Neurodynamics - Fall 2019

BENG 260 / BGGN 260 / PHYS 279

Homework 3: Due October 25

Computational Lab

In this assignment, we will study the single neuron stability dynamics in two variables: membrane potential V, and accommodation (K^+ slow persistent activation gating) variable w. For the neuron you should consider two models: the Morris-Lecar (ML) model and a reduced Hodgkin-Huxley (HH) model (substituting w for n). The ML model is replicated here:

$$\begin{aligned} C_m \frac{dV}{dt} &= -I_{Ca} - I_K - I_L + I_{ext} &(1)\\ I_{Ca} &= g_{Ca}\, m_\infty(V)\,(V - E_{Ca}) &(2)\\ I_K &= g_K\, w\,(V - E_K) &(3)\\ I_L &= g_L\,(V - E_L) &(4)\\ \frac{dw}{dt} &= \frac{w_\infty(V) - w}{\tau_w(V)} &(5) \end{aligned}$$

with voltage dependence [V in units mV] of the kinetics of gating variables:

$$\begin{aligned} m_\infty(V) &= (1 + \tanh((V+1)/15))/2 &(6)\\ w_\infty(V) &= (1 + \tanh(V/30))/2 &(7)\\ \tau_w(V) &= 5\,/\cosh(V/60) \qquad \text{[units } ms\text{]} &(8) \end{aligned}$$

and with electrical parameters:

$$\begin{aligned} C_m &= 1\ \mu F/cm^2 \\ E_{Ca} &= 100\ mV; \quad g_{Ca} = 1.1\ mS/cm^2 \\ E_K &= -70\ mV; \quad g_K = 2.0\ mS/cm^2 \\ E_L &= -50\ mV; \quad g_L = 0.5\ mS/cm^2 \end{aligned} \qquad (9)$$

Note: similar to previous assignments, the external current I_{ext} is varied as a parameter which governs the stability of the dynamics.

1. *Stability Analysis* [40 points].

   Determine the stability of the Morris-Lecar (ML) model when injected with three cases, $I_{ext} = 10\ \mu A/cm^2$, $25\ \mu A/cm^2$ and $40\ \mu A/cm^2$. The following questions evaluated for all three injected currents will guide you through the process:

   (a) Plot the nullclines and stationary point(s) in V–w space.

   (b) Compute the eigenvectors and eigenvalues of the Jacobian around the stationary point(s) and determine the stability.

   (c) Verify the stability by plotting the simulated trajectory of V and w, in the V–w plane, starting from several initial conditions in the vicinity of the stationary point(s).

   (d) If the value of external current is over $200\,\mu A/cm^2$, what is the expected stability and trajectory?

Homework Problems

2. *Reduced Hodgkin-Huxley Model Stability Analysis* [60 points].

   Homework 2 explored the properties of the full and reduced Hodgkin-Huxley (HH) models. To compare the stability of the HH model to the ML model, we need to reduce the dimensionality of the HH model to two variables, V and n. To do this, we will let $h = \lambda - \mu\, n$ as we found by linear regression in Homework 2, and approximate the m gating dynamics with its equilibrium value:

   $$m_\infty(V) = \alpha_m(V)/(\alpha_m(V) + \beta_m(V))$$

   Perform the stability analysis from Problem 1 using the reduced HH model and compare your results with the ML model. Are the reduced HH model results what you would expect for the full HH model? If not, what would account for the differences?

3. *BONUS: Full Hodgkin-Huxley Model Stability Analysis* [Extra 30 points].

   Here we will extend the stability analysis to the full 4-D HH model. We can't visualize null-clines (3-D spaces rather than 1-D contour lines) in four dimensions, but we can still find stationary points by finding the roots of the derivatives in the four variables (V, m, h and n) using the same numerical tools that we used for finding stationary points in Problem 1 extended to four equations in four unknowns (the stationary values in each of the four variables). We then compute the 4×4 Jacobian around the stationary point(s), and find the eigenvalues to determine the stability, for each of the three values of injected current in Problem 1 and 2. Compare your results with those for the reduced HH model in Problem 2.

Submission Guidelines

Solutions without work or explanations where applicable will receive no credit. Submit a single .zip file containing a single PDF with all your solutions, plots, and any handwritten code as well as your Matlab/Python code to both computational lab and homework problems by 3:00pm of due date on Canvas.

The submission file should follow the naming scheme `LastFirst_A12345678_HW2.zip`.