

BENG 186B Winter 2024

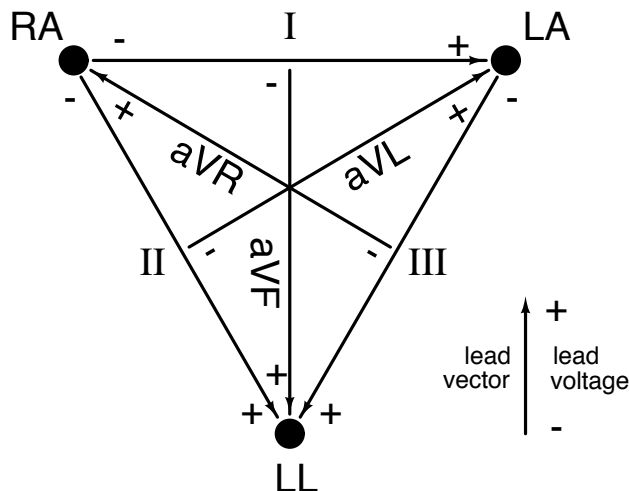
Quiz 3

Friday, March 8, 2024

Last Name, First Name: SOLUTIONS

- This quiz is on-line, open-book, and open-notes. You may use a calculator or an equivalent program, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due March 8, 2024 at 11:59pm, over Canvas. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any quiz-related material among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 4 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

1. [20 pts] Consider Einthoven's triangle of the frontal electrocardiogram (ECG) for the three electrodes RA, LA and LL shown below. The triangle is equilateral and the augmented lead vectors (aVR, aVL, and aVF) bisect the bipolar lead vectors (I, II, and III).



- (a) [8 pts] You are given an instrument that outputs the six-lead frontal ECG. However, four of the leads are malfunctioning, and only leads I and aVF provide useful outputs. Reconstruct the missing leads of the frontal ECG from the lead I and lead aVF outputs:

$$\begin{aligned} \text{II} &= \frac{1}{2} \text{I} + \text{aVF} \\ \text{III} &= -\frac{1}{2} \text{I} + \text{aVF} \\ \text{aVR} &= -\frac{1}{2} (\text{I} + \text{II}) = -\frac{3}{4} \text{I} - \frac{1}{2} \text{aVF} \\ \text{aVL} &= \frac{1}{2} (\text{I} - \text{III}) = \frac{3}{4} \text{I} - \frac{1}{2} \text{aVF} \end{aligned}$$

- (b) [2 pts] Can you reconstruct the Wilson Central Terminal (WCT) from the leads I and aVF as well? Explain.

No! The common-mode of the electrode voltages cannot be reconstructed from differential leads alone.

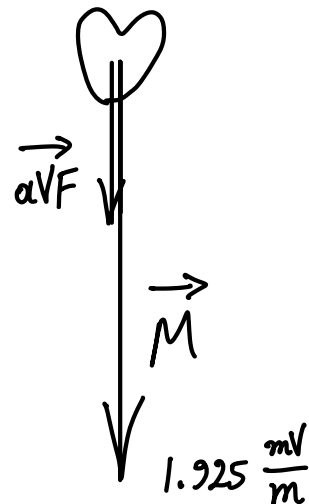
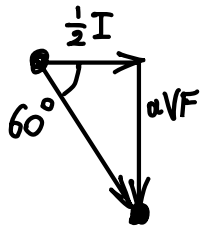
$$\text{WCT} = \frac{1}{3} (\text{LA} + \text{RA} + \text{LL})$$

- (c) [10 pts] You find that lead I measures zero, and lead aVF measures 1 mV. Estimate the direction and magnitude of the cardiac vector in the frontal plane. You may assume the distance between shoulders of the subject is roughly 60 cm.

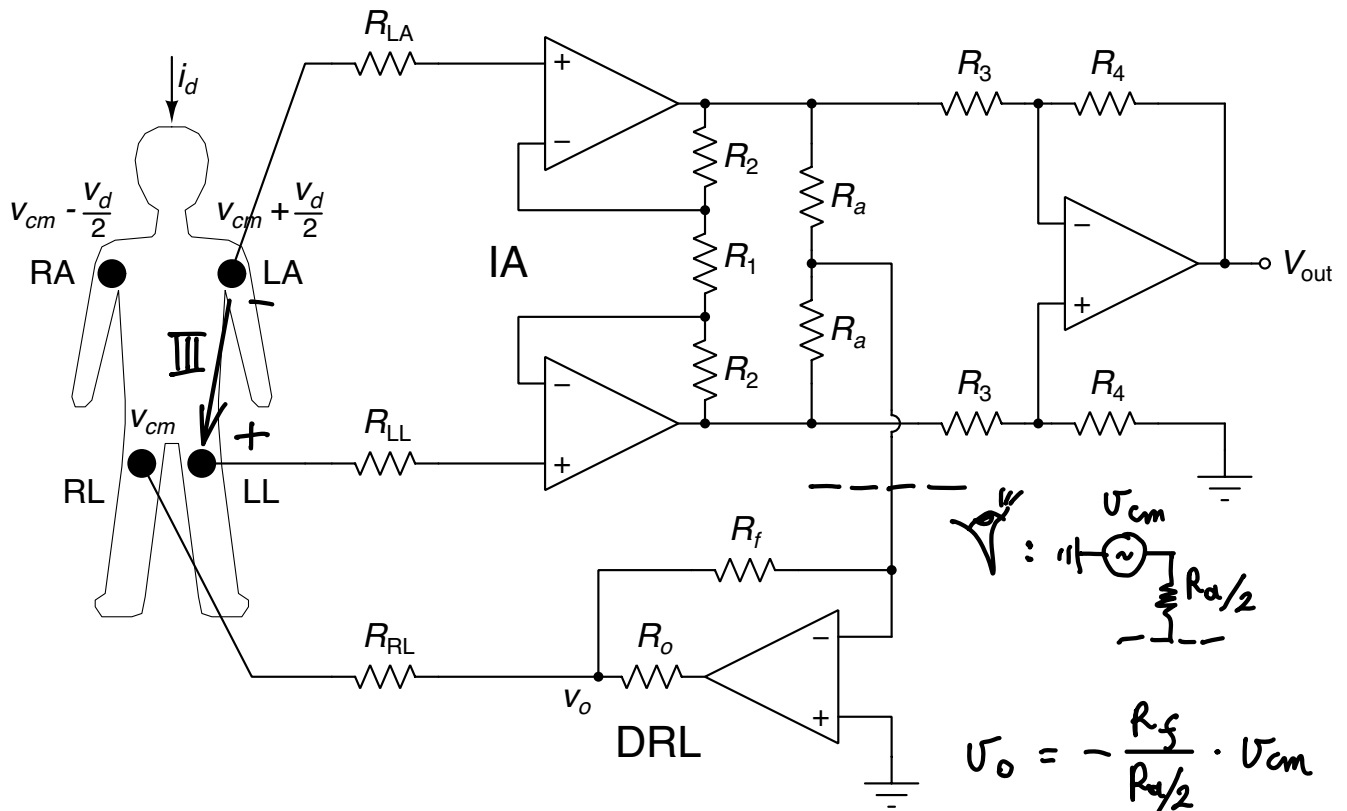
$$\begin{aligned} \vec{M} \cdot \vec{I} &= 0 & \Rightarrow & \vec{M} \perp \vec{I} \\ \vec{M} \cdot aVF &= 1 \text{ mV} & & \vec{M} \parallel aVF \end{aligned}$$

$$\Rightarrow |\vec{M}| \cdot |aVF| = 1 \text{ mV}$$

$$|aVF| = \frac{\sqrt{3}}{2} |\vec{I}| = \frac{\sqrt{3}}{2} 60 \text{ cm} \Rightarrow |\vec{M}| = \frac{1 \text{ mV}}{30\sqrt{3} \text{ cm}} = 1.925 \frac{\text{mV}}{\text{m}}$$



2. [35 pts] An instrumentation amplifier (IA) and driven right leg (DRL) amplifier are connected as shown below to record a single-lead electrocardiogram while actively grounding the body. The electrode-skin interface resistances are $R_{LA} = 100 \text{ k}\Omega$, $R_{LL} = 80 \text{ k}\Omega$, and $R_{RL} = 120 \text{ k}\Omega$. All opamps are ideal with infinite input impedance, and with supply voltages $V^+ = +3\text{V}$ and $V^- = -3\text{V}$. The IA resistances are $R_1 = 1 \text{ k}\Omega$, $R_2 = 499.5 \text{ k}\Omega$, $R_3 = 5 \text{ k}\Omega$ and $R_4 = 50 \text{ k}\Omega$, all with 0.1 % tolerance. The DRL resistances are $R_a = 2 \text{ k}\Omega$, $R_f = 999 \text{ k}\Omega$, and $R_o = 5 \text{ M}\Omega$.



- (a) [5 pts] What ECG lead does the IA output V_{out} represent, and with what voltage gain?

Lead III with differential gain:

$$A_d = \left(1 + 2 \frac{R_2}{R_1}\right) \cdot \frac{R_4}{R_3} = (1 + 999) \cdot 10 = 10,000$$

(80 dB)

(b) [5 pts] Find the common-mode rejection ratio (CMRR) of the IA.

$$\text{CMRR}_{\text{IA}} = \frac{1 + 2 \frac{R_2}{R_1}}{4 \times 0.1\%} = \frac{1,000}{0.004} = 2.5 \times 10^5$$

(108 dB)

(c) [5 pts] Now consider that the IA is subject to an input capacitance of 10 pF on both the non-inverting and inverting nodes. Find the effective CMRR of the system for line noise at 60 Hz frequency.

$$\text{CMRR}_{\text{eff.}} = \frac{|Z_{\text{in}}|}{|R_{\text{LA}} - R_{\text{LL}}|} = \frac{\frac{1}{2\pi \cdot 60 \text{ Hz} \cdot 10 \text{ pF}}}{20 \text{ k}\Omega}$$
$$= \frac{1}{24,000 \pi \cdot 10^{-9}} = 1.33 \times 10^4$$

(82.5 dB)

- (d) [10 pts] For a 60 Hz displacement current i_d with $20 \mu\text{A}$ amplitude entering the body, find the amplitude of the common-mode voltage v_{cm} in the body, with the DRL connection as shown. How much improvement does the DRL active grounding offer over a passive ground connection to the RL electrode?

With the DRL connection as shown:

$$V_o = V_{cm} - R_{RL} i_d = -2 \frac{R_f}{R_{ol}} \cdot V_{cm}$$
$$\Rightarrow V_{cm} = \frac{R_{RL}}{1 + 2 \frac{R_f}{R_{ol}}} \cdot i_d$$
$$= \frac{120 \text{ k}\Omega}{1 + 999} \cdot 20 \mu\text{A} = 2.4 \text{ mV}$$

Passive ground connection on RL:

$$V_{cm} = R_{RL} \cdot i_d$$
$$= 120 \text{ k}\Omega \cdot 20 \mu\text{A} = 2.4 \text{ V}$$

DRL active grounding yields a factor 1000x improvement.

- (e) [5 pts] The amplitude of the differential signal v_d on the ECG lead is 1 mV. Find the corresponding signal-to-noise ratio (SNR) at the output of the instrumentation amplifier.

$$\begin{aligned} \text{SNR}_{\text{out}} &= \frac{A_d \cdot v_d}{A_c \cdot v_{\text{cm}}} = \text{CMRR}_{(\text{eff})} \cdot \frac{v_d}{v_{\text{cm}}} \\ &= 1.33 \cdot 10^4 \cdot \frac{1 \text{ mV}}{2.4 \text{ mV}} = 5.5 \cdot 10^3 \\ &\quad (75 \text{ dB}) \end{aligned}$$

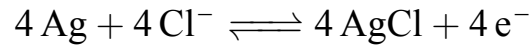
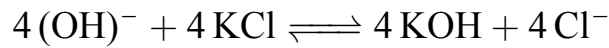
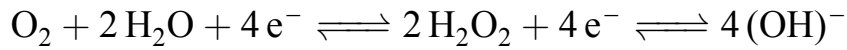
- (f) [5 pts] What is the maximum magnitude of the short-circuit current that may enter the subject's body? Explain.

No current flows in/out the high-impedance input terminals of the IA connected to LA and LL.

The maximum short-circuit current at the DRL output terminal, from its opamp output through R_o , is:

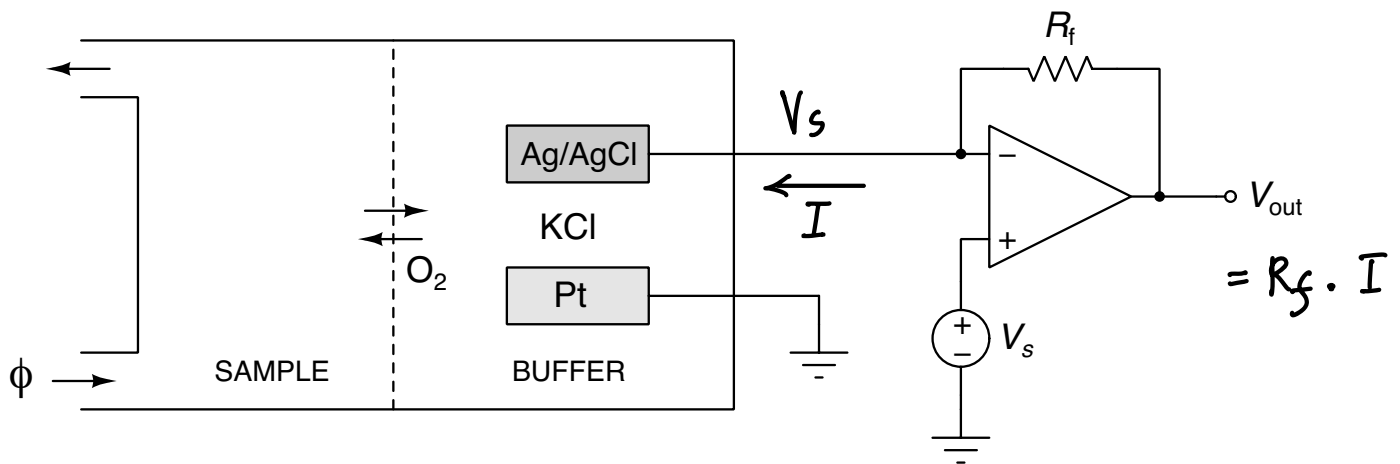
$$\begin{aligned} i_{\text{sc}, \text{max}} &= \frac{1}{R_o} \cdot \max(|V^+|, |V^-|) \\ &= \frac{1}{5 \text{ M}\Omega} \cdot 3 \text{ V} = 0.6 \mu\text{A} \end{aligned}$$

3. [20 pts] Consider the PO_2 sensor below consisting of a Clark electrode and a transimpedance amplifier (TIA). The flow rate ϕ of the sample through the chamber is maintained constant at 1 mL/s, the solution in the buffer is 1 mol/L KCl, and the value of the TIA resistance is $R_f = 1 \text{ M}\Omega$. The following equations may be useful:



$$I = 4 F [\text{O}_2] \phi$$

where $F = 96,500 \text{ C/mol}$ is the Faraday constant.



- (a) [5 pts] Find the source voltage V_s necessary to operate the Clark sensor in its linear region. Explain.

The Ag/AgCl electrode needs to be biased 0.7 V positive with respect to the Pt electrode so that it can supply a stream of electrons into the solution.

$$\Rightarrow V_s = +0.7 \text{ V}$$

- (b) [5 pts] Find the sensitivity of the voltage output V_{out} to oxygen concentration $[O_2]$ in the sample at steady-state.

$$\begin{aligned} S &= \frac{dV_{out}}{d[O_2]} = R_f \frac{dI}{d[O_2]} = R_f 4 F \Phi \\ &= 1 \text{ M}\Omega \cdot 4 \cdot 96,500 \frac{\text{C}}{\text{mol}} \cdot 0.001 \frac{\text{L}}{\text{S}} \\ &= 3.86 \cdot 10^8 \text{ V} / \frac{\text{mol}}{\text{L}} \end{aligned}$$

- (c) [5 pts] What output voltage V_{out} do you expect when there is no oxygen present? Explain.

$$I = 0 \Rightarrow V_{out} = V_s = 0.7 \text{ V}$$

- (d) [5 pts] For every mol of O_2 , how many moles of Ag get consumed in the Ag/AgCl electrode? Explain.

4 moles of electrons exit the Pt electrode, and 4 moles of Cl^- enter the Ag/AgCl electrode, so 4 moles of Ag get converted to AgCl.

4. [25 pts] Circle the **best answer (only one answer per question)**:

- (a) [4 pts] The resonance frequency of the external fluid-filled catheter tube depends on:
- i. Mass density of the fluid
 - ii. Air in the fluid
 - iii. Diameter of the tube
 - iv. All of the above
 - v. None of the above
- (b) [4 pts] An electromagnetic flowmeter measures:
- i. Cardiac output
 - ii. Blood velocity
 - iii. Saturation of carbon dioxide
 - iv. All of the above
 - v. None of the above
- (c) [4 pts] The Severinghaus electrode:
- i. Measures concentration of oxygen in a sample
 - ii. Requires a membrane semi-permeable to carbon dioxide separating the sample from a bicarbonate reference solution
 - iii. Requires a steady flow of the specimen through the sample chamber
 - iv. All of the above
 - v. None of the above
- (d) [4 pts] A chemical fibrosensor:
- i. Is a type of optical sensor
 - ii. Is capable of measuring fluorescence
 - iii. Is subject to Beer's law of volume absorptivity
 - iv. All of the above
 - v. None of the above

(e) [9 pts] Indicate for each statement below whether it is true or false:

- i. **TRUE** **FALSE**: The electric field for a current monopole falls off as one over distance squared, $1/r^2$.
- ii. **TRUE** **FALSE**: Twisted pair wiring between the electrodes and the instrumentation for biopotential measurement reduces differential interference.
- iii. **TRUE** **FALSE**: A microtipped manometer gives a direct measure of the blood pressure waveform.
- iv. **TRUE** **FALSE**: The sphygmomanometer registers systolic and diastolic blood pressure along with heart rate.
- v. **TRUE** **FALSE**: Indicator-dilution methods offer non-invasive means to measure cardiac output.
- vi. **TRUE** **FALSE**: The Doppler flowmeter transduces blood velocity into a frequency shift between transmitter and receiver.
- vii. **TRUE** **FALSE**: Partial pressure and saturation of blood gases are directly related to each other.
- viii. **TRUE** **FALSE**: Double correlated sampling allows to remove a constant measurement offset from a bioinstrument.
- ix. **TRUE** **FALSE**: Pulse oxymetry leverages the pulsative flow of blood in the vasculature to measure both heart rate and oxygen saturation non-invasively.