BEENG 186B: Principles of Bioinstrumentation Design

Lecture 3

Basic Sensors: Displacement, Strain, and Pressure

References

Webster, Ch. 2 (Sec. 2.1-2.4).
http://en.wikipedia.org/wiki/Potentiometer
http://en.wikipedia.org/wiki/Strain_gauge
http://en.wikipedia.org/wiki/Wheatstone_bridge

BASIC SENSORS

Webster, Chap. 2

A sensor is a type of transducer: it converts a physical parameter into an electrical signal.

Here we will first consider:

- Physical parameters:
  - DISPLACEMENT: main parameter used to measure STRAIN, ACCELERATION, FORCE, STRESS, PRESSURE, etc.
  - TEMPERATURE
- Electrical signals:
  - VOLTAGE or CHARGE, directly transduced
  - IMPEDANCE:

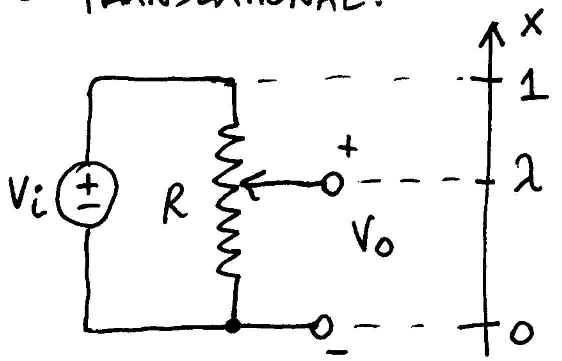
    R : resistive sensor
    L : inductive sensor
    C : capacitive sensor

    indirectly transduced to VOLTAGE through a VOLTAGE DIVIDER or a WHEATSTONE BRIDGE.

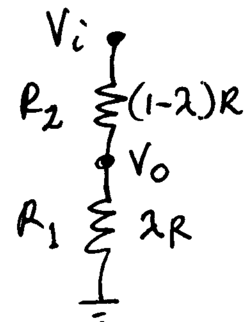
— Resistive displacement sensors

- Potentiometer : a voltage divider which transduces position into voltage through a ratio of resistances:

- TRANSLATIONAL:



linear displacement, $0 \leq \lambda \leq 1$

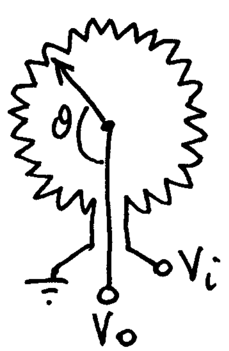


$$V_o = \frac{R_1}{R_1+R_2} V_i$$

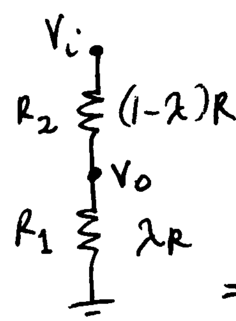
$R_1 = \lambda R$
$R_2 = (1-\lambda)R$

$$\Rightarrow \underbrace{V_o}_{\text{VOLTAGE OUTPUT}} = \underbrace{\lambda}_{\text{TRANSLATION}} . V_i$$

- SINGLE TURN :



angular displacement, $\theta_{min} \leq \theta \leq \theta_{max}$



where $\lambda = \dfrac{\theta - \theta_{min}}{\theta_{max} - \theta_{min}}$

$$\Rightarrow \underbrace{V_o}_{\text{VOLTAGE OUTPUT}} = \underbrace{\frac{\theta - \theta_{min}}{\theta_{max} - \theta_{min}}}_{\text{RELATIVE TURN}} \cdot V_i$$

typically θ_{min} : close to 0
θ_{max} : close to 2π ($360°$)

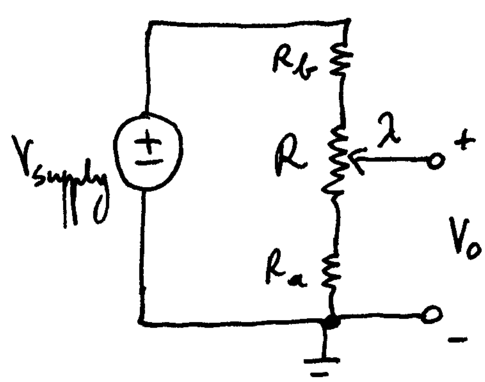
- MULTITURN : N windings $\Rightarrow$
θ_{min} : close to 0
θ_{max} : close to $2\pi N$ ($360° \times N$)

NOTE :

Typically, V_i is a supply voltage V_{supply}, such as a battery. The output V_o then ranges between GND (0 V) and V_{supply}.

Problem: some signal processing circuits can't handle inputs close to the power supply rails (GND and V_{supply}), so that small and large displacements λ (turns) may be cut off.

Solution: add series resistors between the supplies and the potentiometer:



$$\left.\right\} R_2 = R_b + (1-\lambda)R$$

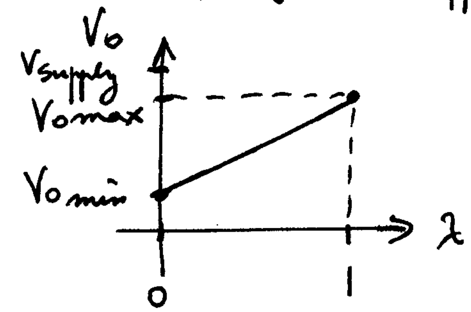
$$\left.\right\} R_1 = R_a + \lambda R$$

$$0 \leq \lambda \leq 1$$

$$V_o = \frac{R_1}{R_1+R_2} \cdot V_{supply}$$

$$\Downarrow$$

$$V_o = \frac{R_a + \lambda R}{R_a + R_b + R} \cdot V_{supply}$$



$$V_{o\,min} : \frac{R_a}{R_a+R_b+R} \cdot V_{supply}$$

$$V_{o\,max} : \frac{R_a+R}{R_a+R_b+R} \cdot V_{supply}$$

Example : $V_{supply} = 5\,V$

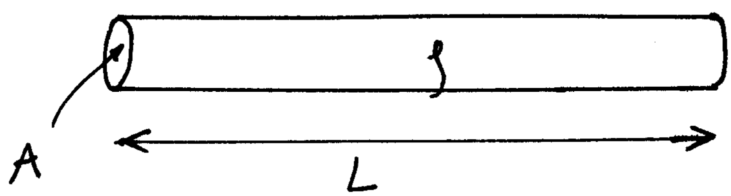
$R = 10\,k\Omega$

Choose R_a and R_b such that the output range is $1\,V \leq V_o \leq 4\,V$ (1 V of margin on both sides).

$$\longrightarrow R_a = R_b = \frac{R}{3} = 3.33\,k\Omega$$

— Strain gauge (or "gage") : more sensitive for very small displacements

Wire resistance depends on geometry that changes with STRAIN



ρ : resistivity $[\Omega . m]$

Resistance : $R = \rho \cdot \frac{L}{A}$ depends on ↗ MATERIAL (ρ) ↘ GEOMETRY (L, A)

A very small change in length $L \to L + dL$ results in a change in resistivity $\rho \to \rho + d\rho$ and area $A \to A + dA$, which combine into a change in resistance $R \to R + dR$:

$$dR = \frac{\rho}{A} dL - \frac{\rho L}{A^2} dA + \frac{L}{A} d\rho \text{ , or:}$$

$$\frac{dR}{R} = \frac{dL}{L} - \frac{dA}{A} + \frac{d\rho}{\rho}$$

Poisson's ratio relates change in diameter to change in length:

$$\frac{dD}{D} = -\mu \cdot \frac{dL}{L} \quad \text{where} \quad A = \frac{\pi D^2}{4} \text{ , or } \frac{dA}{A} = 2 \frac{dD}{D}$$

μ ↓ POISSON'S RATIO (≈ 0.3 for metals)

$$\Rightarrow \frac{dA}{A} = -2\mu \cdot \frac{dL}{L}$$

$$\Rightarrow \quad \frac{dR}{R} = \underbrace{(1+2\mu)\cdot\frac{dL}{L}}_{\text{DIMENSIONAL EFFECT}} + \underbrace{\frac{d\rho}{\rho}}_{\text{PIEZO-RESISTIVE EFFECT}}$$

Gauge factor ("gage" factor) G: ratio of (small) relative changes in resistance and length:

$$G \stackrel{\text{def.}}{=} \frac{\Delta R/R}{\Delta L/L} = 1 + 2\mu + \frac{\Delta\rho/\rho}{\Delta L/L} \qquad (\Delta L/L \ll 1)$$

depends on the material:

- metals: $\Delta\rho/\rho \approx 0$ and $\mu \approx 0.5 \Rightarrow G \approx 2$
- semiconductors: $\Delta\rho/\rho \,/\, \Delta L/L \approx \pm 100 \Rightarrow G \approx \pm 100$, but temperature sensitive $\Rightarrow$ requires temperature compensation such as in a bridge.

<u>NOTE</u>: Several measurands fit the displacement category:

- STRAIN: $\varepsilon \stackrel{\text{def}}{=} \frac{\Delta L}{L} \Rightarrow \frac{\Delta R}{R} = G \cdot \varepsilon$
- STRESS: $\sigma \stackrel{\text{def}}{=} \frac{F}{A}$ force per unit area

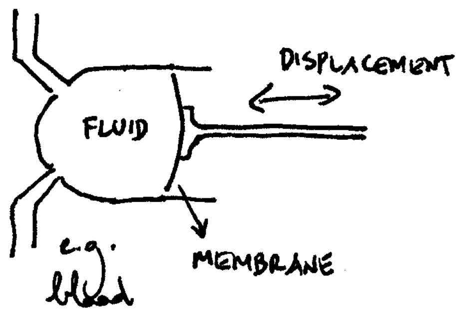
  Young's Modulus: $E = \frac{\sigma}{\varepsilon} = \frac{F/A}{\Delta L/L}$ depends on the material

  $$\Rightarrow \frac{\Delta R}{R} = G\cdot\varepsilon = \frac{G}{E}\cdot\sigma$$
- PRESSURE: like stress, but for fluids

etc...

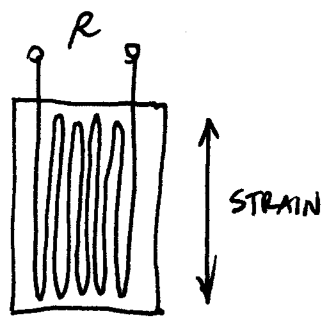
Physical strain gauge:

- unbonded, e.g: pressure sensor



fluid pressure
→ membrane displacement
→ wire elongation
→ resistance change

- bonded

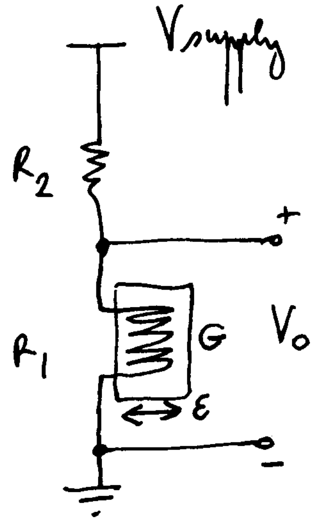


wire on film/foil substrate which is conformal to host surface and hence directly measures its strain.

N turns
→ N times greater resistance R
→ N times greater sensitivity $\Delta R / \varepsilon$

- typically embedded in a voltage divider, or a differential voltage divider (Wheatstone bridge), ideally with temperature compensation.
It is best to match resistance values and their temperature coefficients, at least in pairs.

Example: Voltage divider structure with single strain gauge:



$$V_o = \frac{R_1}{R_1+R_2} \cdot V_{supply}$$

$$dV_o = \left(\frac{dR_1}{R_1+R_2} - \frac{R_1}{(R_1+R_2)^2} \cdot dR_1 \right) V_{supply}$$

$$= \frac{R_2 \cdot dR_1}{(R_1+R_2)^2} \cdot V_{supply}$$

where $dR_1 = G \cdot \varepsilon \cdot R_1$

$$\Rightarrow \quad dV_o = \frac{R_1 R_2}{(R_1+R_2)^2} \cdot G \cdot \varepsilon \cdot V_{supply}$$

$\Rightarrow$ Sensitivity: $\dfrac{dV_o}{\varepsilon} = \dfrac{R_1 R_2}{(R_1+R_2)^2} \cdot G \cdot V_{supply}$

- Find R_2 to maximize sensitivity

$$\Rightarrow \quad R_2 = R_1 \quad \Rightarrow \quad \frac{R_1 R_2}{(R_1+R_2)^2} = \frac{1}{4}$$

- What is the temperature coefficient if R_1 and R_2 have identical temperature coefficients?

$$R_1 \to (1+\alpha\Delta T) R_1$$
$$R_2 \to (1+\alpha\Delta T) R_2$$

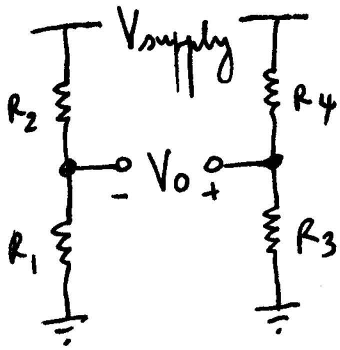
$$\Rightarrow \quad \frac{R_1 R_2}{(R_1+R_2)^2} \to \frac{\cancel{(1+\alpha\Delta T)^2}}{\cancel{(1+\alpha\Delta T)^2}} \cdot \frac{R_1 R_2}{(R_1+R_2)^2}$$

$\Rightarrow$ zero temperature coefficient in the sensitivity

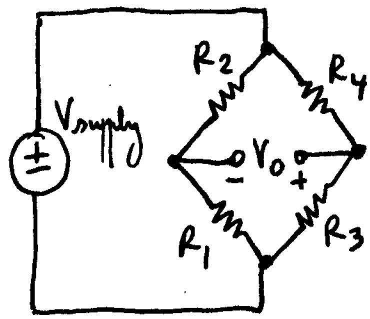
$\rightarrow$ TEMPERATURE COMPENSATED SENSITIVITY

WHEATSTONE BRIDGE (or "bridge" for short):

Differential combination of two voltage dividers
- linear
- zero offset
- high sensitivity
- temp. compensated



or:



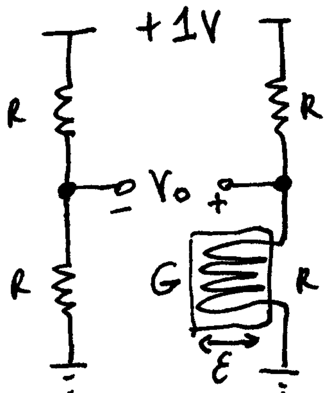
$$\Rightarrow V_0 = \left(\frac{R_3}{R_3+R_4} - \frac{R_1}{R_1+R_2} \right) V_{supply}$$

or $V_0 = 0$ for $\frac{R_1}{R_2} = \frac{R_3}{R_4}$ (balanced)

Ideally $R_1 = R_2 = R_3 = R_4$ for maximum sensitivity.

Any of R_1, R_2, R_3, R_4 or their combinations can be strain gauges, ideally differentially matched in pairs.

Examples:

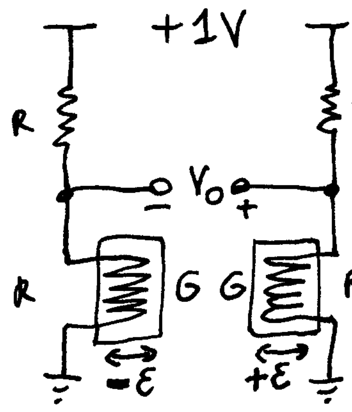


$\Rightarrow$ SENSITIVITY:

$$\frac{V_0}{\varepsilon} = \frac{1}{4} G . 1V$$

$\rightarrow$ LOWEST

not T comp.

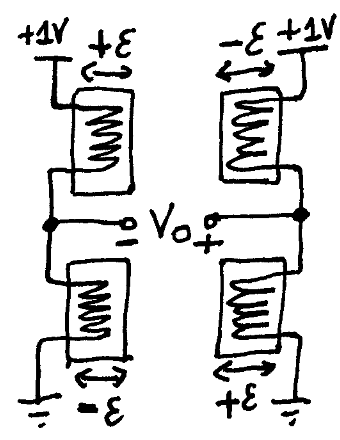


differentially activated (one pulls, the other pushes)

$\Rightarrow$ SENSITIVITY:

$$\frac{V_0}{\varepsilon} = \frac{1}{2} G . 1V$$

T comp.



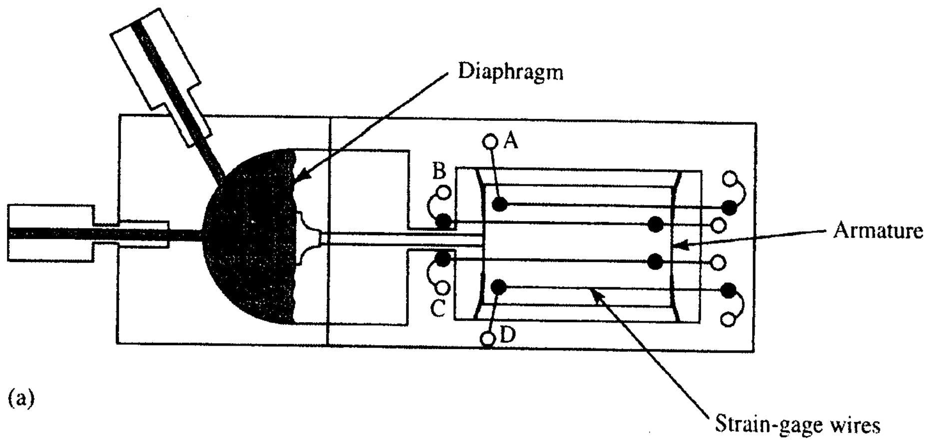
double differentially activated

$\Rightarrow$ SENSITIVITY:

$$\frac{V_0}{\varepsilon} = G . 1V$$

$\rightarrow$ HIGHEST

T comp.



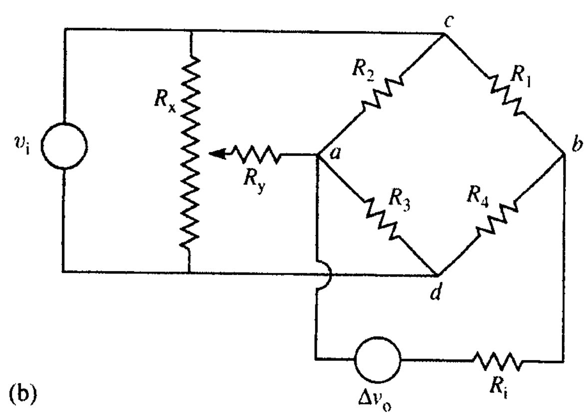


Figure 2.2 (a) Unbonded strain-gage pressure sensor. The diaphragm is directly coupled by an armature to an unbonded strain-gage system. With increasing pressure, the strain on gage pair B and C is increased, while that on gage pair A and D is decreased. (b) Wheatstone bridge with four active elements: $R_1 = \mathrm{B}$, $R_2 = \mathrm{A}$, $R_3 = \mathrm{D}$, and $R_4 = \mathrm{C}$ when the unbonded strain gage is connected for translational motion. Resistor R_y and potentiometer R_x are used to initially balance the bridge, v_i is the applied voltage, and Δv_o is the output voltage on a voltmeter or similar device with an internal resistance of R_i.