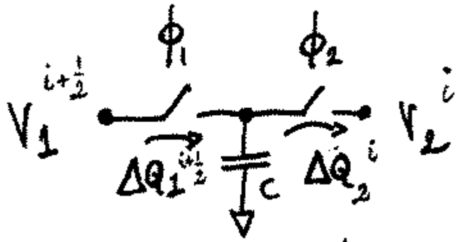
$$\frac{I_{out}}{I_{in}} = \frac{1}{1 + \tau s} \qquad \tau = \frac{C V_{th}}{I_\alpha}$$

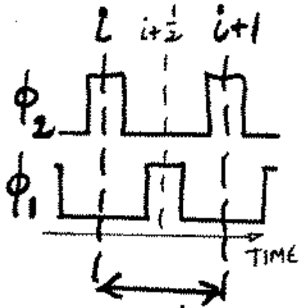
$$\frac{I_{out}}{I_{in}} = \frac{I_a + C_a V_{th} s}{I_b + C_b V_{th} s}$$

ANALOG DISCRETE-TIME IMPLEMENTATION

Switched capacitors:







$T = \frac{1}{f_s}$

$I(t) = \frac{V_1 - V_2}{R}$

Continuous-time

$$\begin{cases} \Delta Q_2^{i+1} = C V_1^{i+\frac{1}{2}} - C V_2^{i+1} \\ \Delta Q_1^{i+\frac{1}{2}} = C V_1^{i+\frac{1}{2}} - C V_2^{i} \end{cases}$$

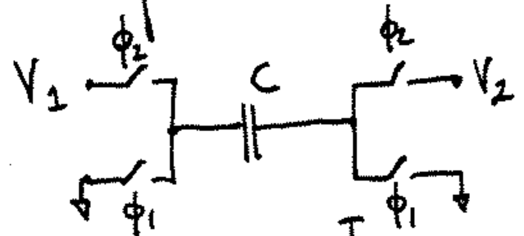
discrete-time (switched)

continuous approximation ($f_N \ll f_s$):

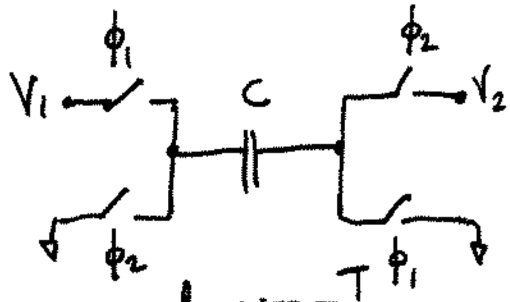
$$V_{1(2)}^{i+1} \approx V_{1(2)}^{i} \Rightarrow I_{aver.} = \frac{\Delta Q}{T} = \frac{C}{T}(V_1 - V_2)$$

$$\boxed{R_{equiv} = \frac{T}{C} = \frac{1}{f_s \cdot C}}$$

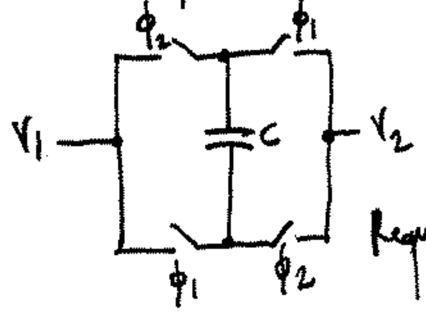
Other implementations:



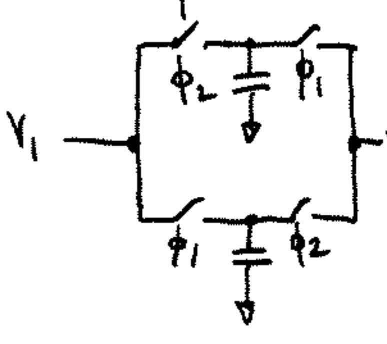
$R_{equiv} = \frac{T}{C}$



$R_{equiv} = -\frac{T}{C}$



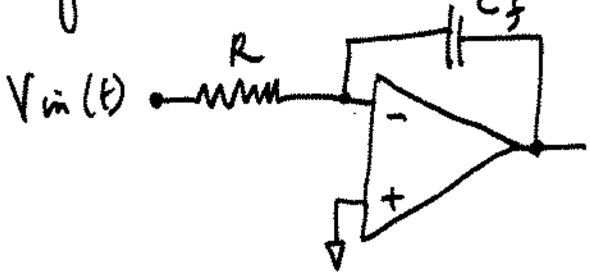
$R_{equiv} = \frac{T}{4C}$



$R_{equiv} = \frac{T}{2C}$

Example: INTEGRATOR $\leftrightarrow$ ACCUMULATOR :

integrator (continuous):

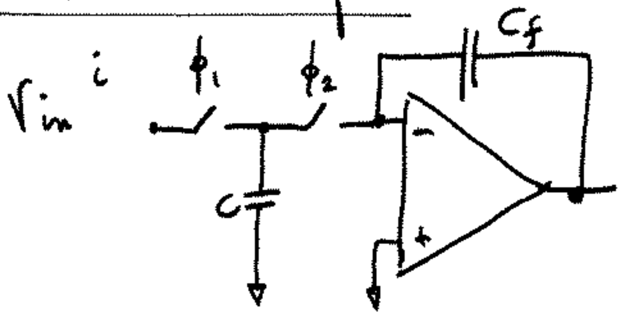


$$V_{out}(t) = V_{out}(0) - \frac{1}{RC_f}\int_0^t V_{in}(\theta)\, d\theta$$

or $$\frac{d}{dt}V_{out} = -\frac{1}{RC_f} V_{in}$$

$$\boxed{H(s) = -\frac{1}{RC_f}\cdot\frac{1}{s}}$$

accumulator (sampled):

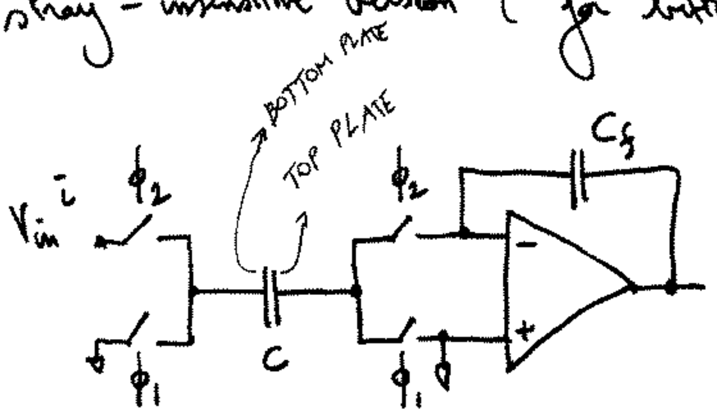


$$V_{out}{}^{i} = V_{out}{}^{0} - \frac{C}{C_f}\sum_{j=1}^{i} V_{in}{}^{j}$$

or $$V_{out}{}^{i} - V_{out}{}^{i-1} = -\frac{C}{C_f} V_{in}{}^{i}$$

$$\boxed{H'(z) = -\frac{C}{C_f}\,\frac{1}{1-z^{-1}}}$$

stray-insensitive version (for better linearity; better clock feedthrough reduction; ...):



$V_{out}{}^{i}$ = (SAME)

Higher frequencies : $f_N \approx f_s$

- <u>case 1</u> : straight mapping $\underset{R}{\text{–WWW–}} \Leftrightarrow S \cdot \frac{1}{T}$ $C = \frac{T}{R}$

  corresponds to $$S \equiv \frac{1 - z^{-1}}{T} \quad \text{or} \quad \frac{d}{dt} f \equiv \frac{f(t) - f(t-T)}{T}$$

  $$\left(z = \frac{1}{1 - sT} \right)$$

  $\Rightarrow$ distortion of spectrum and impulse response

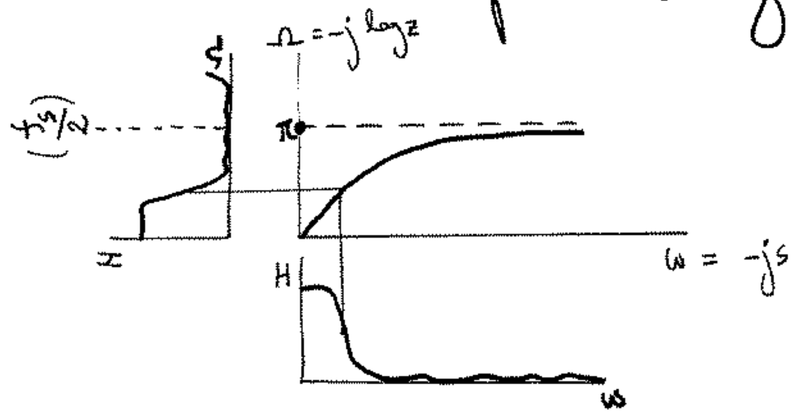
- <u>case 2</u> : find mapping $s \Leftrightarrow z$ to preserve impulse response

  $\Rightarrow \quad z = e^{+sT} \Rightarrow$ synthesize poles / zeros of e^{+sT}

- <u>case 3</u> : bilinear transformation

  $$z = \frac{1 + \frac{sT}{2}}{1 - \frac{sT}{2}} \quad \text{or} \quad f(t) - f(t-T) \approx \underbrace{\frac{\frac{df}{dt}(t) + \frac{df}{dt}(t-\Delta)}{2} \cdot T}_{\approx \int_t^{t+T} \frac{df}{dt}(\theta) \cdot d\theta}$$

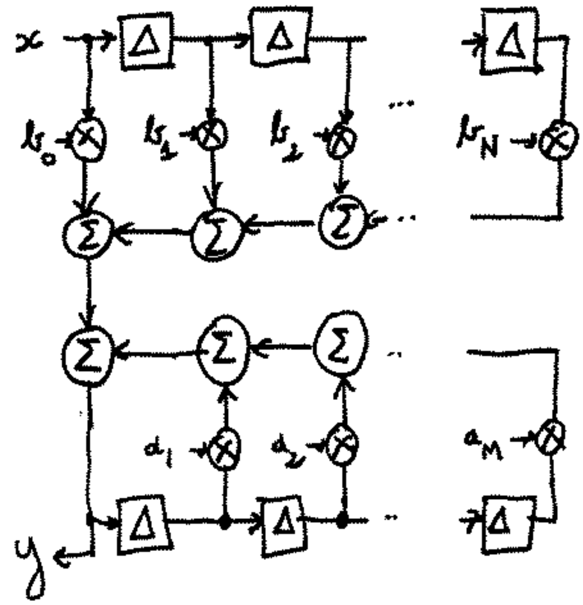
  maps the frequency domain of both representations with a one-to-one correspondence (though distorted)



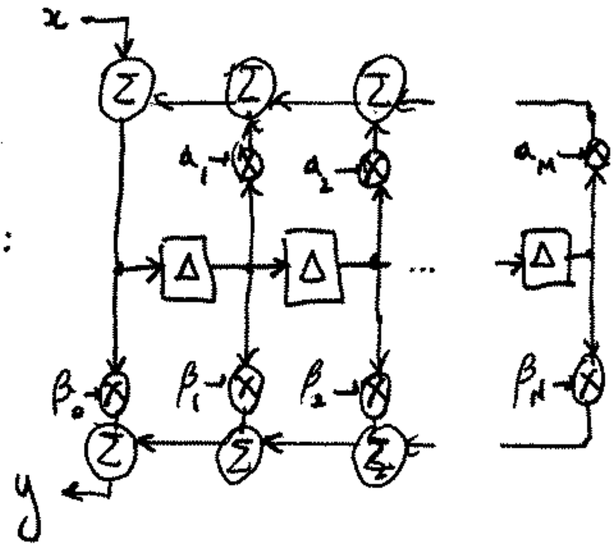
DIGITAL DISCRETE-TIME IMPLEMENTATION

- Infinite Impulse Response (IIR) : $H(z) = \dfrac{\sum_{j=0}^{N} b_j z^{-j}}{1 - \sum_{i=1}^{M} a_i z^{-i}}$

$$\rightarrow \quad y^k = \sum_{i=1}^{M} a_i y^{k-i} + \sum_{j=0}^{N} b_j x^{k-j}$$

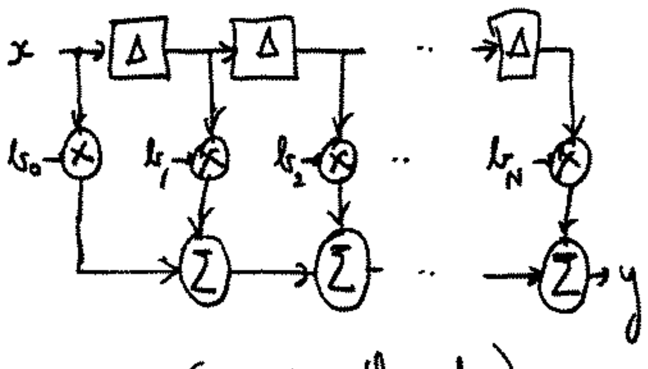


or :



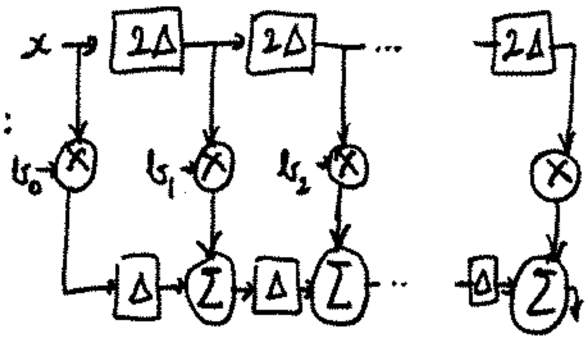
- Finite Impulse Response (FIR) : $H(z) = \sum_{j=0}^{N} b_j z^{-j}$

$$\rightarrow \quad y^k = \sum_{j=0}^{N} b_j x^{k-j}$$



(ripple-through)

or :



PIPELINED (faster)

2× latency (in # cycles)