BENG 186B Winter 2024

Quiz 1

Friday, January 26, 2024

***Name** (Last, First):* SOLUTIONS

- This quiz is on-line, open-book, and open-notes, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due January 26, 2024 at 11:59pm, over Canvas Gradescope. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 3 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

1. **[20 pts]** Circle the **best answer (only one answer per question)**:

   (a) [4 pts] The Thévenin and Norton equivalents of an electrical circuit:

      i. represent the circuit as a source in series with an impedance.
      ii. assume a perfect load with infinite impedance.
      [✓ iii.] have identical impedance.
      iv. none of the above.

   (b) [4 pts] The accuracy of a bioinstrument:

      i. is independent of precision.
      ii. can be improved by calibration.
      iii. is half of its resolution at worst.
      [✓ iv.] all of the above.

   (c) [4 pts] At zero frequency an inductor:

      i. is an open circuit.
      ii. induces a voltage.
      iii. has zero magnetic field.
      [✓ iv.] none of the above.

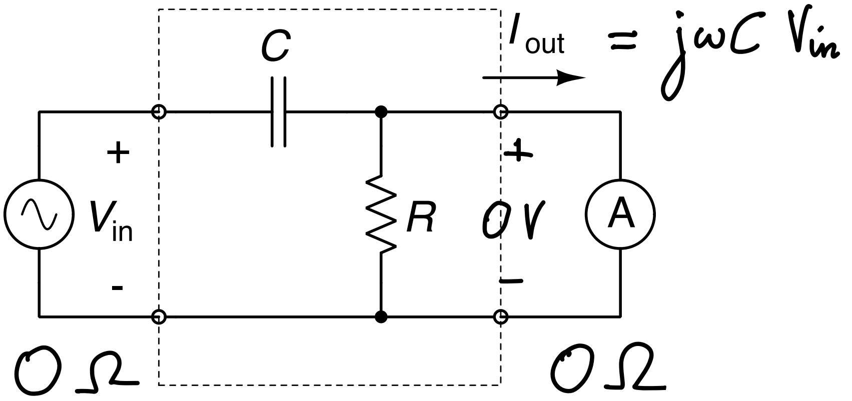
   (d) [4 pts] The piezoelectric effect causes:

      [✓ i.] a change in voltage with pressure.
      ii. a change in resistance with strain.
      iii. a stress proportional to strain.
      iv. all of the above.

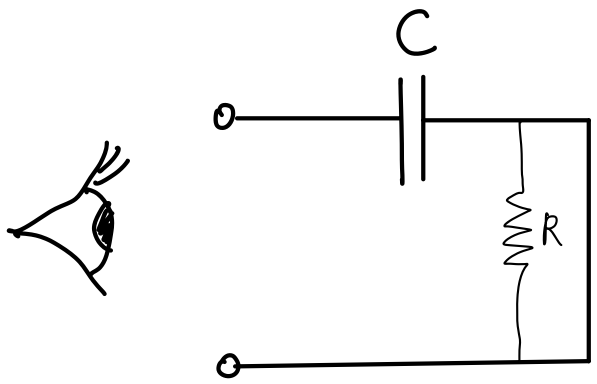
   (e) [4 pts] The sensitivity of a Wheatstone bridge sensor is maximized by:

      i. choosing equal resistances.
      ii. maximizing the supply voltage.
      iii. adding complementary sensors in a double differential configuration.
      [✓ iv.] all of the above.

2. **[40 pts]** Consider the *voltage-input*, *current-output* filter circuit below. You may assume an ideal voltage source for V_{in}, and an ideal ammeter for I_{out}.

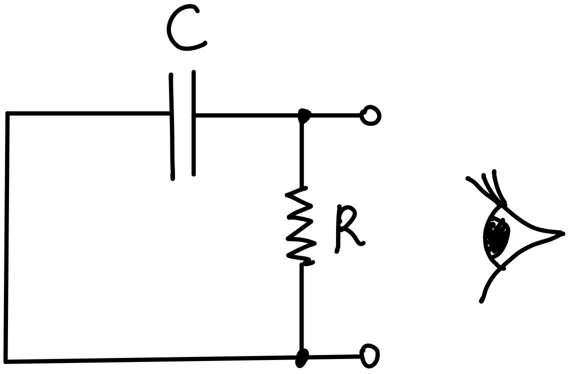


(a) [10 pts] Find the input impedance $Z_{in}(j\omega)$.



$$Z_{in}(j\omega) = \frac{1}{j\omega C}$$

(b) [10 pts] Find the output impedance $Z_{out}(j\omega)$.

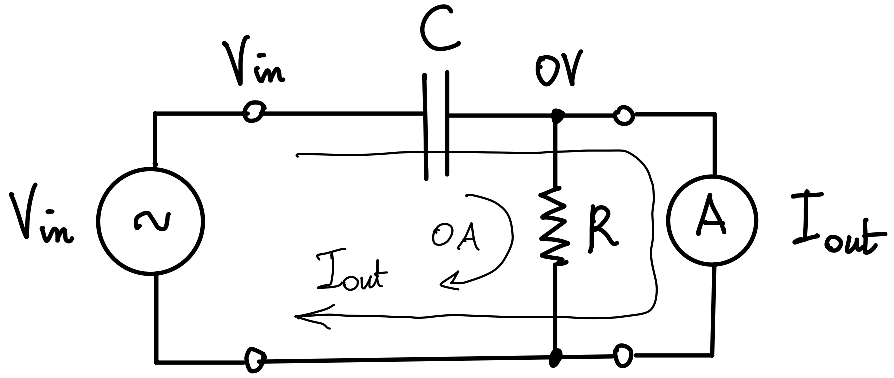


$$Z_{out}(j\omega) = R \parallel \frac{1}{j\omega C}$$

$$= \frac{R\,\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$= \frac{R}{1 + j\omega RC}$$

(c) [10 pts] Find the transfer function $H(j\omega) = I_{out}(j\omega) \,/\, V_{in}(j\omega)$. Verify the units.



$$V_C = V_{in} - 0V = \frac{1}{j\omega C} \cdot I_{out}$$

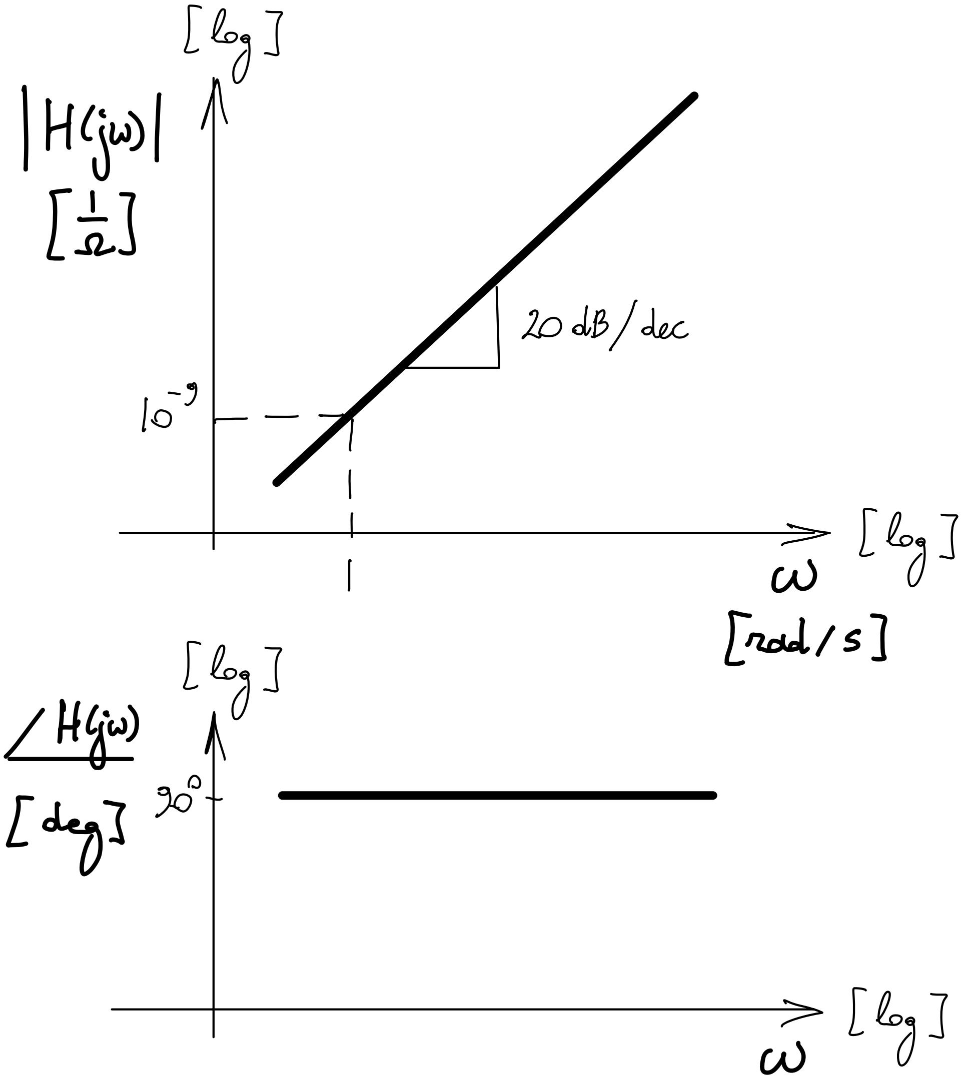
$$\Rightarrow I_{out} = j\omega C \, V_{in}$$

$$H(j\omega) = \frac{I_{out}(j\omega)}{V_{in}(j\omega)} = j\omega C$$

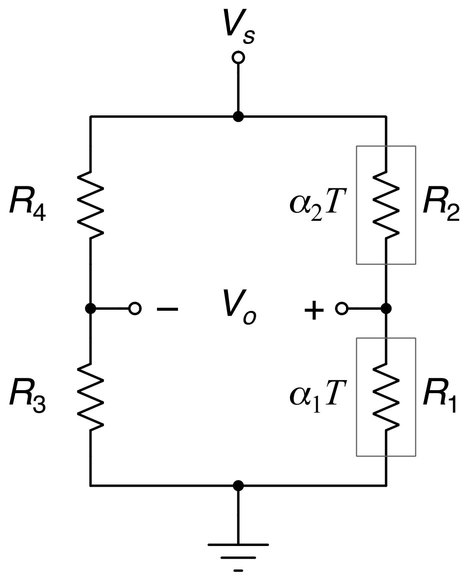
units : $\frac{1}{\Omega}$

(d) [10 pts] Sketch the Bode plot of the transfer function $H(j\omega)$ for $C = 1$ nF and $R = 100$ kΩ. Be sure to label the axes and indicate the units.

$$H(j\omega) = j\omega C$$



3. **[40 pts]** Consider the temperature transducer below, with constant supply voltage $V_s = 1$ V, a first thermistor R_1 with nominal resistance R_{nom} and temperature coefficient α_1, a second thermistor R_2 with same nominal resistance but with complementary temperature coefficient $\alpha_2 = -\alpha_1$, and two identical resistors $R_3 = R_4 = R$. The transducer produces a differential output voltage V_o in response to temperature T acting on both thermistors.



$$\begin{aligned} R_1 &= R_{\text{nom}}\,(1+\alpha_1 T) \\ R_2 &= R_{\text{nom}}\,(1+\alpha_2 T) \end{aligned}$$

(a) [10 pts] Find the output voltage V_o as a function of temperature T. Is the voltage response linear in temperature, and why?

$$V_o = \frac{R_1}{R_1+R_2} V_s - \frac{R_3}{R_3+R_4} V_s$$

$$= \left(\frac{\cancel{R_{nom}}(1+\alpha_1 T)}{\cancel{R_{nom}}(2+\underbrace{(\cancel{\alpha_1+\alpha_2})}_{=0}T)} - \frac{1\cancel{R}}{2\cancel{R}} \right) V_s$$

$$= \left(\frac{1}{2}(\cancel{1}+\alpha_1 T) - \cancel{\frac{1}{2}} \right) V_s = \frac{1}{2}\alpha_1 V_s \cdot T$$

linear in T!

($\alpha_2 = -\alpha_1$)

(b) [10 pts] Find the sensitivity and the offset of the transducer.

$$V_0 = \underbrace{\frac{1}{2}\alpha_1 V_s}_{S = \frac{1}{2}\alpha_1 V_s} \cdot T + \underbrace{0}_{\text{Offset} = 0}$$

(c) [10 pts] The flexible wearable temperature sensor is mounted on the skin of a patient to monitor body temperature. Due to stretching during movement the thermistors and resistors are all subject to the same strain ϵ. If all four resistances have the same strain gauge factor G, show that the voltage response of the transducer is insensitive to strain.

The voltage output only depends on ratios of resistances. All resistances change by the same relative amount under strain, and so their ratios are independent of strain, so that the transducer is insensitive to strain.

$$R_1 = R_{nom}\left(1 + \alpha_1 T\right)\left(1 + G\varepsilon\right)$$

$$R_2 = R_{nom}\left(1 + \alpha_2 T\right)\left(1 + G\varepsilon\right)$$

$$R_3 = R_4 = R\left(1 + G\varepsilon\right)$$

$$V_0 = \frac{R_1}{R_1 + R_2} V_s - \frac{R_3}{R_3 + R_4} V_s$$

$$= \left(\frac{R_{nom}\left(1 + \alpha_1 T\right)\cancel{\left(1 + G\varepsilon\right)}}{R_{nom}\left(2 + (\alpha_1 + \alpha_2)T\right)\cancel{\left(1 + G\varepsilon\right)}} - \frac{R\cancel{\left(1 + G\varepsilon\right)}}{2R\cancel{\left(1 + G\varepsilon\right)}} \right) V_s$$

independent of ε

(d) [10 pts] The wearable sensor is powered by a lithium-ion battery which is subject to voltage variations. How do the offset and sensitivity change for a 10 % drop in the voltage supplied by the battery? Explain.

The sensitivity is linearly proportional to the supply voltage Vs, and the offset is zero no matter what. So, a 10% drop in the supply Vs causes the sensitivity to drop by the same 10%, and the offset remains at zero.

$$V_0 = \underbrace{\frac{1}{2}\alpha_1 V_s}_{} \cdot T + \underbrace{0}_{}$$

$$S = \frac{1}{2}\alpha_1 V_s \qquad \text{Offset} = 0$$

$$\uparrow 10\% \qquad \uparrow 10\%$$