Gradient Flow Source Separation and Localization

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Gradient Flow Source Separation and Localization

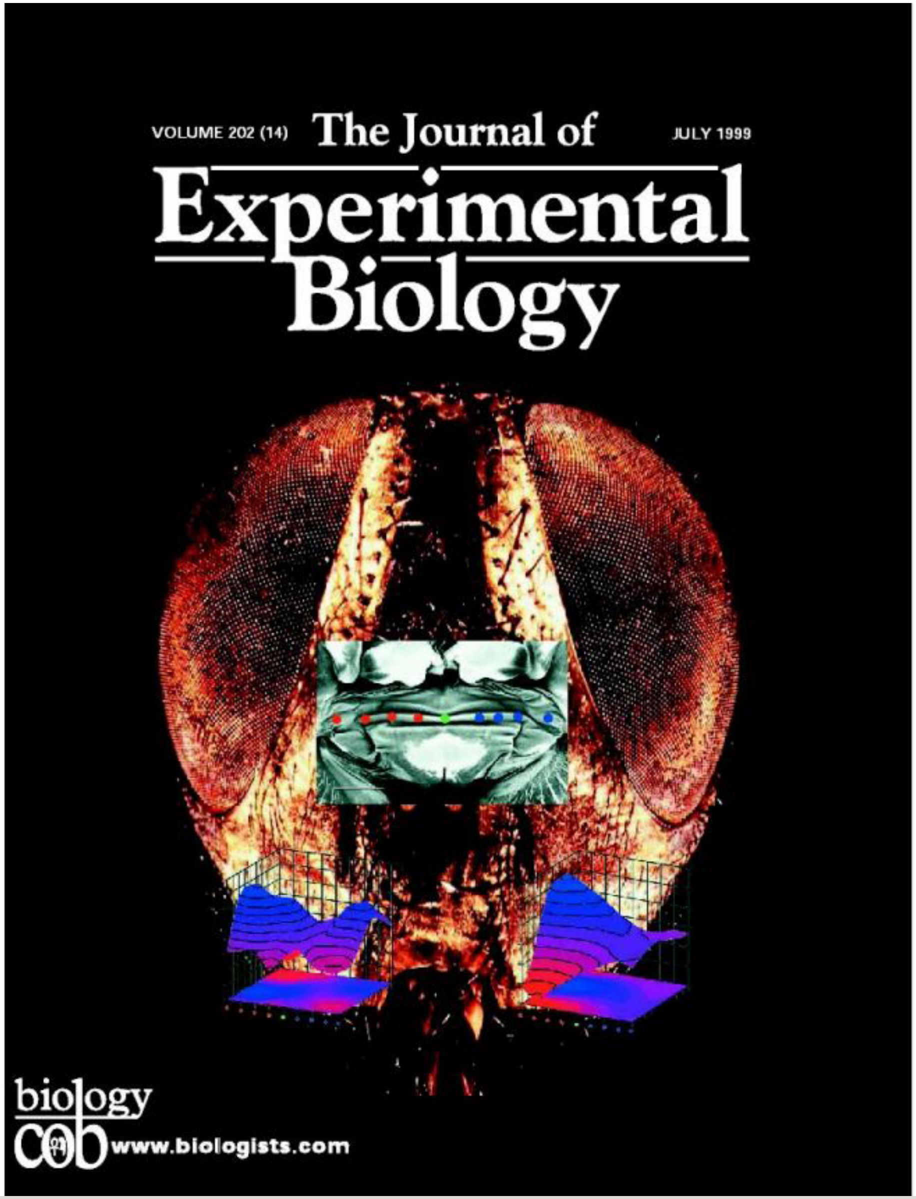
- **Introduction**
  - Spatial diversity in array signal processing
  - Directional hearing at sub-wavelength scale
- **Broadband Localization and Separation**
  - From delays to temporal derivatives
  - *Gradient Flow*
  - Equivalent static linear ICA problem
  - Multipath extension and convolutive ICA
- **Performance Analysis**
  - Scaling properties
  - Cramer-Rao bounds
  - Differential sensitivity
- **Bearing Estimation**
  - Micropower mixed-signal VLSI implementation
  - Experimental *GradFlow* / ASU acoustic bearing estimation
- **Independent Component Analysis**
  - Micropower mixed-signal VLSI implementation
  - Experimental acoustic source separation
- **Hearing Aid Implications**

Blind Separation and Beamforming Localization

- **Modeling**
  - Source signals propagate as traveling waves
  - Spatially diverse sensor array receives linear mixtures of time-delayed sources
  - The time delays determine the direction coordinates of the waves relative to the sensor geometry

- **Methods**
  - Super-resolution techniques estimate the time delays in the spectral domain, assuming narrowband sources
  - Joint estimation of multiple broadband sources and their time delays is possible in an extended ICA framework, but requires non-convex optimization leading to unpredictable performance

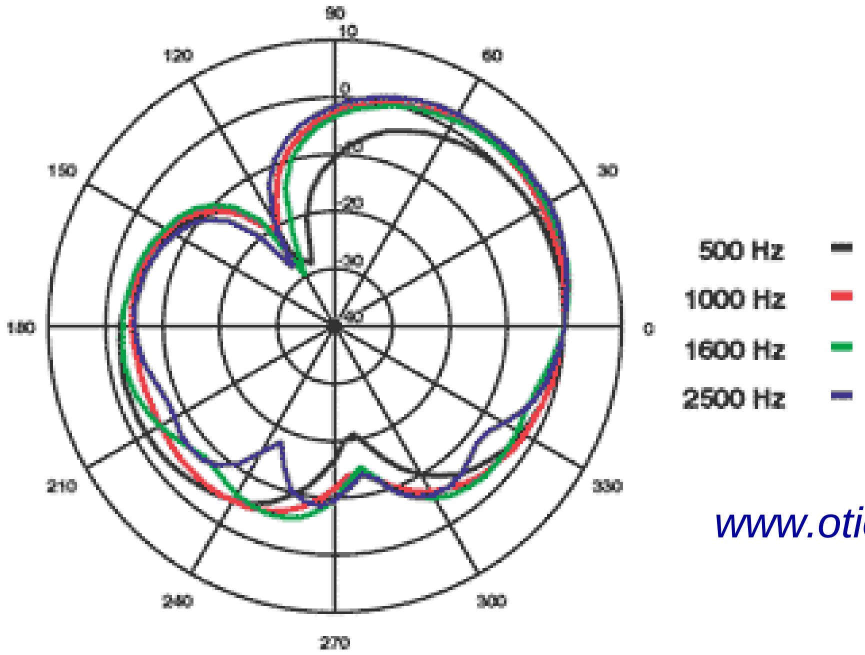
Biomechanics of Tympanal Directional Hearing



- Parasitoid fly localizes sound-emitting target (cricket) by a beamforming acoustic sensor of dimensions a factor 100 smaller than the wavelength.
- Tympanal beamforming organ senses acoustic pressure gradient, rather than time delays, in the incoming wave

Robert, D., Miles, R.N. and Hoy, R.R., "Tympanal hearing in the sarcophagid parasitoid fly *Emblemasoma sp*.: the biomechanics of directional hearing," *J. Experimental Biology*, v. 202, pp. 1865-1876, 1999.

Directional Selectivity in Hearing Aids



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- Two microphones allow for one null angle in directionality pattern
- Adaptive beamforming allows to steer the null to noise source
- Presence of multiple noise sources requires source localization and separation with multiple microphones

Wave Propagation

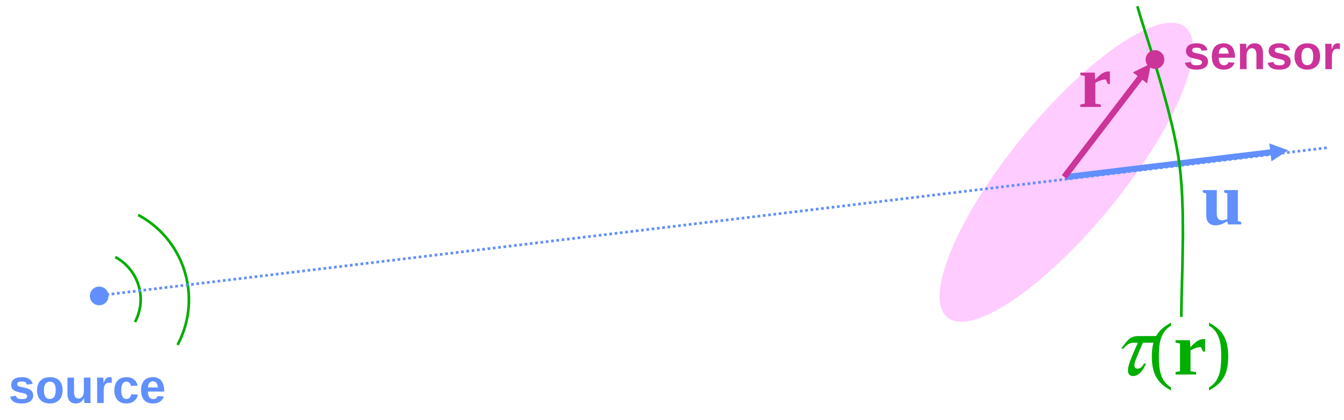
Traveling wave (e.g., acoustic, sonar, RF, …) in free space:

$$S(\mathbf{r},t) = A(\mathbf{r})s(t+\tau(\mathbf{r}))$$

In the *far field* limit:

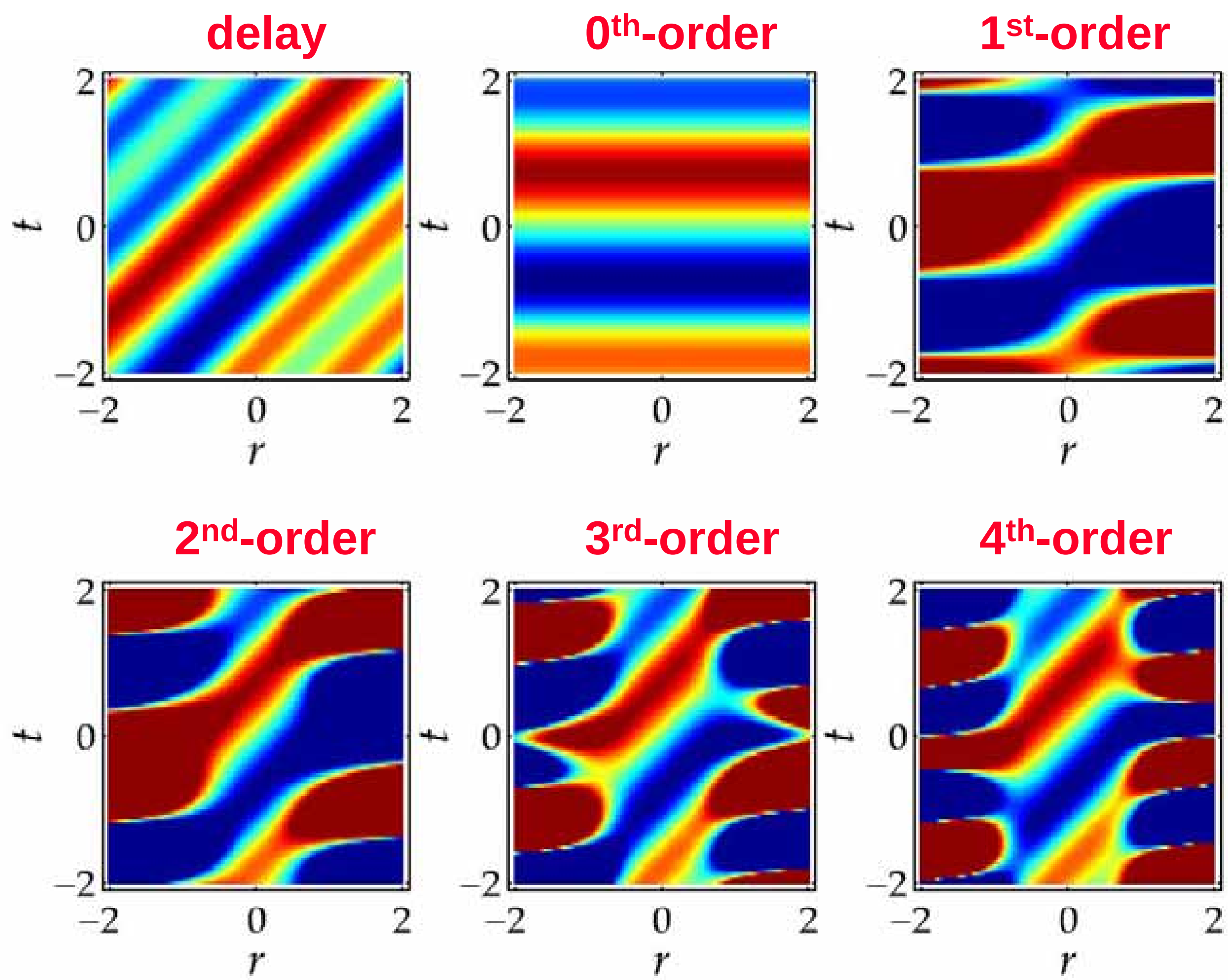
$$A(\mathbf{r}) \equiv 1$$

$$\tau(\mathbf{r}) = \frac{1}{c}\mathbf{r}\cdot\mathbf{u}$$



Temporal Series Expansion

$$s(t+\tau(\mathbf{r})) = s(t) + \tau(\mathbf{r})\dot{s}(t) + \frac{1}{2}\tau(\mathbf{r})^2\ddot{s}(t) + \ldots$$



– Reduces the problem of identifying time delayed source mixtures to that of separating static mixtures of the time-differentiated sources

– Implies sub-wavelength geometry of the sensor array

Spatial Sensing

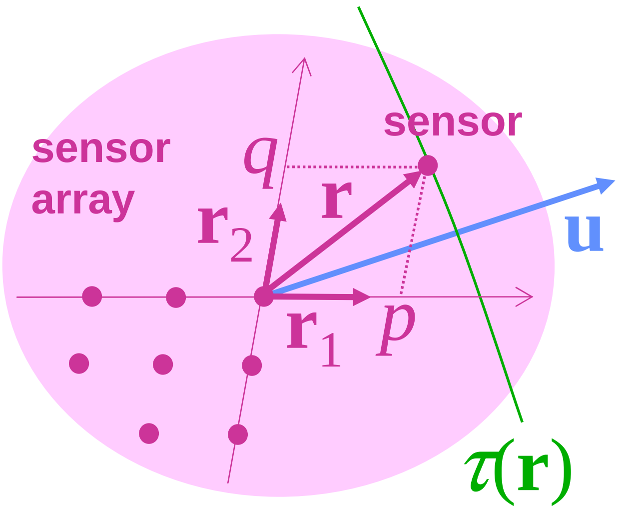
Sensor distribution:

e.g., for a planar sensor geometry:

$$\mathbf{r}_{pq} = p\mathbf{r}_1 + q\mathbf{r}_2$$

– sensor *array*: p, q discrete

– *distributed* sensor: p, q continuous



Source delays:

$$\tau_{pq} = p\tau_1 + q\tau_2$$

with:

$$\tau_1 = \frac{1}{c}\mathbf{r}_1 \cdot \mathbf{u}$$

$$\tau_2 = \frac{1}{c}\mathbf{r}_2 \cdot \mathbf{u}$$

the direction coordinates of source relative to sensor geometry

Wave Flow: Spatial and Temporal Gradients

Linear flow:

Sensor signals:

$$x_{pq}(t) = s(t+\tau_{pq}) = s(t) + (p\tau_1 + q\tau_2)\dot{s}(t) + \ldots$$

Gradients:

$$\dot{\xi}_{00} = \dot{x}_{pq}\Big|_{p=q=0} = \dot{s}(t)$$

$$\xi_{10} = \frac{\partial x_{pq}}{\partial p}\Big|_{p=q=0} = \tau_1\dot{s}(t)$$

$$\xi_{01} = \frac{\partial x_{pq}}{\partial q}\Big|_{p=q=0} = \tau_2\dot{s}(t)$$

$$= \begin{bmatrix} 1 \\ \tau_1 \\ \tau_2 \end{bmatrix} \dot{s}(t)$$

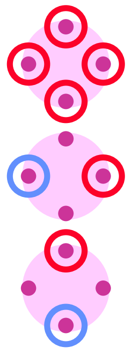
Higher-order flow:

$$\xi_{ij} = \frac{\partial^{i+j} x_{pq}}{\partial^i p \partial^j q}\Big|_{p=q=0} = (\tau_1)^i (\tau_2)^j s^{(i+j)}(t)$$

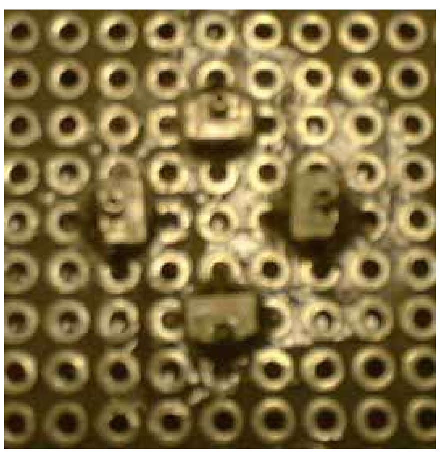
Miniature Sensor Arrays

Finite-difference gradient approximation on a grid:

e.g., planar array of 4 sensors:

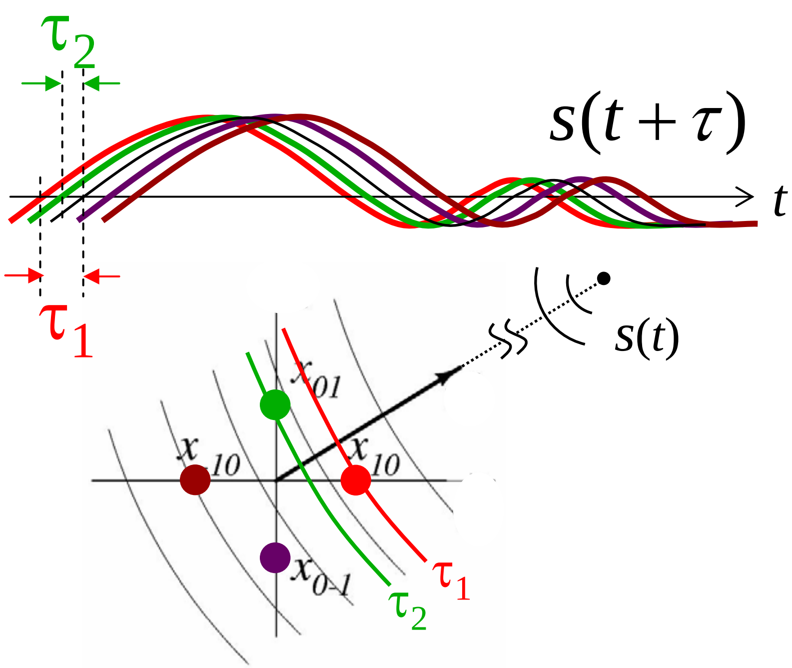


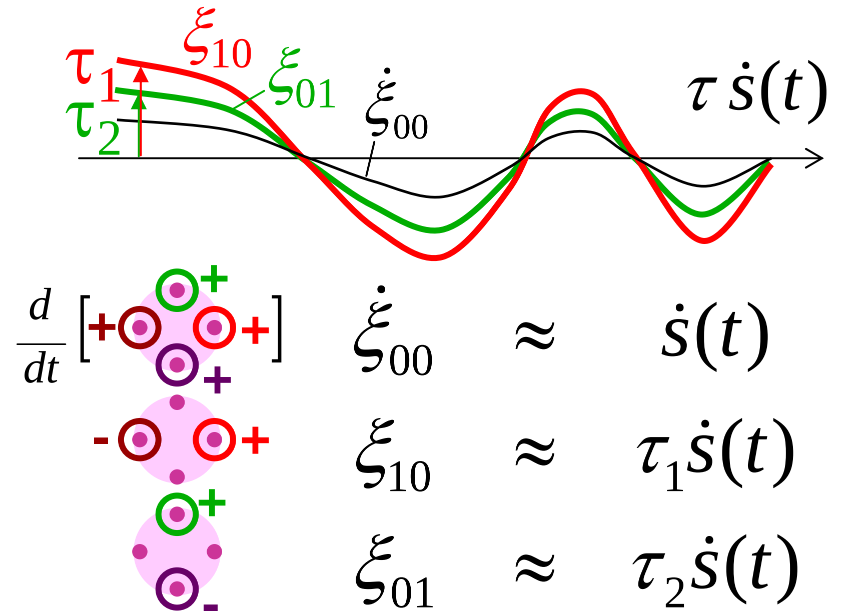
$$
\begin{aligned}
\tfrac{1}{4}\left(x_{-1,0} + x_{1,0} + x_{0,-1} + x_{0,1}\right) &\approx \xi_{00} \approx s(t) \\
\tfrac{1}{2}\left(x_{1,0} - x_{-1,0}\right) &\approx \xi_{10} \approx \tau_1 \dot{s}(t) \\
\tfrac{1}{2}\left(x_{0,1} - x_{0,-1}\right) &\approx \xi_{01} \approx \tau_2 \dot{s}(t)
\end{aligned}
$$



1cm

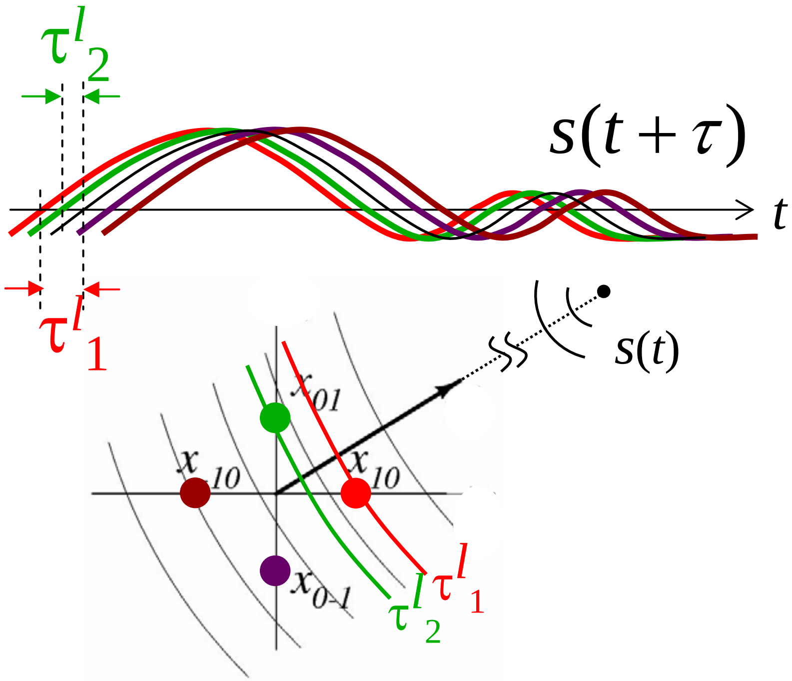
Gradient Flow Localization

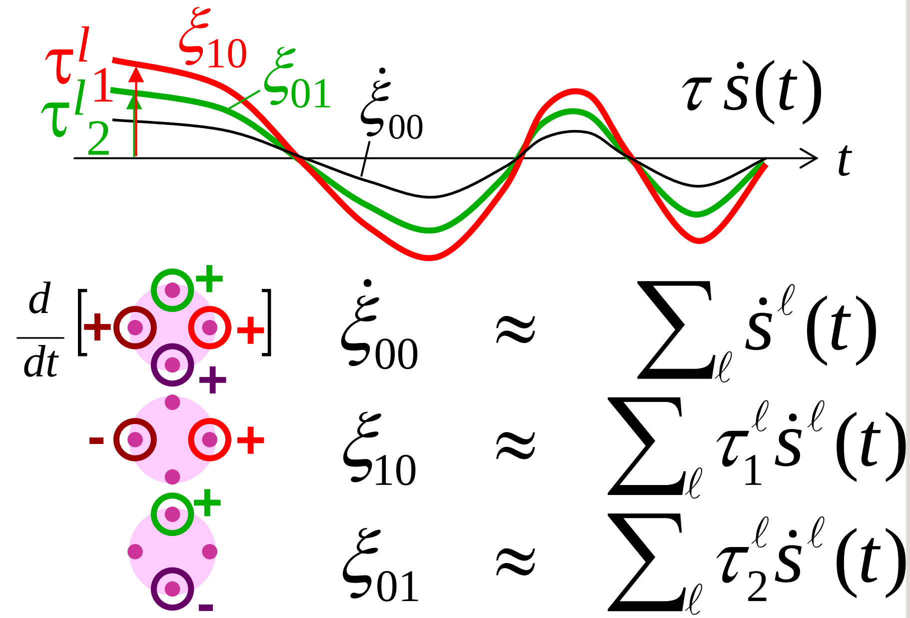




- **Gradient flow bearing resolution is fundamentally independent of aperture**
- **Resolution is determined by sensitivity of gradient acquisition**
  - *Mechanical differential coupling (Miles et al.)*
  - *Optical differential coupling (Degertekin)*
  - *Analog VLSI differential coupling*

Gradient Flow Localization and Separation





- **Gradient flow bearing resolution is fundamentally independent of aperture**
- **Resolution is determined by sensitivity of gradient acquisition**
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  - *Optical differential coupling (Degertekin)*
  - *Analog VLSI differential coupling*
- ***Multiple target tracking with independent component analysis (ICA)***

Separation and Localization

Source mixtures are observed with additive sensor noise:

$$x_{pq}(t) = \sum_{\ell=1}^{L} s^{\ell}(t + \tau_{pq}^{\ell}) + n_{pq}(t)$$

Gradient flow reduces to a static (noisy) mixture problem:

$$\begin{bmatrix} \dot{\xi}_{00} \\ \xi_{10} \\ \xi_{01} \end{bmatrix} = \begin{bmatrix} 1 & \cdots & 1 \\ \tau_1^1 & \cdots & \tau_1^L \\ \tau_2^1 & \cdots & \tau_2^L \end{bmatrix} \begin{bmatrix} \dot{s}^1(t) \\ \vdots \\ \dot{s}^L(t) \end{bmatrix} + \begin{bmatrix} \dot{\nu}_{00} \\ \nu_{10} \\ \nu_{01} \end{bmatrix}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$\mathbf{x} = \mathbf{A} \qquad \mathbf{s} + \mathbf{n}$$

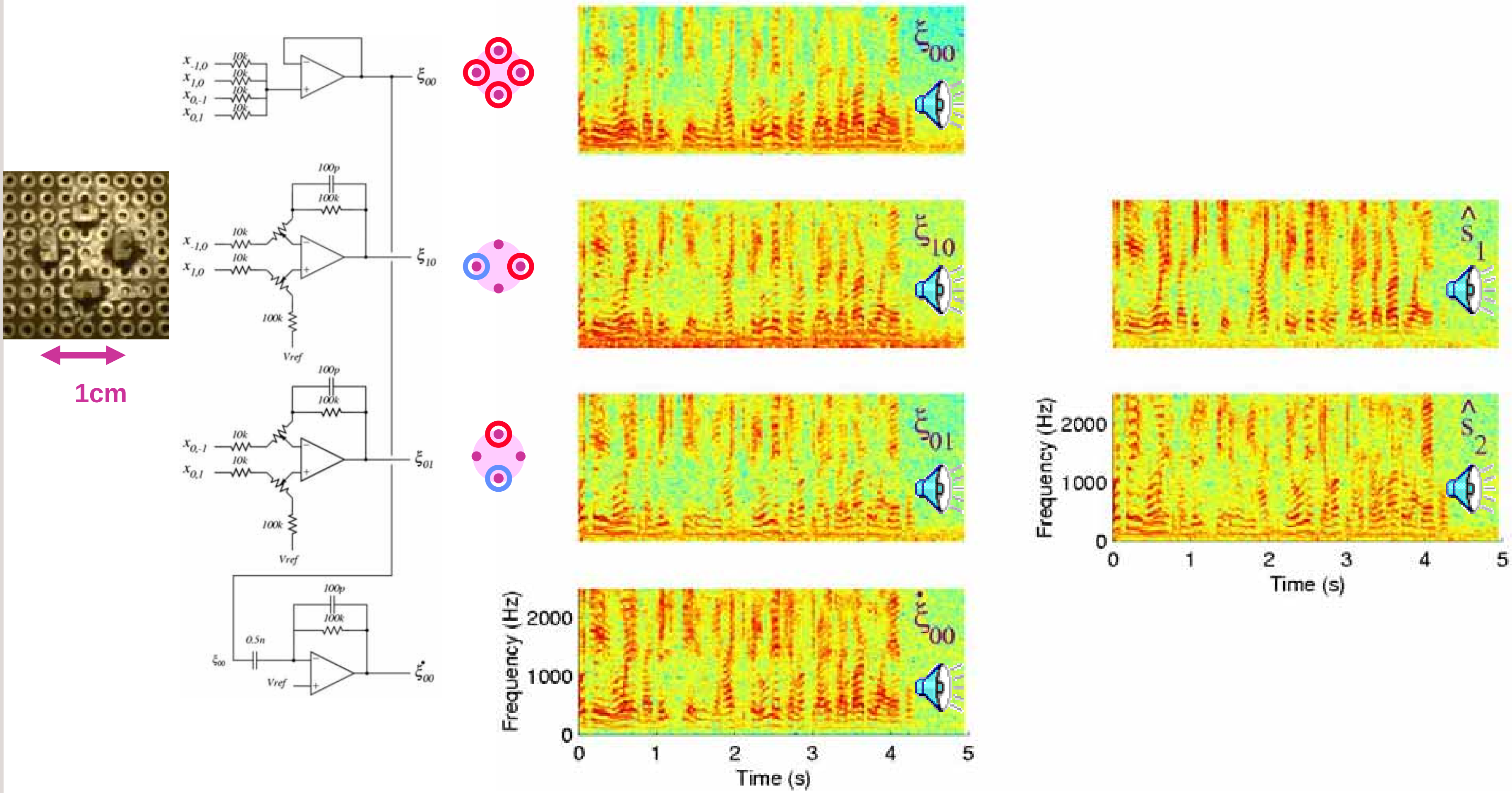
observations	direction	sources	noise
(gradients)	*vectors*	*(time-differentiated)*	*(gradients)*

solved by means of linear *static* ICA

Gradient Flow Acoustic Separation

Outdoors Environment

- 4 microphones within 5 mm radius
- 2 male speakers at 0.5 m, lawn surrounded by buildings at 30 m

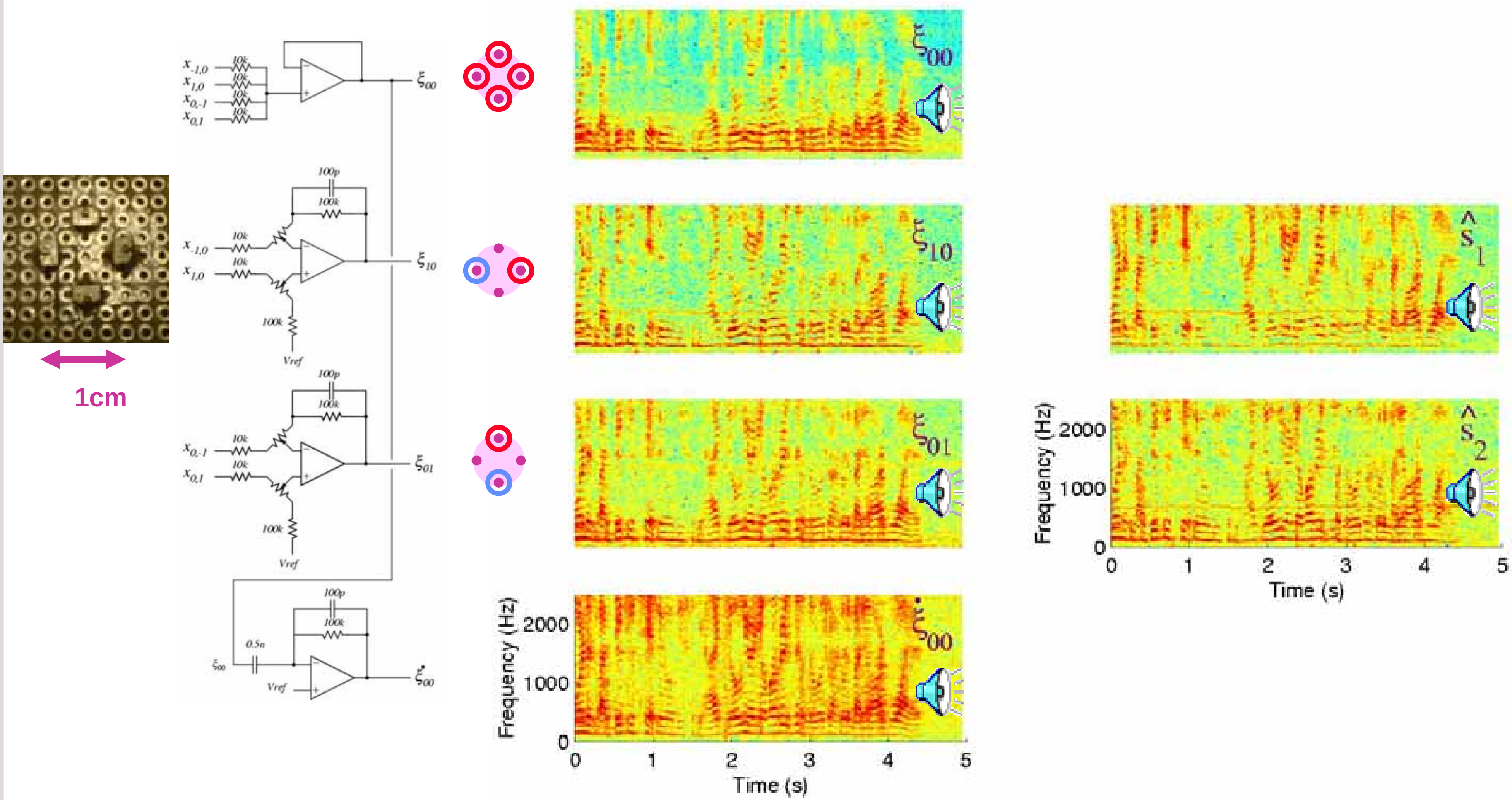


Gradient Flow Acoustic Separation

Indoors Environment

– 4 microphones within 5 mm radius

– 2 male speakers at 0.5 m, reverberant room of dimensions 3, 4 and 8 m



Multipath Wave Propagation

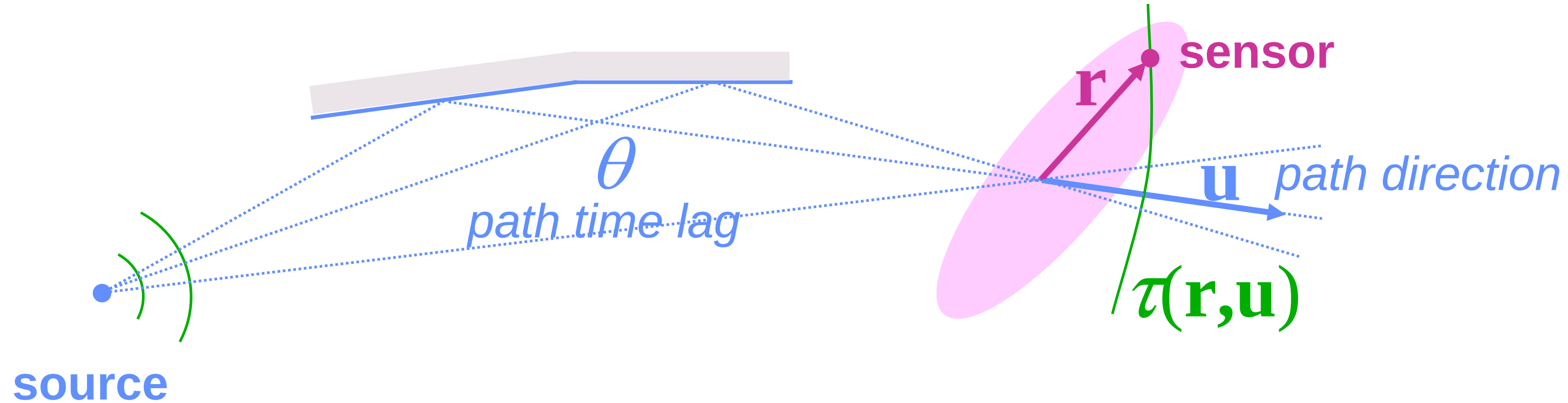
Multipath convolutive wave expansion:

$$S(\mathbf{r},t) = \iint d\mathbf{u}\, d\theta\, A(\mathbf{r},\mathbf{u},\theta)\, s(t-\theta+\tau(\mathbf{r},\mathbf{u},\theta))$$

In the far field:

$$A(\mathbf{r},\mathbf{u},\theta) \equiv A(\mathbf{u},\theta)$$

$$\tau(\mathbf{r},\mathbf{u},\theta) \equiv \tau(\mathbf{r},\mathbf{u}) = \frac{1}{c}\mathbf{r}\cdot\mathbf{u}$$



Multipath Gradient Flow Separation and Localization

Gradient Flow, uniformly sampled above the Nyquist rate:

$$\dot{\xi}_{00}[i] \approx \sum_{\ell} \sum_{j} \alpha^{\ell}[j] \dot{s}^{\ell}[i-j] + \dot{\nu}_{00}[i]$$

$$\xi_{10}[i] \approx \sum_{\ell} \sum_{j} \tau_1^{\ell}[j] \dot{s}^{\ell}[i-j] + \nu_{10}[i]$$

$$\xi_{01}[i] \approx \sum_{\ell} \sum_{j} \tau_2^{\ell}[j] \dot{s}^{\ell}[i-j] + \nu_{01}[i]$$

yields a mixing model of general convolutive form:

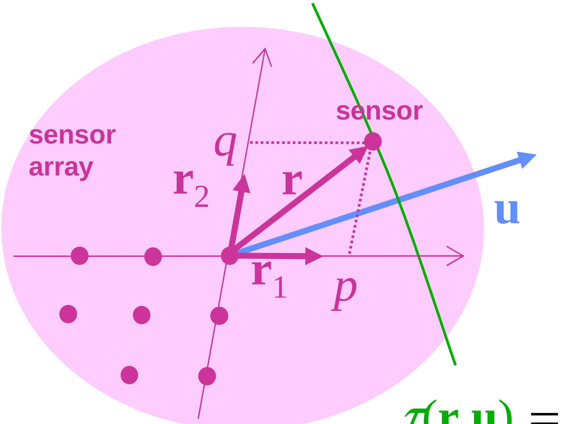
$$\mathbf{x}[i] = \sum_{j} \mathbf{A}[j] \cdot \mathbf{s}[i-j] + \mathbf{n}[i]$$

with moments of multipath distributions over sensor geometry:

$$\alpha^{\ell}[j] = \int_{\mathbf{u}} \int_{\theta=(n-\frac{1}{2})T_s}^{(n-\frac{1}{2})T_s} d\mathbf{u}\, d\theta A^{\ell}(\mathbf{u},\theta)$$

$$\tau_1^{\ell}[j] = \int_{\mathbf{u}} \int_{\theta=(n-\frac{1}{2})T_s}^{(n-\frac{1}{2})T_s} d\mathbf{u}\, d\theta A^{\ell}(\mathbf{u},\theta) \tau(\mathbf{r}_1,\mathbf{u})$$

$$\tau_2^{\ell}[j] = \int_{\mathbf{u}} \int_{\theta=(n-\frac{1}{2})T_s}^{(n-\frac{1}{2})T_s} d\mathbf{u}\, d\theta A^{\ell}(\mathbf{u},\theta) \tau(\mathbf{r}_2,\mathbf{u})$$



$$\tau(\mathbf{r},\mathbf{u}) = \frac{1}{c}\mathbf{r} \cdot \mathbf{u}$$

Scaling Properties

Order *k*, dimension *m*:

$$\underbrace{\xi_{ij\ldots h}}_{m} \approx \sum_{\ell=1}^{L} (\tau_1{}^{\ell})^i (\tau_2{}^{\ell})^j \ldots (\tau_n{}^{\ell})^h \, s_\ell^{(\underbrace{i+j+\ldots+h}_{\leq k})}(t) + \nu_{ij\ldots h}$$

$$\mathbf{x} = \mathbf{A} \cdot \mathbf{s} + \mathbf{n}$$

Maximum separable number of sources L_{max}:

m \ k	0	1	2	3
0	1	1	1	1
1	1	2	3	4
2	1	3	6	10
3	1	4	10	20

- Assumes full-rank **A** with linearly independent mixture combinations
- Depends on the geometry of the source direction vectors relative to the array
- More sources can be separated in the overcomplete case by using prior information on the sources

Noise Characteristics

Mixing model:

$$\mathbf{x} = \mathbf{A} \cdot \mathbf{s} + \mathbf{n}$$

Signal and bearing estimates:

$$\hat{\mathbf{s}} = \hat{\mathbf{A}}^{-1}\mathbf{x} = \underbrace{\hat{\mathbf{A}}^{-1}\mathbf{A} \cdot \mathbf{s}}_{\text{bias}} + \underbrace{\hat{\mathbf{A}}^{-1}\mathbf{n}}_{\text{variance}} \approx \mathbf{s} + \underbrace{\mathbf{A}^{-1}\mathbf{n}}_{= \mathbf{e}}$$

Error covariance:

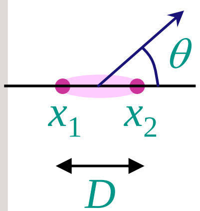
$$E[\mathbf{e}\mathbf{e}^T] = \mathbf{A}^{-1} E[\mathbf{n}\mathbf{n}^T](\mathbf{A}^{-1})^T$$

- Angular directions of the sources (matrix **A**), besides sensor noise, affect the error variance of the estimated sources.
- Determinant of square matrix **A** measures the volume (area) spanned by the direction vectors. When direction vectors are co-planar (co-linear), error variance becomes singular.
- For two sources in the plane with angular separation $\Delta\theta$, the error variance scales as $1/\sin^2(\Delta\theta)$.

Cramer-Rao Lower Bounds on Bearing Estimation

$$\Delta\theta \geq \frac{1}{\sqrt{J}} \qquad J = -\mathrm{E}\left[\frac{\partial L(\theta)}{\partial\theta}\frac{\partial L(\theta)}{\partial\theta}\right]$$ Fisher information

Time Delay:



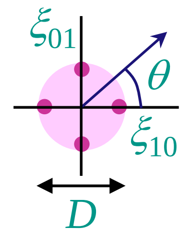
$$\begin{aligned} x_1 &= s(t+\tau\cos\theta)+n_1 \\ x_2 &= s(t-\tau\cos\theta)+n_2 \end{aligned}$$

$2a^2\sin^2\theta\, S$ $\quad$ $N+E$

$$J = T\int df \frac{\left(a\sin\theta\dfrac{2S}{N+E}\right)^2}{1+\dfrac{2S}{N+E}}$$

(Friedlander, 1984)

Gradient Flow:



$$\begin{aligned} \xi_{10} &= \tau\cos\theta\,\dot{s}+\nu_{10} \\ \xi_{01} &= \tau\sin\theta\,\dot{s}+\nu_{01} \end{aligned}$$

a^2S $\quad$ $\frac{1}{2}(a^2N+E)$

$$J = T\int df \frac{\left(\dfrac{2S}{N+E/a^2}\right)^2}{1+\dfrac{2S}{N+E/a^2}}$$

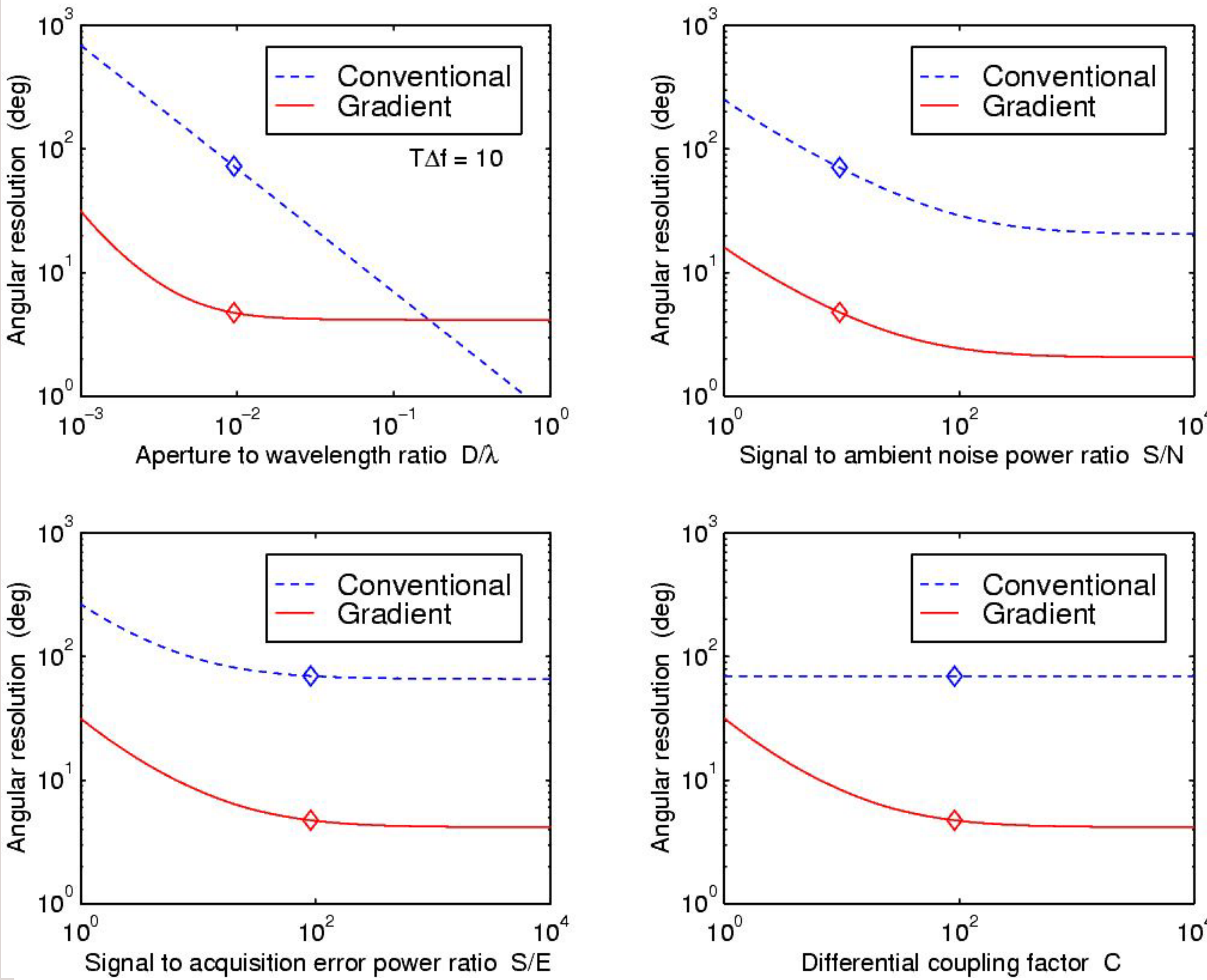
S Signal power

N Ambient noise power

E Acquisition error power

$$a = \omega\tau = 2\pi\frac{|r|}{\lambda} = \pi\frac{D}{\lambda}$$ Aperture

Cramer-Rao Lower Bounds on Bearing Estimation

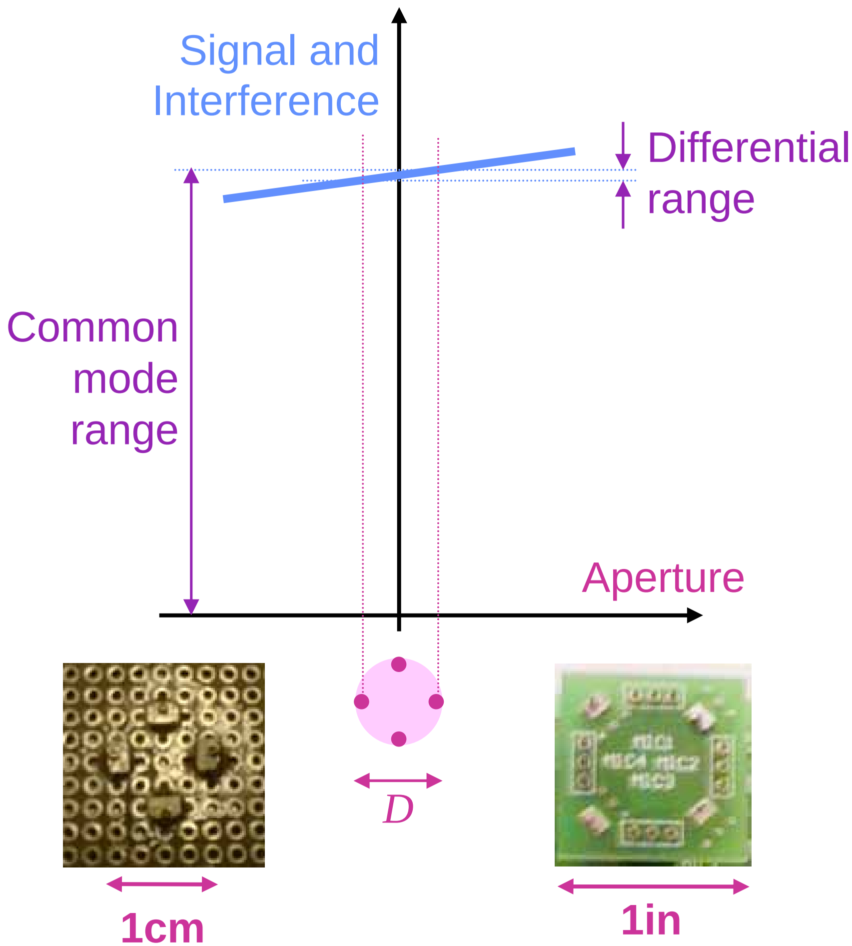


– Conventional:
  - *time delayed source*
  - *uncorrelated noise*

– Gradient:
  - *spatial gradients (ξ_{10} and ξ_{01})*
  - *ambient noise is highly correlated*
  - *mechanical or electrical coupling enhances differential spatial sensitivity*

– Further refinements:
  - *non-Gaussian source statistics*
  - *non-stationary source dynamics*

Differential Sensitivity



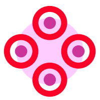
$$\frac{a^2 S}{a^2 N + E} \xrightarrow{diff.coupling} \frac{a^2 S}{a^2 N + \frac{E}{C}}$$

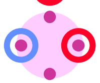
- Cramer-Rao bound on angular precision is fundamentally independent of aperture.
- The sensor and acquisition design challenge is to resolve small signal gradients amidst a large common-mode signal pedestal.
- Differential coupling eliminates the common mode component and boosts the differential sensitivity by a factor *C*, the ratio of differential to common mode signal amplitude range.
- Signal to acquisition error power ratio *S/E* is effectively enhanced by the differential coupling factor *C*.
- *Mechanical* (sensor) and *electrical* (amplifier) differential coupling can be combined to yield large gain *C* > 1,000.

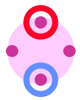
Adaptive Common-Mode Suppression

Systematic common-mode error in finite-difference gradients:

due to gain mismatch across sensors in the array:

$\hat{\xi}_{00}$  $\frac{1}{4}(x_{-1,0} + x_{1,0} + x_{0,-1} + x_{0,1}) \approx \xi_{00} \approx \sum_\ell s^\ell(t)$

$\hat{\xi}_{10}$  $\frac{1}{2}(x_{1,0} - x_{-1,0}) \approx \xi_{10} + \varepsilon_1 \xi_{00} \approx \sum_\ell \tau_1{}^\ell \dot{s}^\ell(t) + \varepsilon_1 \sum_\ell s^\ell(t)$

$\hat{\xi}_{01}$  $\frac{1}{2}(x_{0,1} - x_{0,-1}) \approx \xi_{01} + \varepsilon_2 \xi_{00} \approx \sum_\ell \tau_2{}^\ell \dot{s}^\ell(t) + \varepsilon_2 \sum_\ell s^\ell(t)$

can be eliminated using second order statistics only:

$$\mathrm{E}[\dot{s}^\ell(t) s^m(t)] = 0, \quad \forall \ell, m \quad \Rightarrow \quad \begin{cases} \mathrm{E}[\xi_{00}\xi_{10}] = 0 \\ \mathrm{E}[\xi_{00}\xi_{01}] = 0 \end{cases}$$

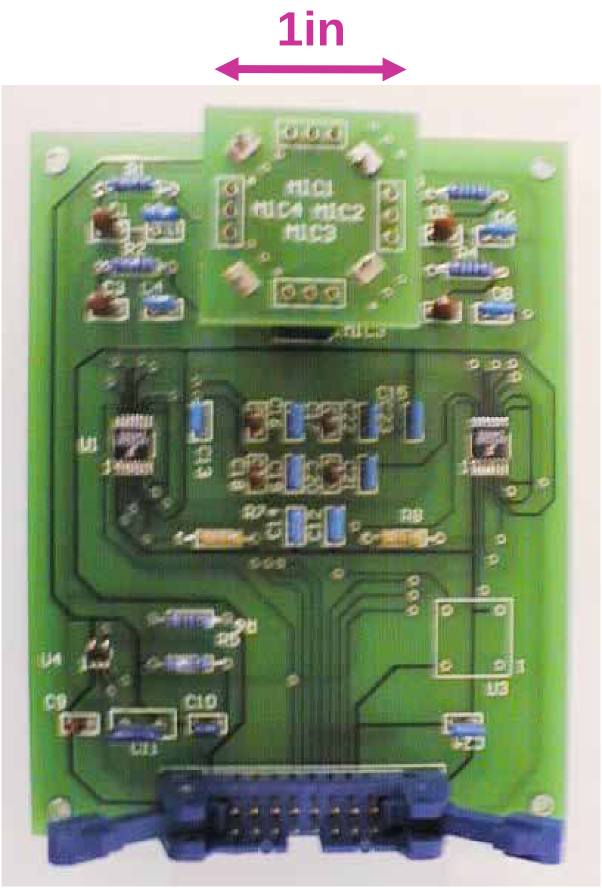
Adaptive LMS calibration:

$$\xi_{10} \approx \hat{\xi}_{10} - \frac{\mathrm{E}[\hat{\xi}_{00}\hat{\xi}_{10}]}{\mathrm{E}[\hat{\xi}_{00}{}^2]} \hat{\xi}_{00}$$

$$\xi_{01} \approx \hat{\xi}_{01} - \frac{\mathrm{E}[\hat{\xi}_{00}\hat{\xi}_{01}]}{\mathrm{E}[\hat{\xi}_{00}{}^2]} \hat{\xi}_{00}$$

Gradient Flow DSP Frontend

Milutin Stanacevic



- 4 miniature microphones
  - *Knowles IM-3246*
  - *100mV/Pa sensitivity (w/ internal preamp)*
  - *100Hz-8kHz audio range*
  - *0.2mW each*
- 2 stereo audio Δ–Σ ADCs
  - *Cirrus Logic CS5333A*
  - *24bit, 96kHz*
  - *11mW active, 0.2mW standby*
- Low-power DSP backend
  - *Texas Instruments TMS320C5204*
  - *100MIPS peak*
  - *0.32mW/MIPS*
- Benchmark, and prototyping testbed, for micropower VLSI miniaturized integration

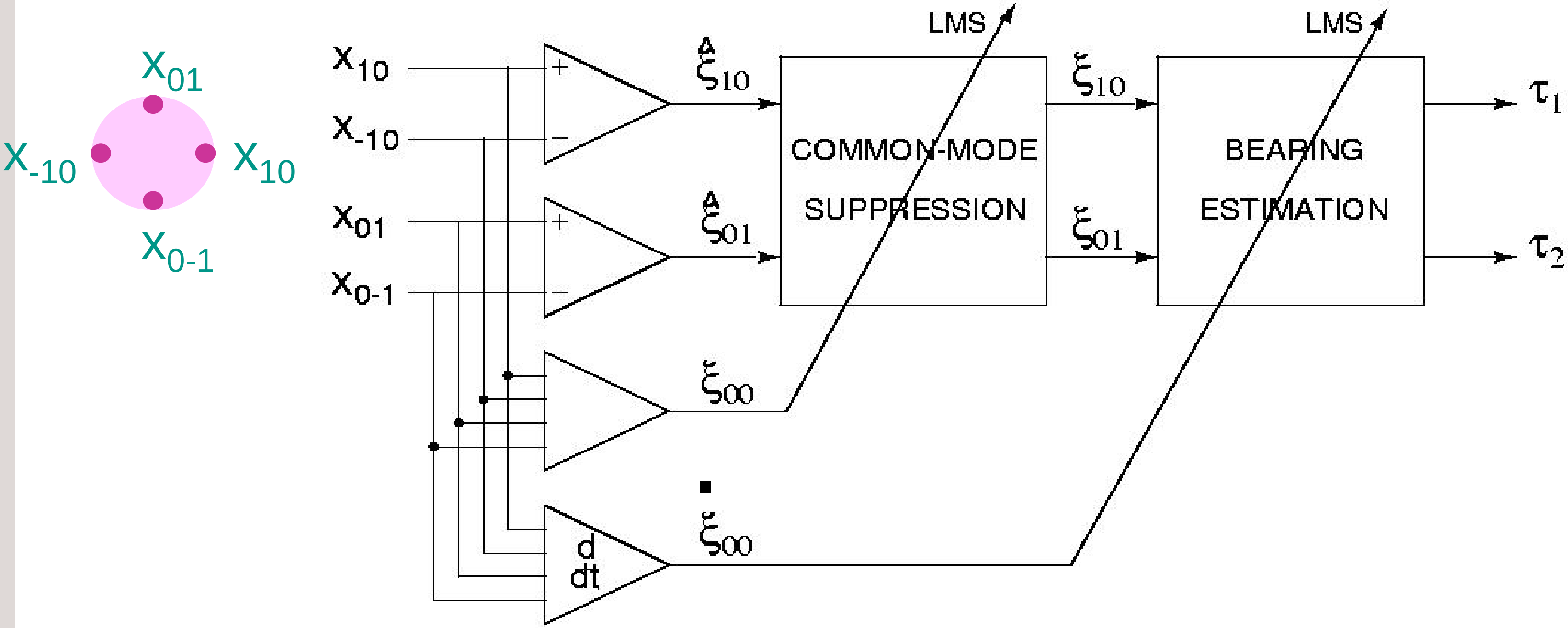
Gradient Flow System Diagram

Analog inputs

Average, temporal derivative and estimated spatial gradients

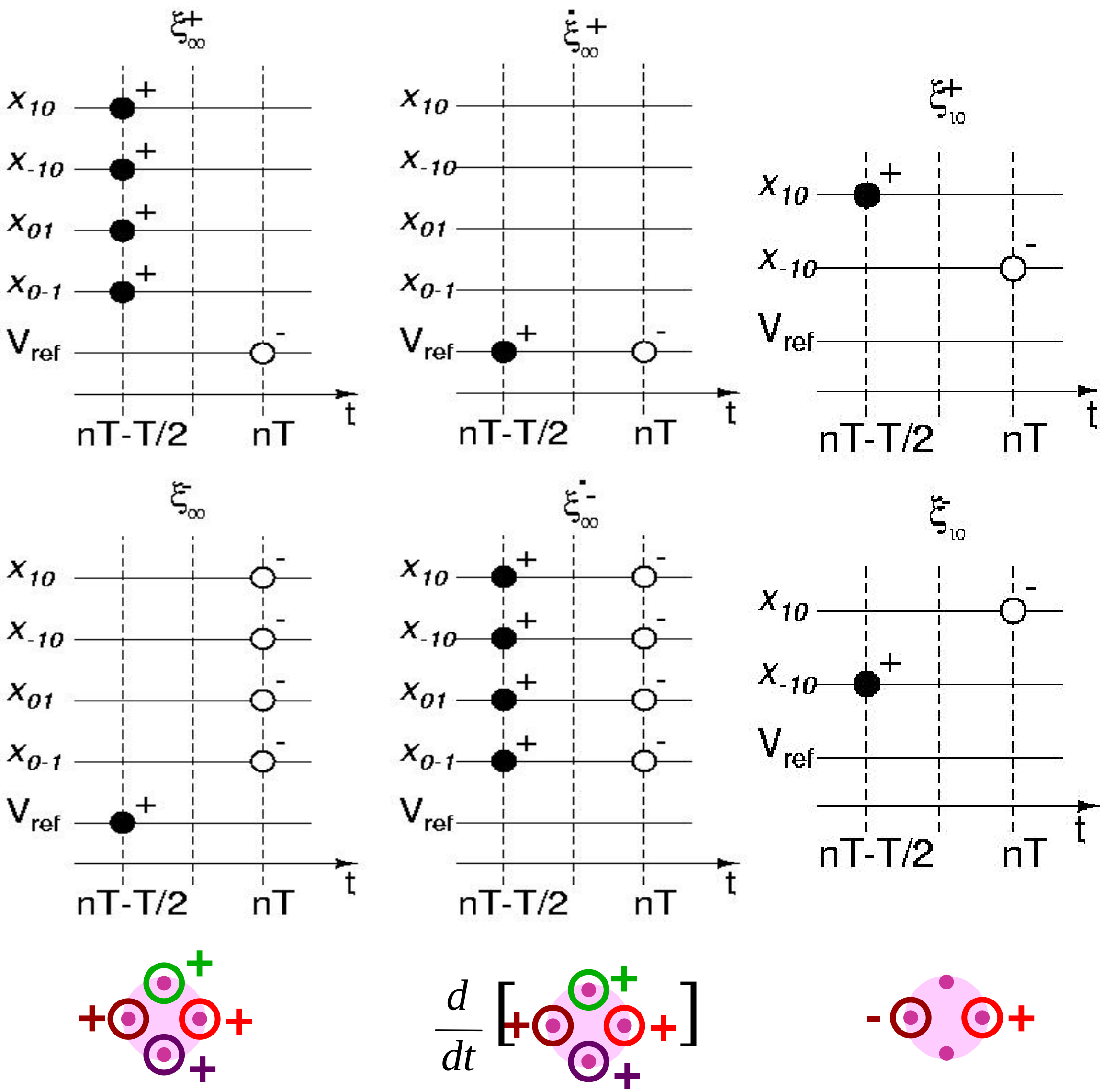
Spatial gradients with suppressed common-mode

Digital estimated delays



- **Least Mean Squares (LMS) digital adaptation**
  - Common mode offset correction for increased sensitivity in the analog differentials
  - 3-D bearing direction cosines
- ***Analog in, digital out (A/D smart sensor)***

CDS Differential Sensing



Switched-capacitor, discrete-time analog signal processing

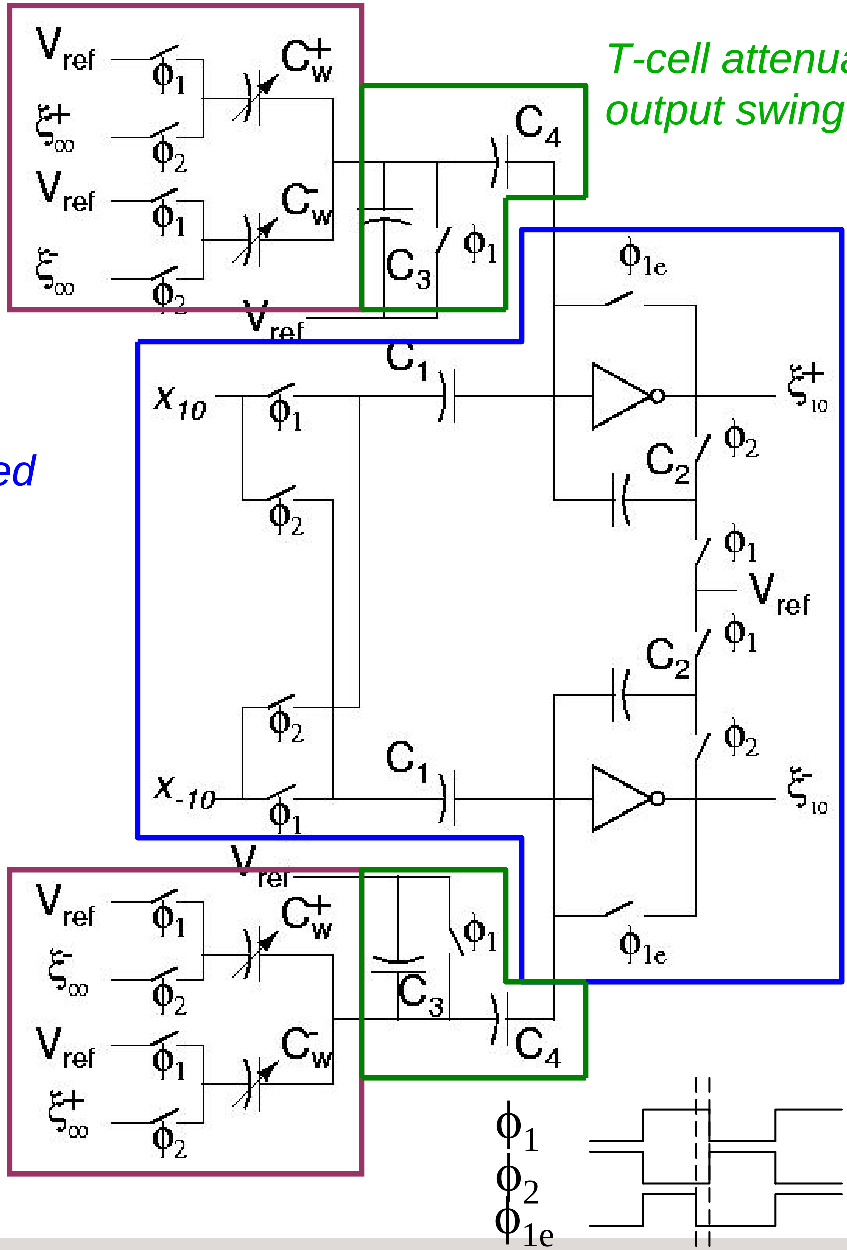
- Correlated Double Sampling (CDS)
  - *Offset cancellation and 1/f noise reduction*
- Fully differential
  - *Clock and supply feedthrough rejection*

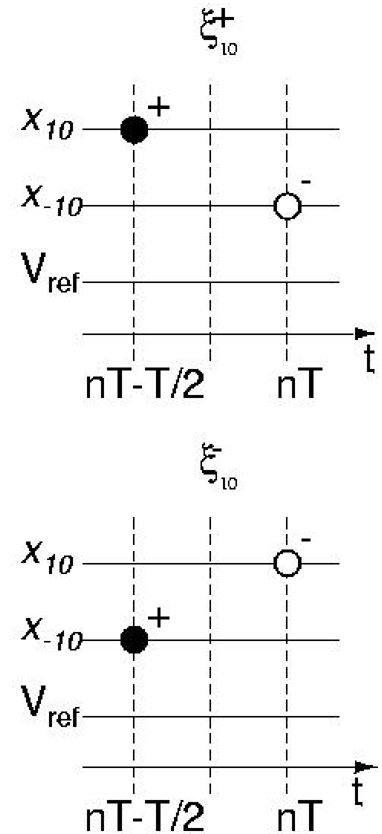
Spatial Gradient Acquisition

Multiplying DAC for common-mode compensation

T-cell attenuates output swing

Uncompensated spatial finite difference computation





$$\hat{\xi}^{+}_{10}[n] = x_{10}[n-\frac{1}{2}] - x_{-10}[n]$$

$$\hat{\xi}^{-}_{10}[n] = x_{-10}[n-\frac{1}{2}] - x_{10}[n]$$

Mixed-Signal LMS Adaptation

- **Two stages**
  - Common mode compensation
  - Delay parameter estimation
- **Sign-sign LMS differential on-line adaptation rule**
  - Delay parameter estimation :

$$e_{10}^{+}[n] = \xi_{10}^{+}[n] - (\tau_1^{+}\,\dot{\xi}_{00}^{+}[n] + \tau_1^{-}\,\dot{\xi}_{00}^{-}[n])$$

$$e_{10}^{-}[n] = \xi_{10}^{-}[n] - (\tau_1^{-}\,\dot{\xi}_{00}^{+}[n] + \tau_1^{+}\,\dot{\xi}_{00}^{-}[n])$$

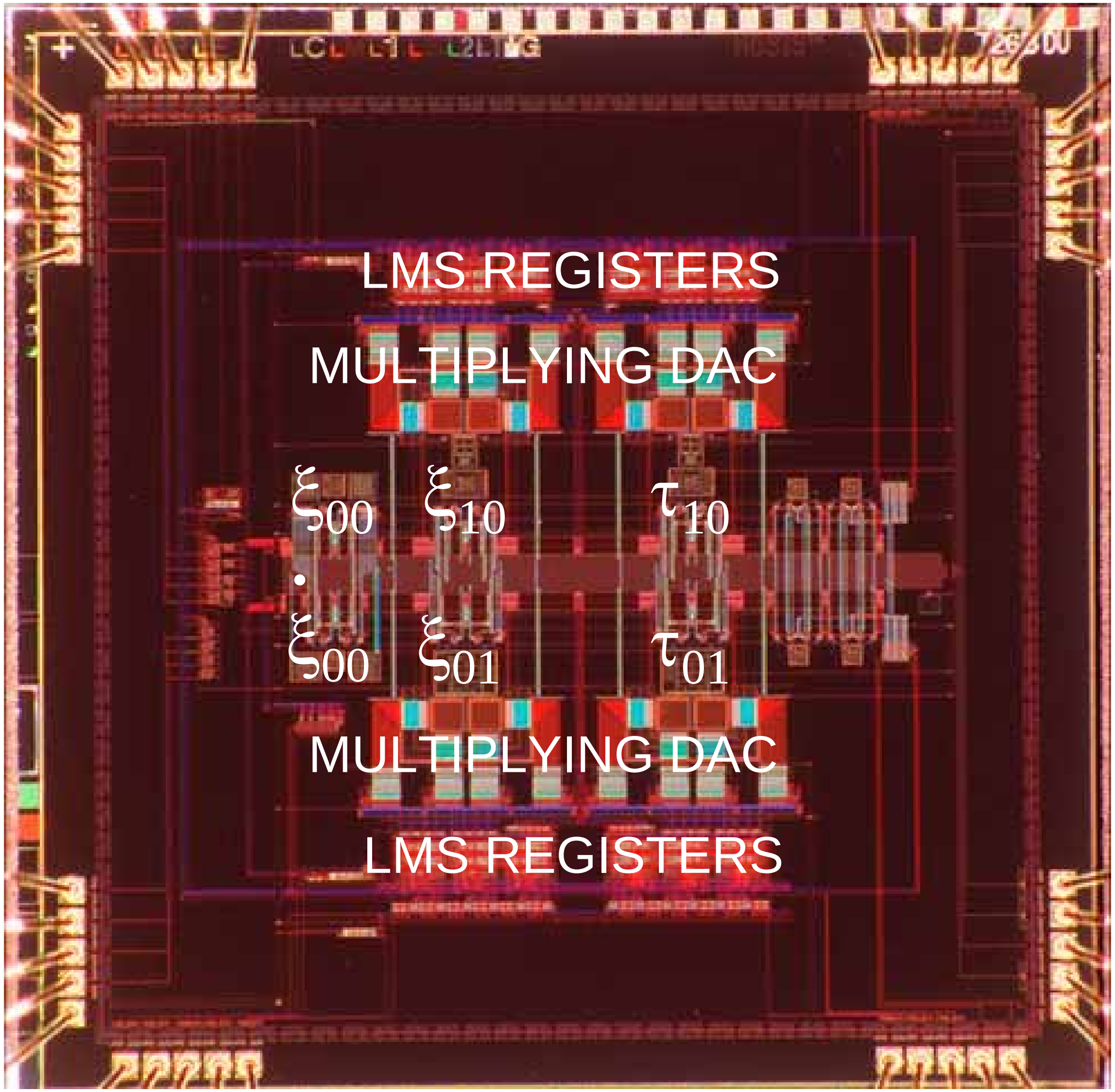
$$\tau_1^{+}[n+1] = \tau_1^{+}[n] + \mathrm{sgn}(e_{10}^{+}[n] - e_{10}^{-}[n])\,\mathrm{sgn}(\dot{\xi}_{00}^{+}[n] - \dot{\xi}_{00}^{-}[n])$$

$$\tau_1^{-} = 2^{n} - 1 - \tau_1^{+}$$

- **Digital storage and update of parameter estimates**
  - 12-bit counter
  - 8-bit multiplying DAC to construct LMS error signal

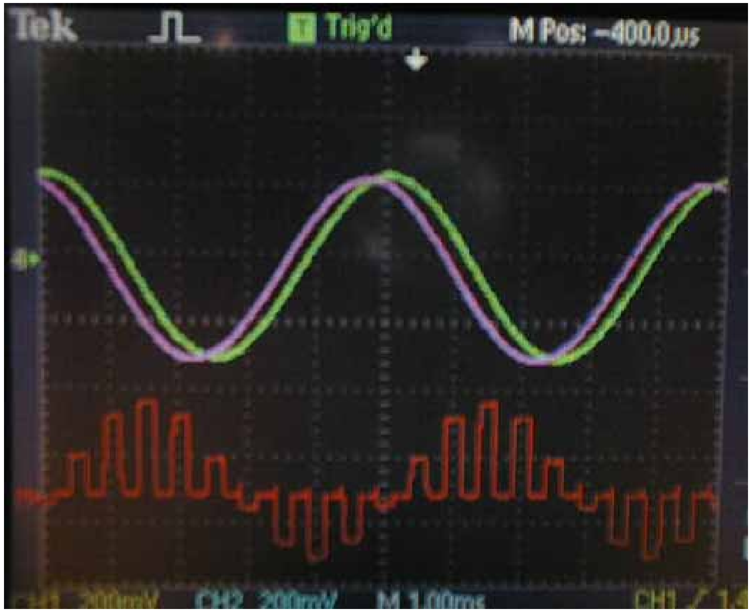
Gradient Flow Processor

Stanacevic and Cauwenberghs (2003)

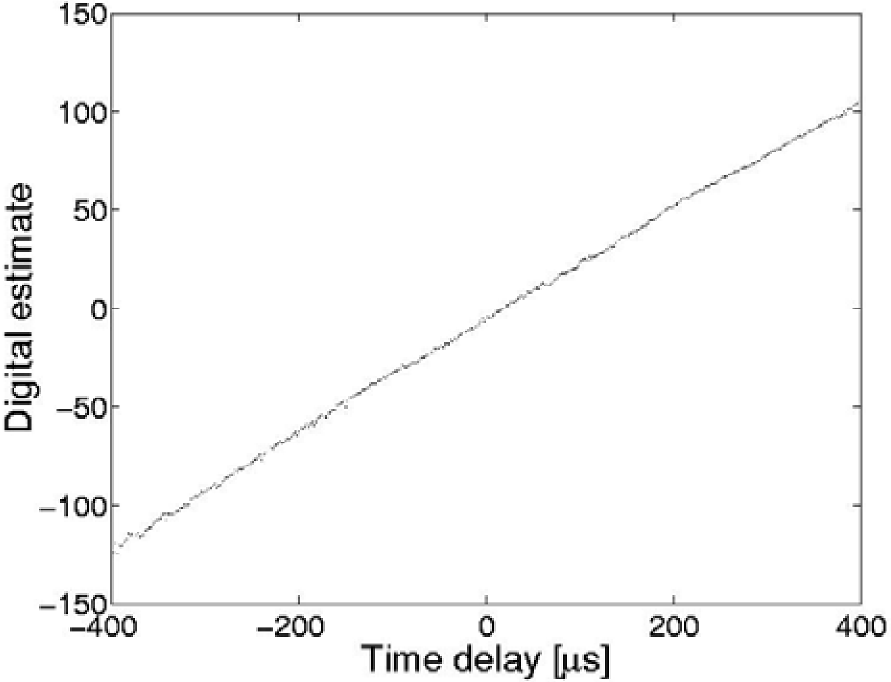


- **Digital LMS adaptive 3-D bearing estimation**
  - ***Analog microphone inputs***
  - ***Digital bearing outputs***
  - ***Analog gradient outputs***
- **8-bit effective digital resolution**
  - ***0.5μs at 240Hz input***
- **3*mm* x 3*mm* in 0.5μ*m* 3M2P CMOS**
- **32μW power dissipation at 10 kHz clock**

GradFlow Delay Estimation



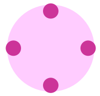
Sinewave inputs and spatial gradient



Digital output - estimated delays

sin(ω t)

sin(ω (t-τ))

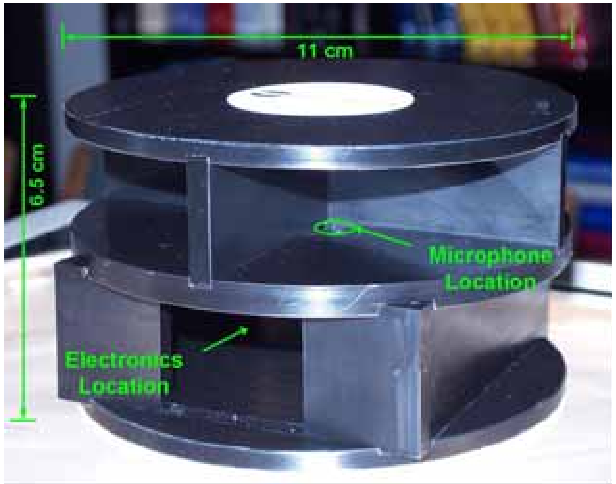


sin(ω t)

sin(ω (t-τ))

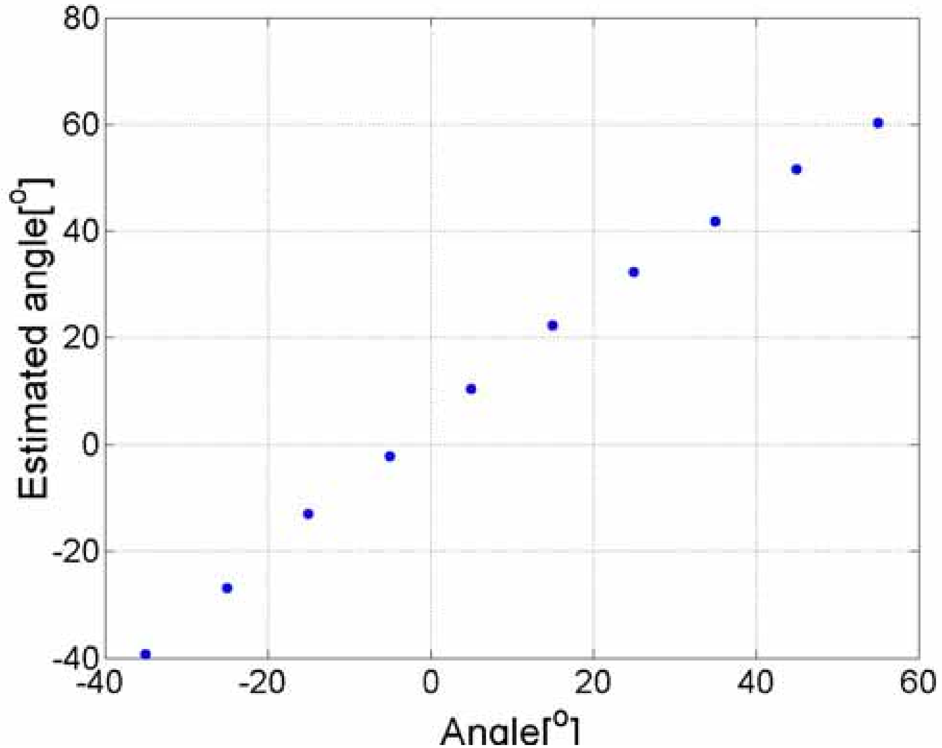
- **200 Hz synthetic sine wave input signals**
- **2 kHz sampling frequency**
- **Time delay varied from -400µs to 400µs in 2µs increments**

GradFlow/ASU Localization



Acoustic Surveillance Unit

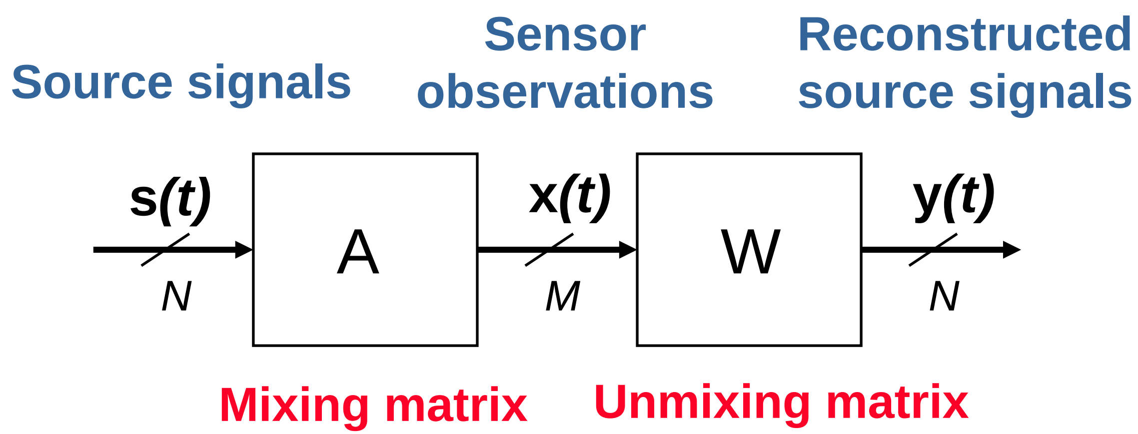
courtesy of Signal Systems Corporation



- **One directional source in open-field environment**
  - ***Band-limited (20-300Hz) Gaussian signal presented through loudspeaker***
- **16cm effective distance between microphones**
- **18m distance between loudspeaker and sensor array**

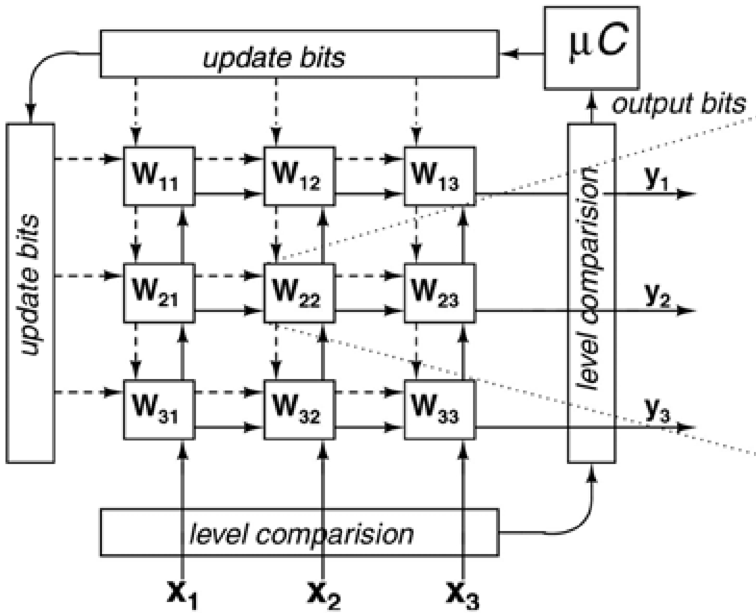
Independent Component Analysis

- The task of blind source separation (BSS) is to separate and recover independent sources from mixed sensor observations, where both the sources and mixing matrix are unknown.

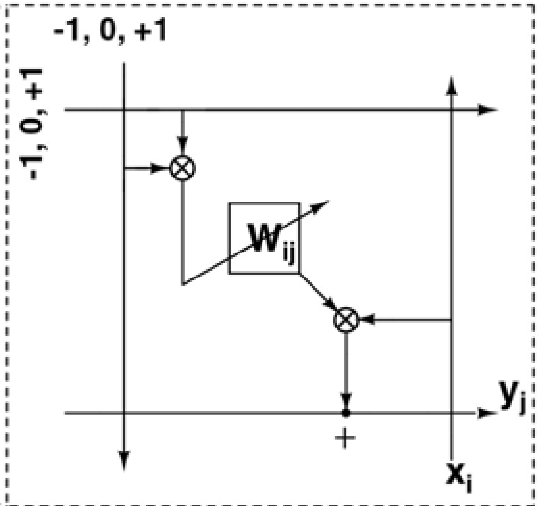


- Independent component analysis (ICA) minimizes higher-order statistical dependencies between reconstructed signals to estimate the unmixing matrix.

ICA System Diagram



System block diagram



Cell functionality

- **Digitally reconfigurable ICA update rule**
  - Jutten-Herault
  - InfoMax
  - SOBI
- **Digital storage and update of weight coefficients**
  - 14-bit counter
  - 8-bit multiplying DAC to construct output signal

Example Mixed-Signal ICA Implementation

- **Implemented ICA update rule :**

$$W[n+1] = W[n] + \mu(I - f(y)y^T)$$

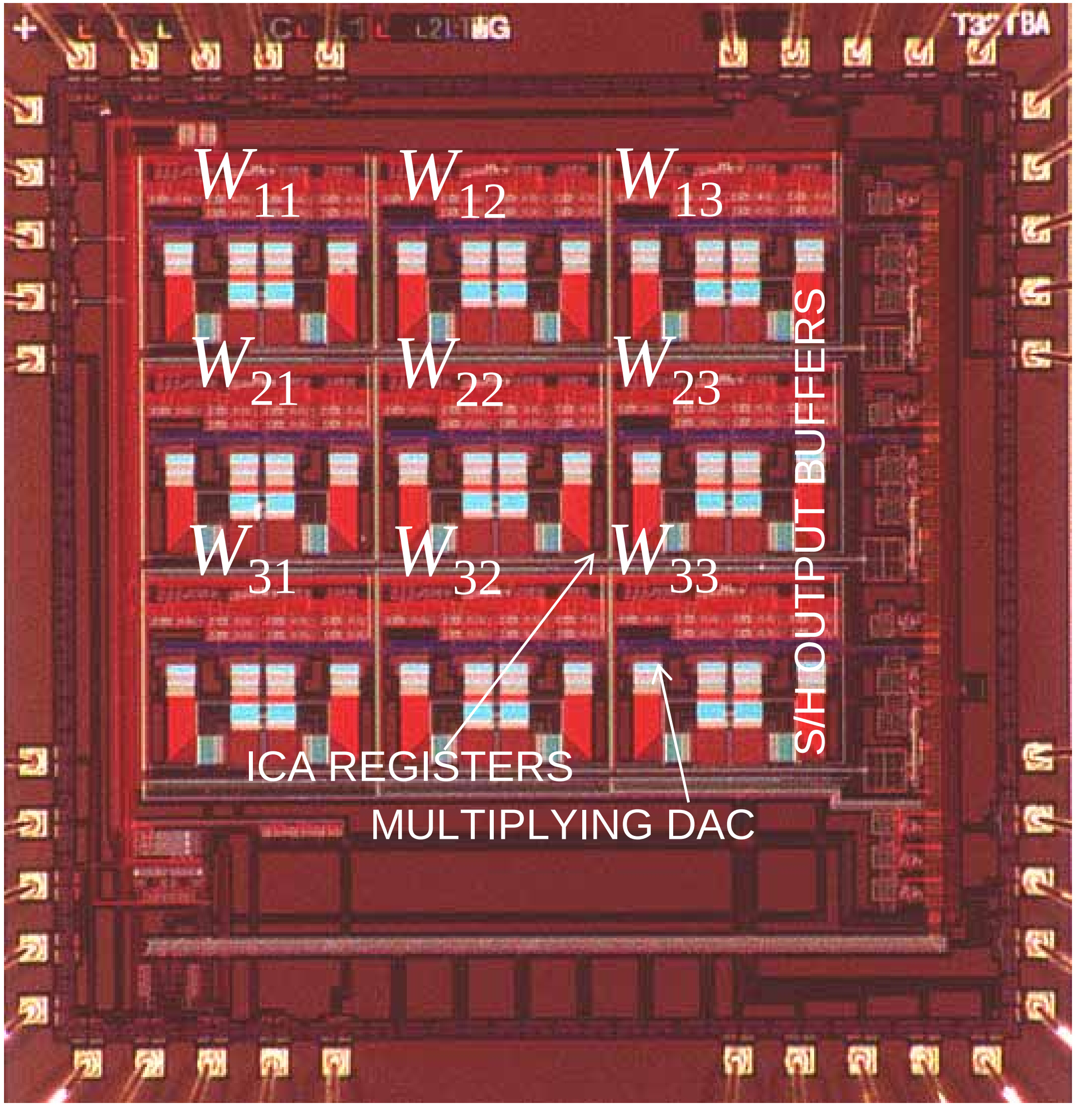
  - Corresponds to the feed-forward version of the *Jutten-Herault* network.
  - Implements the ordinary gradient of the *InfoMax* cost function, multiplied by W^T.

- **For source signals with Laplacian probability density, the distribution optimal function $f(\mathbf{y})$ is $sign(\mathbf{y})$, implemented with a single comparator.**

- **The linear term in the output signal in the update rule is quantized to two bits.**

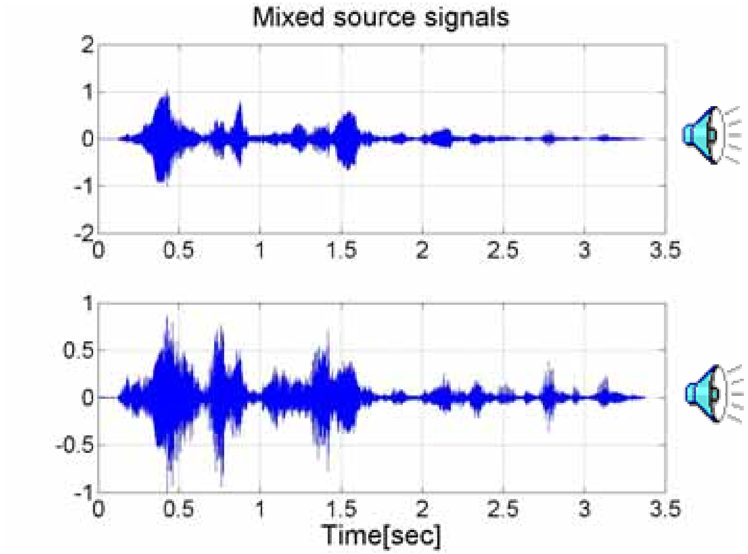
ICA VLSI Processor

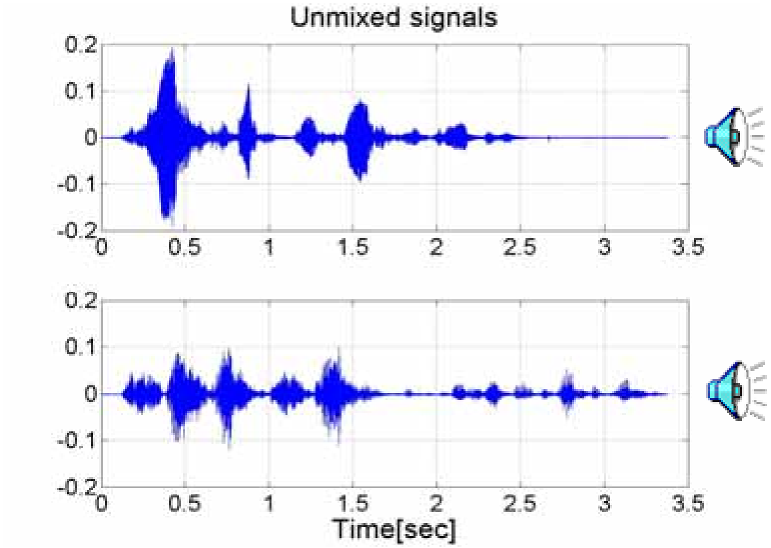
Celik, Stanacevic and Cauwenberghs (2004)

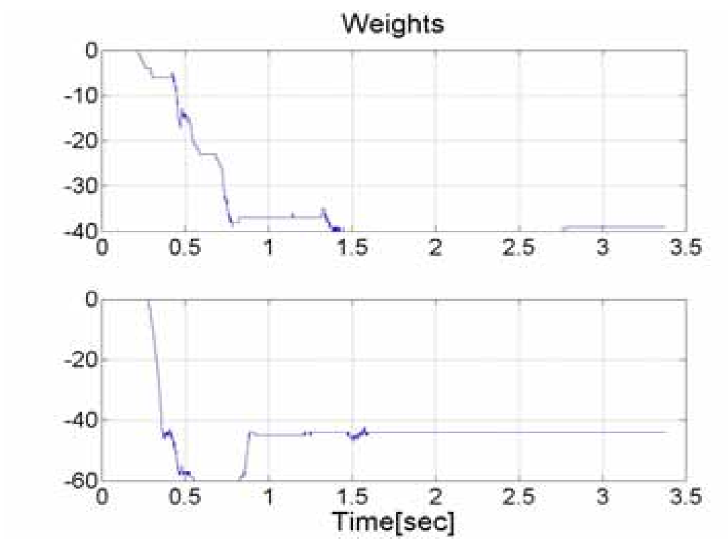


- **3 inputs – sensor signals or gradient flow signals**
- **3 outputs – estimated sources**
- **14-bit digital estimates of unmixing coefficients**
- **3*mm* x 3*mm* in 0.5μ*m* CMOS**
- **180μW power consumption at 16kHz**

ICA Experimental Results







- Two mixed speech signals presented at 16kHz
- InfoMax ICA implemented in VLSI
- 30dB separation in this case

Hearing Aid Implications

- Gradient flow method provides estimates of three independent acoustic sources along with the cosines of the angles of arrival.
- Directional hearing aids amplify the signals in the front plane and suppress the signals in the back plane of the microphone array.
- Gradient flow offers more flexibility in choice of the signal that will be amplified and presented to the listener. The signal can be chosen based on the direction of arrival with respect to microphone array or based on the power of the signal. The estimation of independent sources leads to adaptive suppression of number of noise sources independent of their angle of arrival.

Conclusions

- Wave gradient "flow" converts the problem to that of static ICA, with unmixing coefficients yielding the direction cosines of the sources.
- The technique works for arrays of dimensions smaller than the shortest wavelength in the sources.
- Localization and separation performance is independent of aperture, provided that differential sensitivity be large enough so that ambient interference noise dominates acquisition error noise.
- High resolution delay estimation for source localization using miniature sensor arrays and blind separation of artificially mixed signals with reconfigurable adaptation has been demonstrated.
- System allows integration with sensor array for small, compact, battery-operated "smart" sensor applications in surveillance and hearing aids.

Further Reading

[1] G. Cauwenberghs, M. Stanacevic and G. Zweig, “Blind Broadband Source Localization and Separation in Miniature Sensor Arrays,” *ISCAS’2001*, Sydney Australia, May 2001.

http://bach.ece.jhu.edu/pub/papers/iscas01_ica.pdf

[2] M. Stanacevic, G. Cauwenberghs and G. Zweig, “Gradient Flow Broadband Beamforming and Source Separation,” *ICA’2001*, La Jolla CA, Dec. 2001.

http://bach.ece.jhu.edu/pub/papers/ica2001_gradflow.pdf

[3] M. Stanacevic, G. Cauwenberghs and G. Zweig, “Gradient Flow Adaptive Beamforming and Signal Separation in a Miniature Microphone Array” *ICASSP’2002*, Orlando FL, May 2002.

http://bach.ece.jhu.edu/pub/papers/icassp2002_gradflow.pdf

[4] M. Stanacevic and G. Cauwenberghs, “Mixed-Signal Gradient Flow Bearing Estimation” *ISCAS’2003*, Bangkok, Thailand, May 2003.

http://bach.ece.jhu.edu/pub/papers/iscas03_bearing.pdf

[5] M. Stanacevic and G. Cauwenberghs, “Micropower Mixed-Signal Acoustic Localizer” ESSCIRC*’2003*, Estoril, Portugal, Sept. 2003.

http://bach.ece.jhu.edu/pub/papers/esscirc2003.pdf