BENG 186B Winter 2024

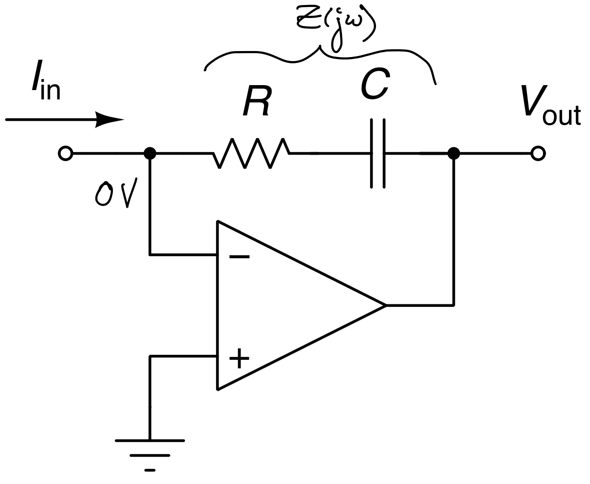
Quiz 2

Friday, February 16, 2024

***Name** (Last, First):* SOLUTIONS

- This quiz is on-line, open-book, and open-notes, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due February 16, 2024 at 11:59pm, over Canvas Gradescope. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 4 problems. Points for each problem are given in **[brackets]**. There are 100 points total, but also a bonus +10 points challenge.

1. **[25 pts]** Consider the *current-in*, *voltage-out* active filter circuit below. Assume the operational amplifier is ideal and unsaturated.



(a) [15 pts] Derive the transfer function $H(j\omega) = V_{out}(j\omega) \,/\, I_{in}(j\omega)$. Find any poles and zeros, and describe what this filter does at low frequencies, and at high frequencies. What is the cut-off frequency where the filter transitions between these two modes of operation?

$$0V - V_{out} = Z(j\omega) \cdot I_{in}$$

$$\Rightarrow H(j\omega) = \frac{V_{out}(j\omega)}{I_{in}(j\omega)} = -Z(j\omega) = -\left(R + \frac{1}{j\omega C}\right)$$

$$= -\frac{1 + j\omega RC}{j\omega C} \quad \begin{array}{l} \longrightarrow \text{ZERO @ } s = -\frac{1}{RC} \\ \longrightarrow \text{POLE @ } s = 0 \end{array}$$

$$\omega \ll \omega_c = \frac{1}{RC} \Rightarrow H(j\omega) \approx -\frac{1}{j\omega C}$$ Inverting integrator

$$\omega \gg \underset{\downarrow}{\omega_c = \frac{1}{RC}} \Rightarrow H(j\omega) \approx -R$$ Inverting gain

Cut-off (radial) frequency

(b) [5 pts] What is the input impedance at the I_{in} node?

0 because I_{in} is at a virtual ground.

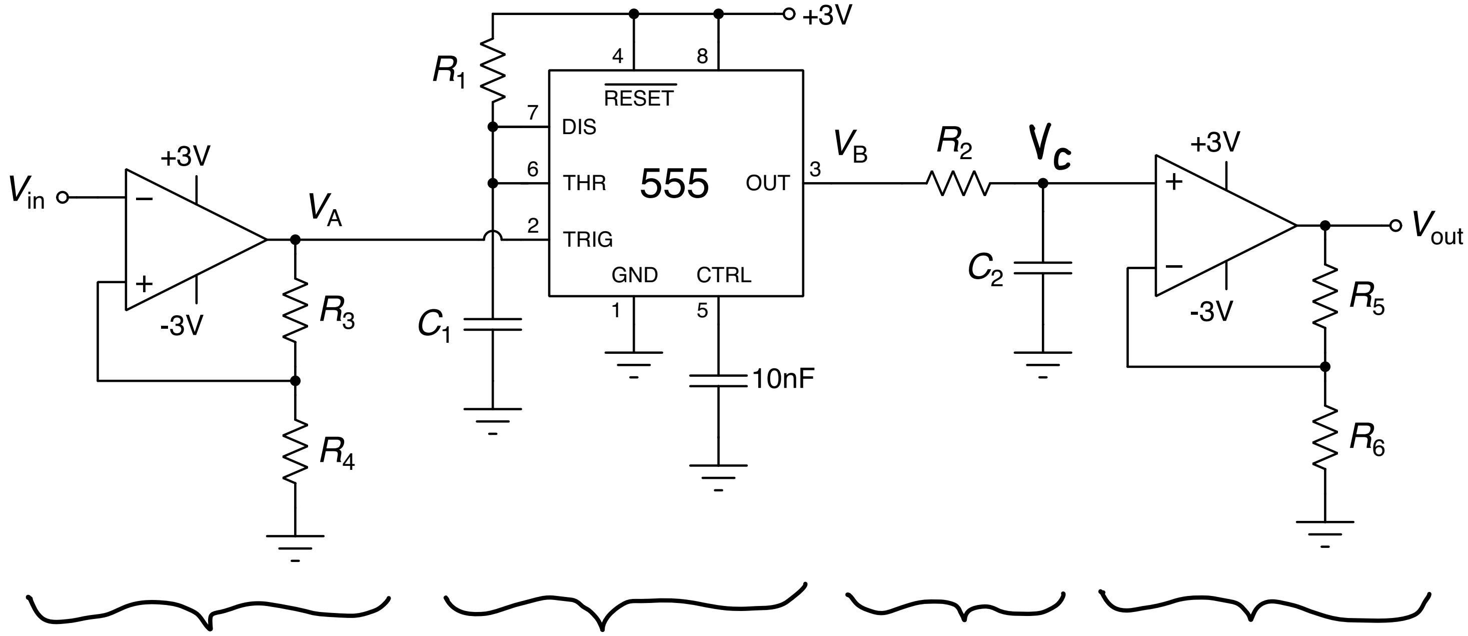
(c) [5 pts] What is the output impedance at the V_{out} node?

0 because V_{out} feeds back to the inverting input of the opamp.

2. **[30 pts]** Consider the heart rate monitor circuit below with ideal 555 and opamp components, and passive components $R_1 = 909\,\text{k}\Omega$, $C_1 = 0.5\,\mu\text{F}$, $R_2 = 10\,\text{M}\Omega$, $C_2 = 10\,\mu\text{F}$, $R_3 = 999\,\text{k}\Omega$, $R_4 = 1\,\text{k}\Omega$, and $R_5 = R_6 = 100\,\text{k}\Omega$. You may also find these equations useful for the 555 timer ($\ln(3) \approx 1.1$ and $\ln(2) \approx 0.7$):

$$T = \ln(3) \times RC \qquad T_{lo} = \ln(2) \times R_2 C \qquad T_{hi} = \ln(2) \times (R_1 + R_2)C$$

On the diagrams on the next page sketch the waveforms for the voltages $V_A(t)$, $V_B(t)$, and $V_{out}(t)$ for the given waveform for $V_{in}(t)$. Note the different voltage scales! Ignore any initial transients, and assume that the circuit has been running forever and settled into steady state by time $t = 0$. Show your work below.



Inverting hysteretic comparator

Monostable single-shot

Low pass filter

Non-inverting amplifier

$$V_{ref}^{\pm} = \frac{R_4}{R_3+R_4}(\pm 3\text{V})$$
$$= \pm 3\,\text{mV}$$

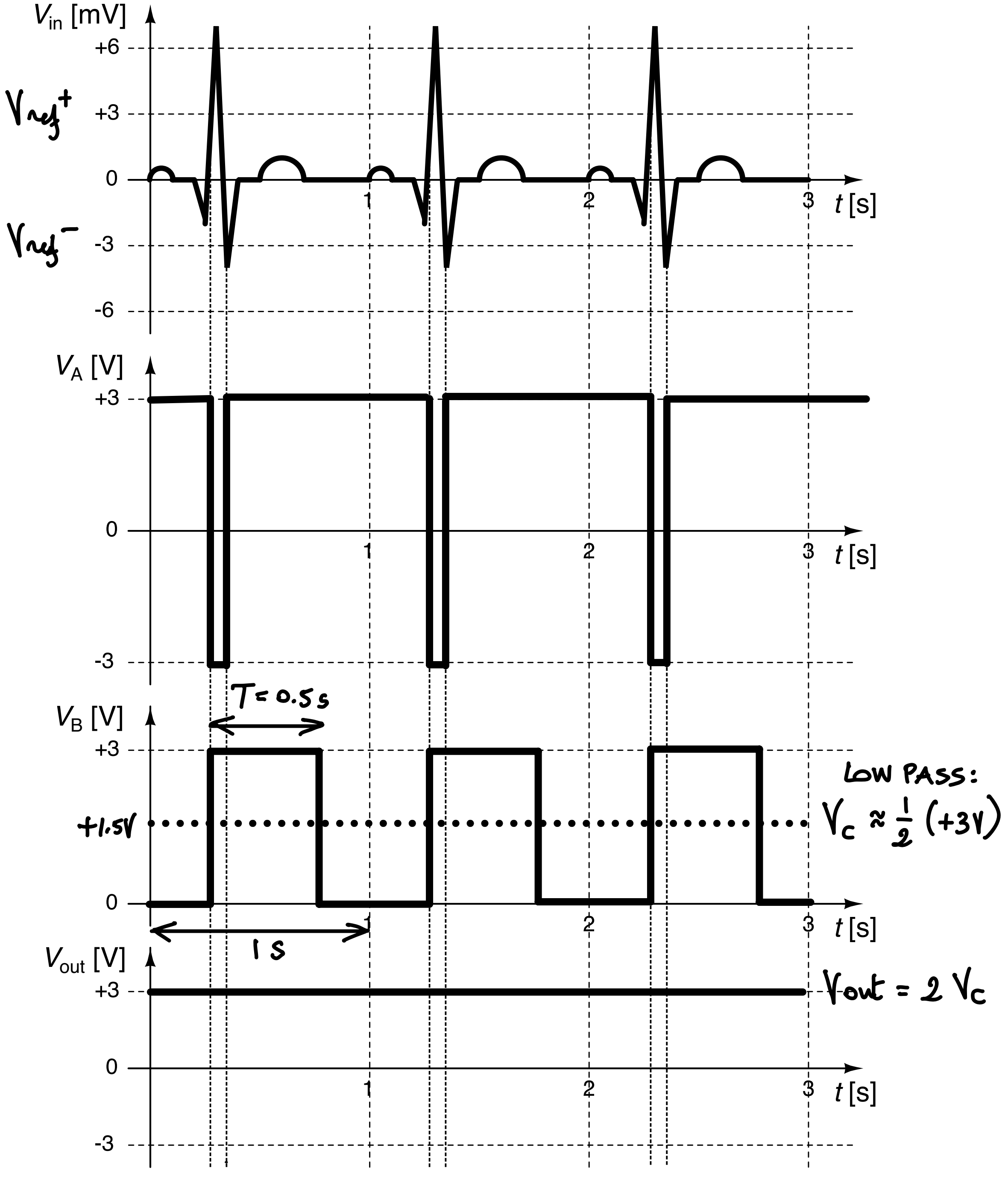
$$T = 1.1\,R_1 C_1$$
$$= 0.5\text{ s}$$

$$\tau = R_2 C_2$$
$$= 100\text{ s}$$

$$f_c = \frac{1}{2\pi\tau}$$
$$= 0.0016\text{ Hz} \ll 1\text{ Hz}$$

$\Rightarrow$ Low pass only lets DC through.

$$A = 1 + \frac{R_5}{R_6}$$
$$= 2$$



V_{in} [mV]
+6
V_{ref}^{+}
+3
0
V_{ref}^{-}
-3
-6
1
2
3
t [s]
V_A [V]
+3
0
-3
1
2
3
t [s]
T = 0.5 s
V_B [V]
+3
LOW PASS:
+1.5V
$V_C \approx \frac{1}{2}(+3V)$
0
1
2
3
t [s]
1 s
V_{out} [V]
+3
$V_{out} = 2\,V_C$
0
1
2
3
t [s]
-3

Bonus [extra +10 pts]: What is V_{out} when the heart rate decreases to 40 bpm? What is it when the heart rate increases to 90 bpm? Explain.

$$V_C \approx DC(V_B) = T \cdot \underset{\substack{\downarrow \\ \text{HEART RATE}}}{HR} \cdot (+3V) = 1.5\,Vs\ HR$$

$$HR = 40\text{ bpm} = \frac{40}{60}\text{ Hz} \Rightarrow V_C = 1\,V$$

$$V_{out} = 2V_C = 2V$$

$$HR = 90\text{ bpm} = \frac{90}{60}\text{ Hz} \Rightarrow V_C = \frac{9}{4}\,V = 2.25\,V$$

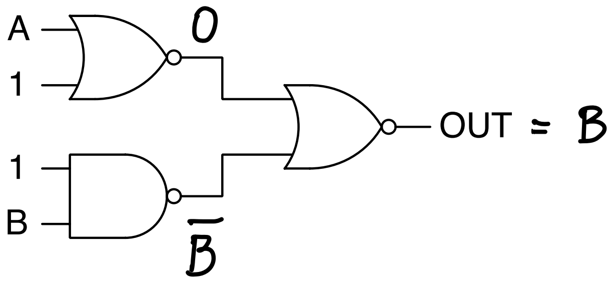
$$V_{out} = \cancel{2V_C = 4.5\,V}$$

$$= 3V$$

due to clipping at the positive supply rail

3. **[20 pts]** Circle the **best answer (only one answer per question)**:

(a) [4 pts] Find the simplest logical expression for the output of the circuit shown at right:



i. 0

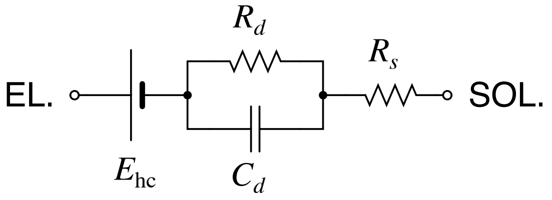
ii. 1

iii. A

[✓ iv.] B

v. AND (A, B)

(b) [4 pts] A non-contact electrode has a circuit model on the right with:



i. zero half-cell potential

ii. zero series resistance

[✓ iii.] infinite double-layer resistance

iv. infinite double-layer capacitance

v. all of the above

(c) [4 pts] The voltage generated by a current dipole is:

i. positive in the direction of the dipole.

ii. negative in the direction opposite to the dipole.

iii. zero in the plane perpendicular to the dipole.

iv. none of the above.

[✓ v.] all of the above.

(d) [1 pt ea.] Indicate for each statement below whether it is true or false:

i. **TRUE** / [✓ **FALSE**]: The resting potential of a cell with impermeable membrane is zero.

ii. [✓ **TRUE**] / **FALSE**: A myelinated axon allows action potentials to travel at a faster speed.

iii. [✓ **TRUE**] / **FALSE**: Ventrical depolarization and atrial repolarization coincide in the QRS wave complex.

iv. **TRUE** / [✓ **FALSE**]: ERG measures the resting state of visual cortex.

v. **TRUE** / [✓ **FALSE**]: The signal magnitude of ENG is larger than that of EMG.

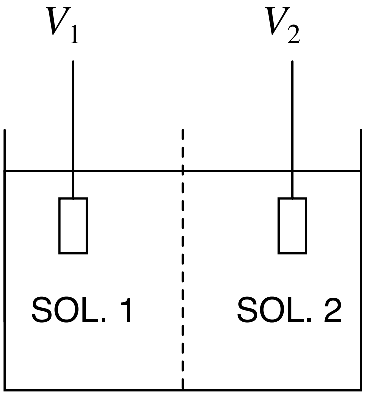
vi. [✓ **TRUE**] / **FALSE**: Scalp EEG measures brain activity primarily originating from the surface of the brain.

vii. [✓ **TRUE**] / **FALSE**: The Ag/AgCl electrode allows current to flow by exchanging electrons with chloride ions in the electrolyte.

viii. [✓ **TRUE**] / **FALSE**: Electrode arrays penetrating the brain may become ineffective over time due to immune reaction induced glial scar accumulation.

4. **[25 pts]** Consider an electrochemical cell at room temperature, with two compartments of ionic solutions each containing KCl and NaCl with concentrations given in the table below. The two compartments are separated by a membrane that is permeable to Na^+, Cl^-, and/or K^+, but no other ion types. Two identical Ag/AgCl electrodes are inserted, one in each compartment. The GHK equation at room temperature is:

$$V_m = 60\,\text{mV} \times \log_{10}\left(\frac{P_{\text{Na}}[\text{Na}^+]_o + P_{\text{K}}[\text{K}^+]_o + P_{\text{Cl}}[\text{Cl}^-]_i}{P_{\text{Na}}[\text{Na}^+]_i + P_{\text{K}}[\text{K}^+]_i + P_{\text{Cl}}[\text{Cl}^-]_o}\right)$$



	SOL. 1	SOL. 2
KCl	10 mmol/L	1 mmol/L
NaCl	1 mmol/L	10 mmol/L
K^+	10 mmol/L	1 mmol/L
Na^+	1 mmol/L	10 mmol/L
Cl^-	11 mmol/L	11 mmol/L

(a) [15 pts] Find the voltage $V_2 - V_1$ between the electrodes measured by an ideal voltmeter when the membrane is (*i*) only permeable to Cl^-, (*ii*) equally and only permeable to Na^+ and K^+, and (*iii*) equally permeable to Na^+, K^+, and Cl^-.

$$V_2 - V_1 = 60\,\text{mV} \log_{10} \frac{P_{Na}[Na^+]_1 + P_K[K^+]_1 + P_{Cl}[Cl^-]_2}{P_{Na}[Na^+]_2 + P_K[K^+]_2 + P_{Cl}[Cl^-]_1}$$

$$(i)\quad V_2 - V_1 = 60\,\text{mV} \log_{10} \frac{11}{11} = 0$$

$$(ii)\quad V_2 - V_1 = 60\,\text{mV} \log_{10} \frac{1+10}{10+1} = 0$$

$$(iii)\quad V_2 - V_1 = 60\,\text{mV} \log_{10} \frac{1+10+11}{10+1+11} = 0$$

(b) [10 pts] Now consider that the Ag/AgCl electrodes each contain 1 mol of Ag uniformly coated with 1 mmol of AgCl. The cell contains 1 L each of the two solutions, and the membrane is permeable to Cl^-. What is the maximum amount of charge that can be passed through the cell from one electrode to the other? Explain. The value of the Faraday constant is approximately $F = 96{,}500$ C/mol.

For every 96,500 C of electrons flowing between the electrodes, 1 mol of AgCl gets converted to Ag on one electrode, 1 mol of Ag gets converted to AgCl on the other electrode, and 1 mol of Cl- crosses the membrane.

There are 1 mol Ag and 11 mmol Cl- plentiful available, but only 1 mmol of AgCl is available on either electrode, so the maximum charge that can flow in either direction is 1 mmol F = 96.5 C.