Neurodynamics - Fall 2019
BENG 260 / BGGN 260 / PHYS 279

Midterm: Due November 11

- Open book, open notes.
- Take the midterm wherever and whenever is most convenient to you. No communication except with the instructor and TAs.
- Submit a single .zip file containing a single PDF with all your solutions, plots, and any handwritten code as well as your Matlab/Python code by 11:59pm on the due date on Canvas.

1 Analytical Part

Consider two Hodgkin-Huxley (HH) neurons V_1 and V_2, and a single excitatory chemical synapse that couples the first neuron to the second, *i.e.,* V_1 is presynaptic, and V_2 is postsynaptic. Assume the standard form for the HH neural dynamics identical for the two neurons, each using three gating variables describing sodium fast activation, sodium slow inactivation, and potassium slow activation. For the chemical synapse assume a standard rate-based postsynaptic conductance, where the gating variable $r(t)$ is activated at a rate proportional to neurotransmitter concentration $[T]$ with coefficient α_r, and deactivated at a constant rate β_r. For simplicity, you may assume that the neurotransmitter concentration $[T]$ reaches a nominal value $[T]_{max}$ whenever the presynaptic voltage V_1 goes above zero, and is zero whenever the presynaptic voltage V_1 goes below zero.

1. *Excitation* [5 points]. What is required for this synapse to be excitatory?

2. *Regular spiking* [10 points]. Show that a sufficiently large steady current I_{ext} injected into the first neuron causes it to fire action potentials at a constant rate, and the firing rate increases monotonically with current level. Find an approximate expression for the firing rate ν_1 in terms of the injected current I_{ext}.

3. *Postsynaptic conductance* [10 points]. Find an analytical expression for the postsynaptic membrane conductance as a function of time, in terms of the timing of each presynaptic spike.

4. *EPSC and EPSP* [10 points]. Find the corresponding excitatory postsynaptic current EPSC as a function of time and the postsynaptic voltage V_2. Find also an approximate expression for the excitatory postsynaptic voltage EPSP, as the step change in V_2, assuming this is small. What does "small" mean, in relation to biophysical potentials of interest that govern the neurodynamics?

5. *Synaptic weight* [15 points]. For a constant presynaptic firing rate ν_1, show that the postsynaptic neuron also fires regularly at a constant rate ν_2. Find an approximate expression for ν_2 as a function of ν_1. Find the corresponding effective synaptic weight W_{ij} as the peak firing rate sensitivity $d\nu_2 \,/\, d\nu_1$, and interpret this quantity in terms of the biophysical parameters in the HH and synaptic models.

2 Numerical Part

1. *Simulate and validate your analytical observations* [50 points]. Use the HH and synapse parameters of your computational labs and assignments (Homework 4), with the first neuron supplied a constant current $I_{ext} = 10$ μA/cm^2, and the second neuron no external current other than the postsynaptic current. Validate your assumptions in deriving the analytical expressions for firing rate as a function of current, EPSC and EPSP as a function of postsynaptic voltage, and postsynaptic firing rate as a function of presynaptic firing rate. Discuss and interpret your results.