520.492 Mixed-Signal VLSI Systems

Week 5

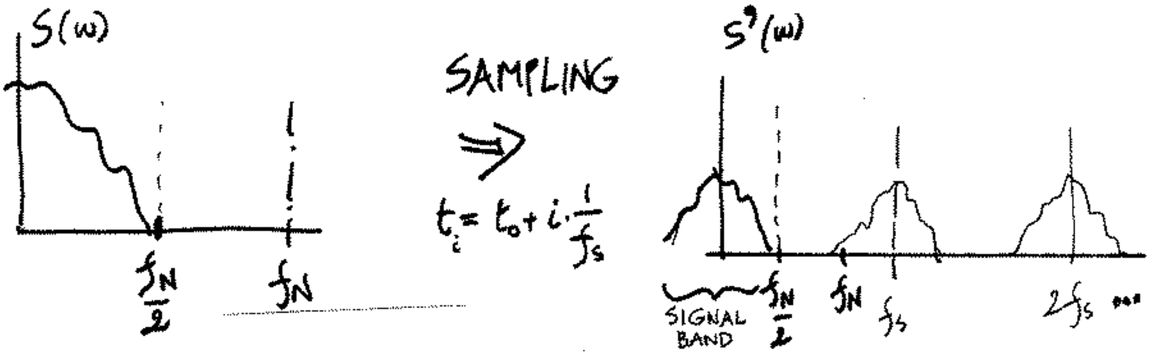
Delta-Sigma Modulation

References

1. Franca and Tsividis, Chapter 10.
2. Oversampling Delta-Sigma Data Converters, J.C. Candy and G.C. T emes, Eds., IEEE Press, 1992.

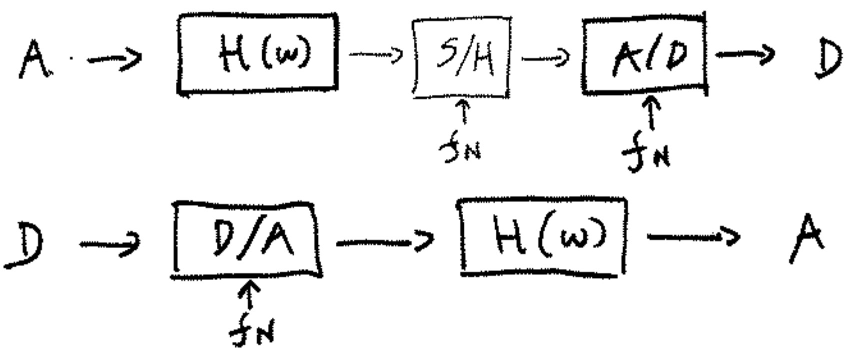
DELTA-SIGMA MODULATION

Nyquist:



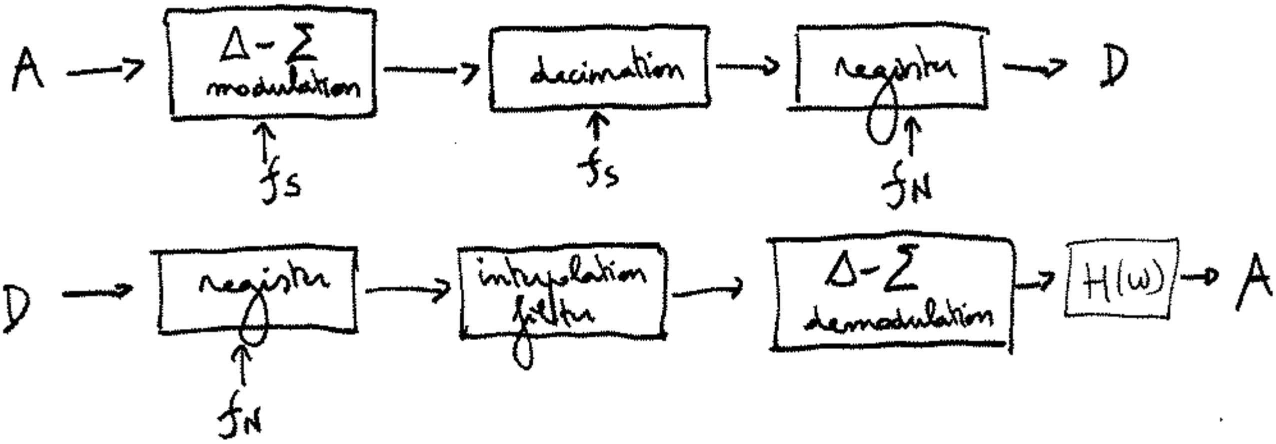
Nyquist Sampling theorem: $\begin{cases} f_s \geq f_N & \text{to preserve signal content in signal band} \\ f_N = 2 f_{max}(S) \end{cases}$

Nyquist-rate converters: $f_s \equiv f_N$ (Precision Conversion)

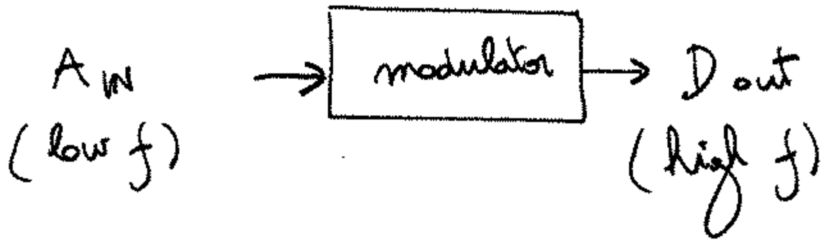


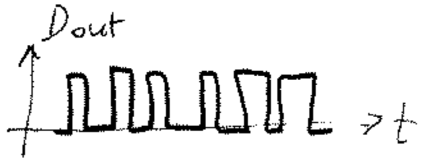
Oversampled Converters: $f_s \gg f_N$ (Using low-precision conversion)

$f_s = OSR \times f_N$ (OSR: over sampling rate)



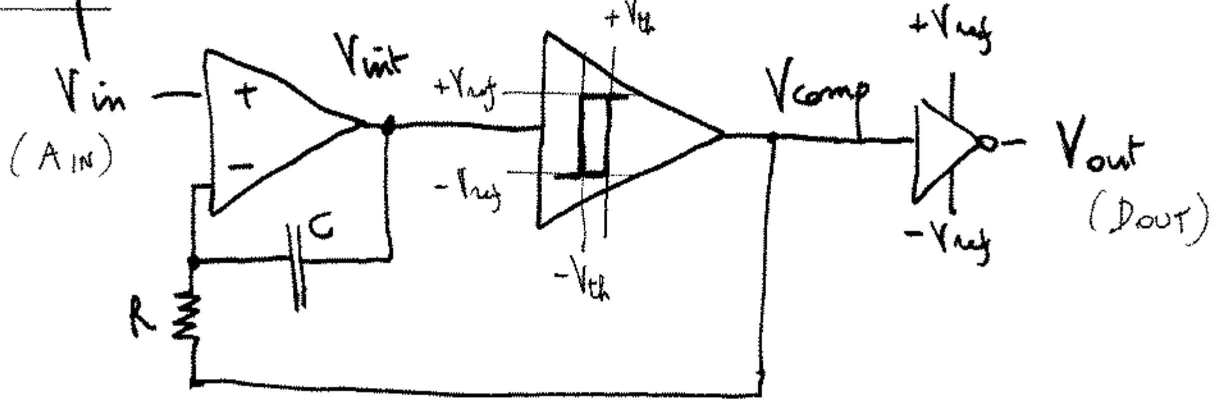
Continuous-Time Pulse Modulation



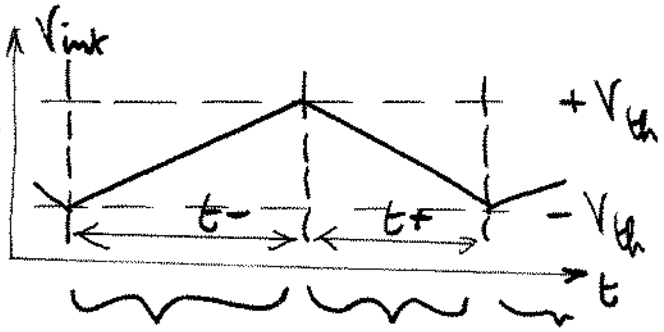


$\rightarrow \langle D_{out} \rangle = A_{out}$

example:



$\Rightarrow$



$$t^- = 2RC \frac{V_{th}}{V_{ref} - V_{in}}$$

$$t^+ = 2RC \frac{V_{th}}{V_{ref} + V_{in}}$$

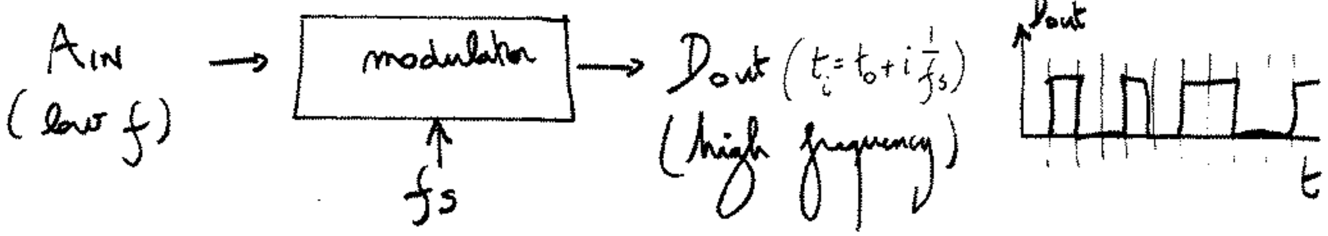
$V_{comp} = \quad +V_{ref} \quad -V_{ref} \quad +V_{ref}$

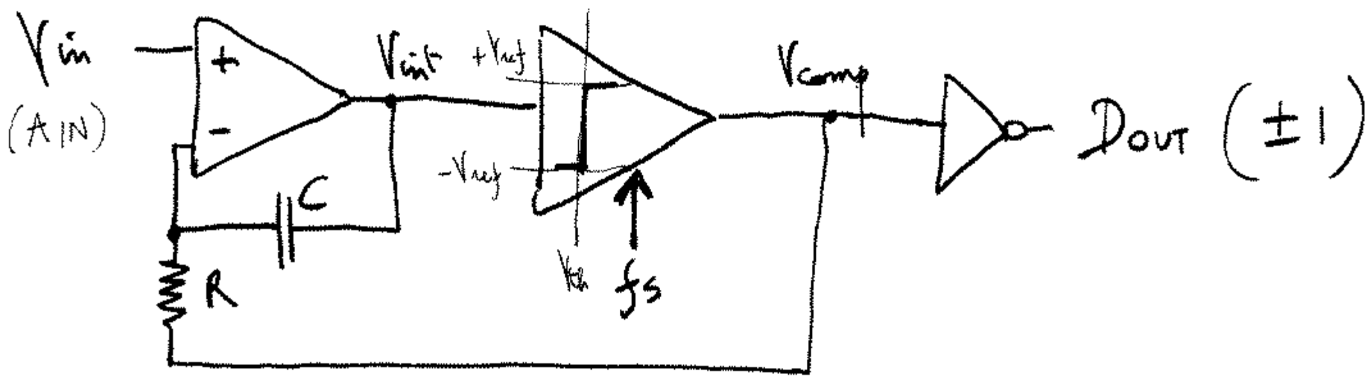
$V_{out} = \quad -V_{ref} \quad +V_{ref} \quad -V_{ref}$

<u>Duty cycle</u>: $\delta = \dfrac{t^+}{t^+ + t^-} = \dfrac{V_{ref} + V_{in}}{(V_{ref} + \cancel{V_{in}}) + (V_{ref} - \cancel{V_{in}})} = \dfrac{1}{2} + \dfrac{1}{2}\dfrac{V_{in}}{V_{ref}}$

$$\langle V_{out} \rangle = \delta V_{ref} + (1-\delta)(-V_{ref}) = V_{in}$$

Clocked Pulse Modulation (First-Order Delta-Sigma)





When $D_{out} = +1$ $(V_{comp} = -V_{ref})$: $V_{int}^{(k)} - V_{int}^{(k-1)} = \frac{1}{RCf_S}(V_{in} - V_{ref})$

When $D_{out} = -1$ $(V_{comp} = +V_{ref})$: $V_{int}^{(k)} - V_{int}^{(k-1)} = \frac{1}{RCf_S}(V_{in} + V_{ref})$

⇒ ON AVERAGE : $r_{+1} \times \frac{1}{RCf_S}(V_{ref} - V_{in}) = r_{-1} \times \frac{1}{RCf_S}(V_{in} + V_{ref})$

r_{+1} ↓ rate of $D_{out} = +1$

r_{-1} ↓ rate of $D_{out} = -1$ $(= 1 - r_{+1})$

⇒ Duty Cycle : $\delta = \frac{r_{+1}}{r_{+1} + r_{-1}} = \frac{1}{2} + \frac{1}{2}\frac{V_{in}}{V_{ref}}$

$\langle D_{out} \rangle = r_{+1} \times (+1) + r_{-1} \times (-1) = \frac{V_{in}}{V_{ref}}$

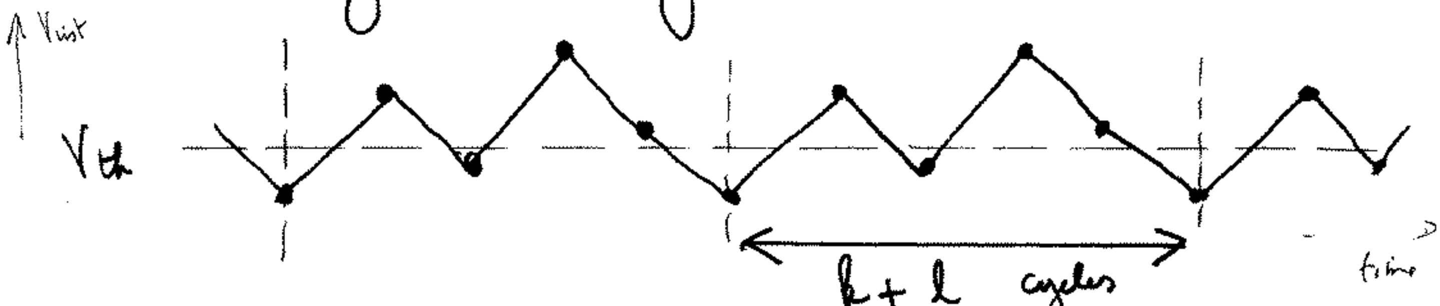
What about the deviation from the mean? (→ NOISE)

Again, consider DC input (AC later)

Case ① : $\frac{V_{ref} - V_{in}}{k} = \frac{V_{ref} + V_{in}}{l}$

with k and l integers with no common factors (unless = 1)

⇒ converges to limit cycle:

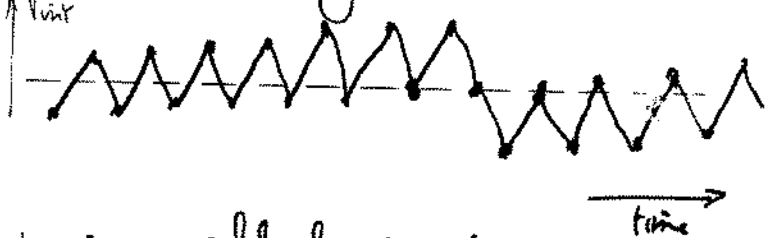


l cycles up } back to same
k cycles down } value

⇒ low noise, especially for $k+l \ll OSR$

Case ② : $\nexists\, k, l$ but in the neighborhood of a k, l

e.g.: $V_{in} \approx 0$ but $\neq 0$:



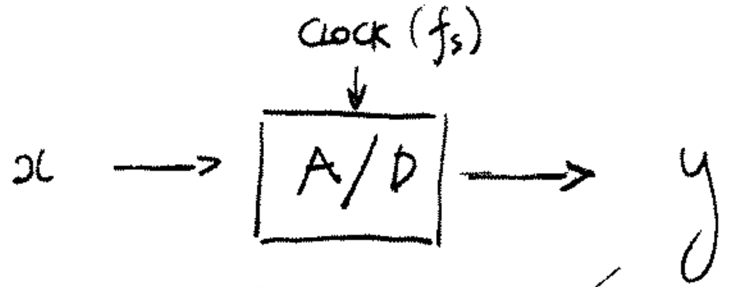
⇒ noisy

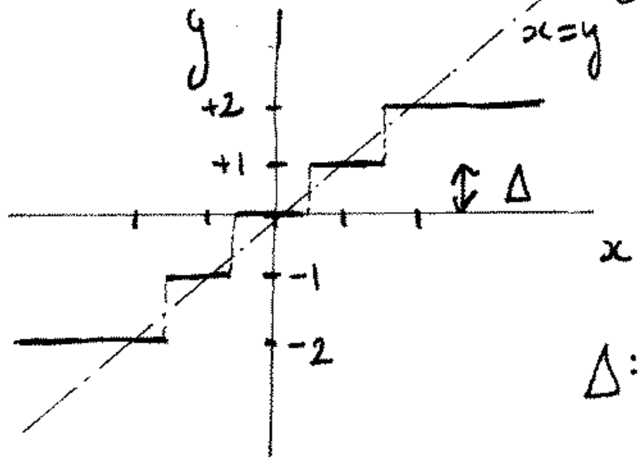
Case ③ : $\nexists\, k, l$, and not in neighborhood either

⇒ somewhere in between

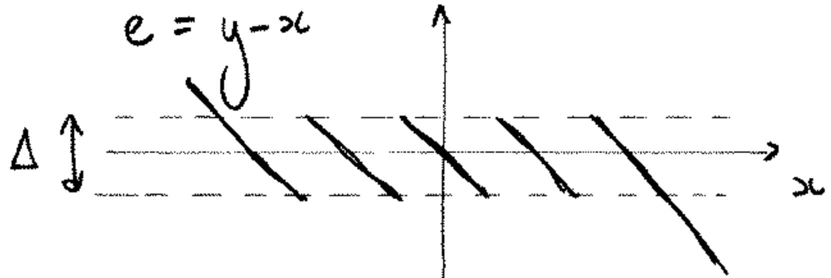
FORMAL ANALYSIS for "busy" input signals

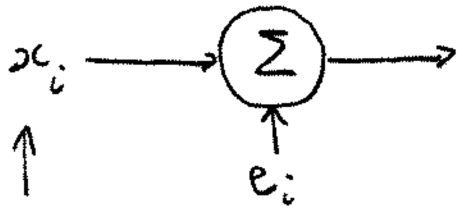
Quantization





Δ: quantization step (resolution)





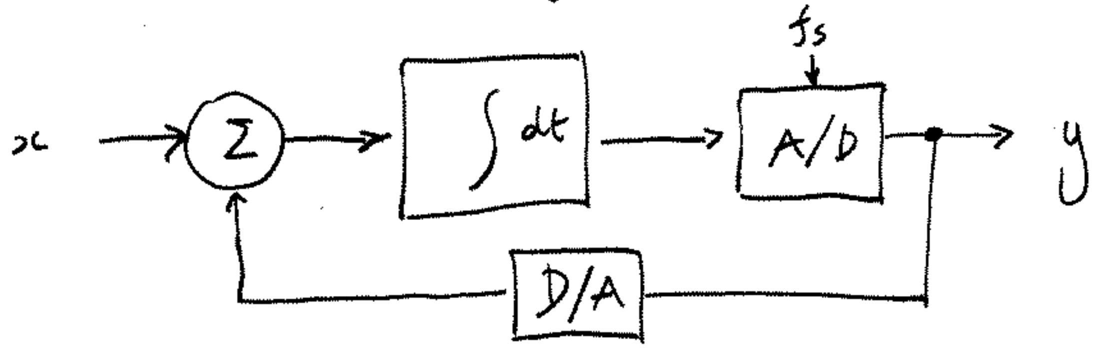
$$y_i = x_i + e_i$$

x_i: "busy" (or: add DITHER signal outside base band)

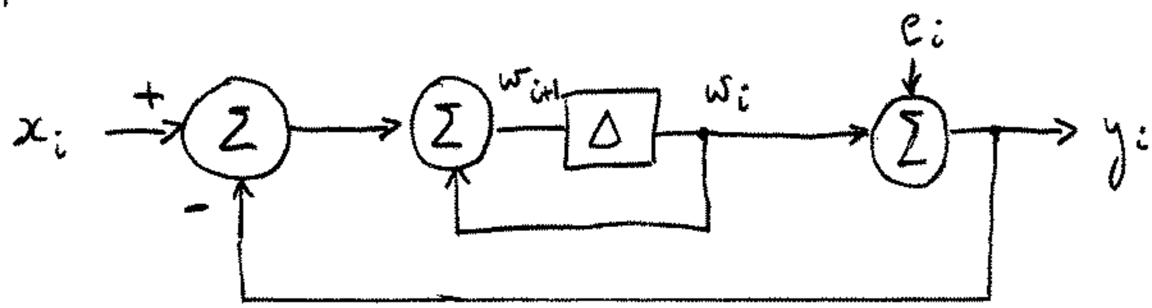
e_i: pseudo-white random noise (quantization noise)

$$\Rightarrow \left| N(\omega) \right| = \left| E(\omega) \right|$$

First-Order Delta-Sigma Modulator:



equivalent diagram ("busy" input; or dither input added)



$$\left\{ \begin{array}{l} w_{i+1} = w_i + (x_i - y_i) \\ y_i = w_i + e_i \end{array} \right. \Rightarrow \quad w_{i+1} = x_i - e_i$$

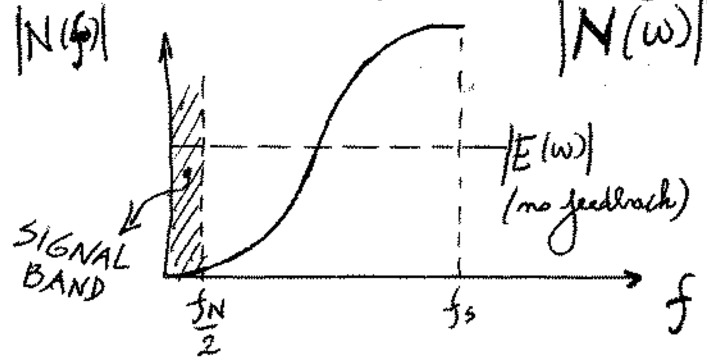
$$\Downarrow$$

$$y_{i+1} = w_{i+1} + e_{i+1} = x_i + (e_{i+1} - e_i)$$

$$y_i = x_{i-1} + (e_i - e_{i-1})$$

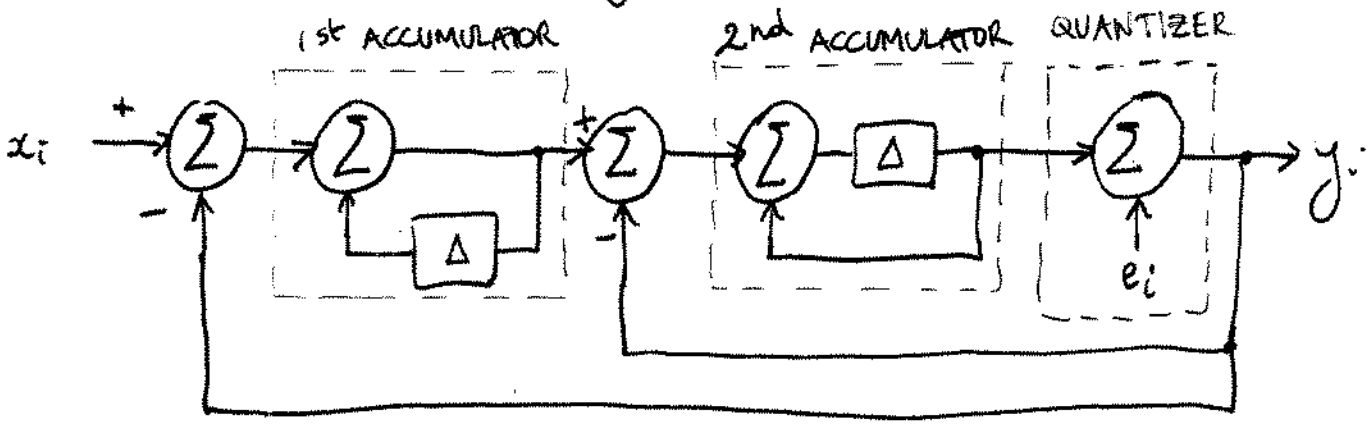
x_{i-1} ↓ SIGNAL — low frequency

$(e_i - e_{i-1})$ ↓ QUANTIZATION NOISE (differentiated) — high frequency



$$|N(\omega)|^2 = \underbrace{\left|1 - e^{-j\omega \frac{1}{f_s}}\right|}_{2\sin\left(2\pi \frac{f}{f_s}\right)} \cdot |E(\omega)|$$

Second-Order Delta-Sigma Modulator:

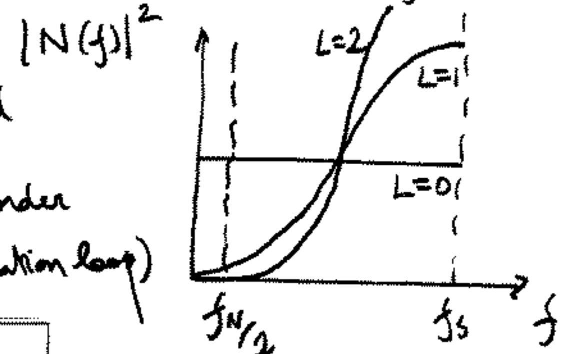


$$y_i = x_{i-1} + (e_i - 2e_{i-1} + e_{i-2})$$

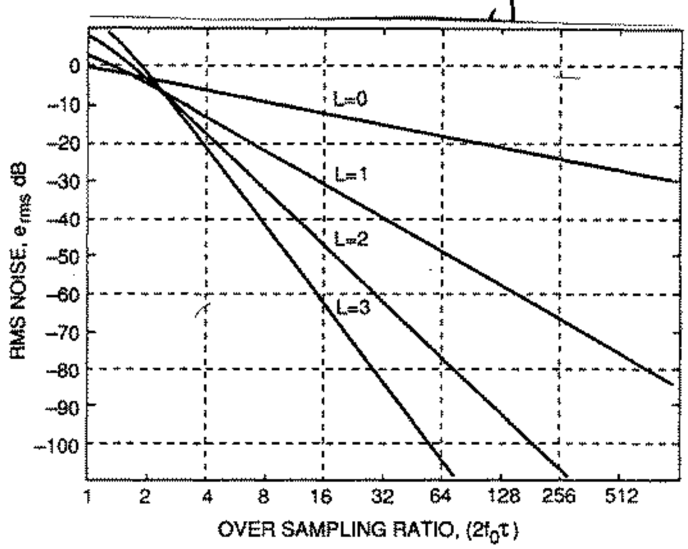
SIGNAL — low frequency

QUANTIZATION NOISE (2nd derivative) — high frequency

$$|N(\omega)| = \underbrace{\left|1 - e^{-j\omega \frac{1}{f_s}}\right|^2}_{4\sin^2\left(2\pi \frac{f}{f_s}\right)} \cdot |E(\omega)|$$

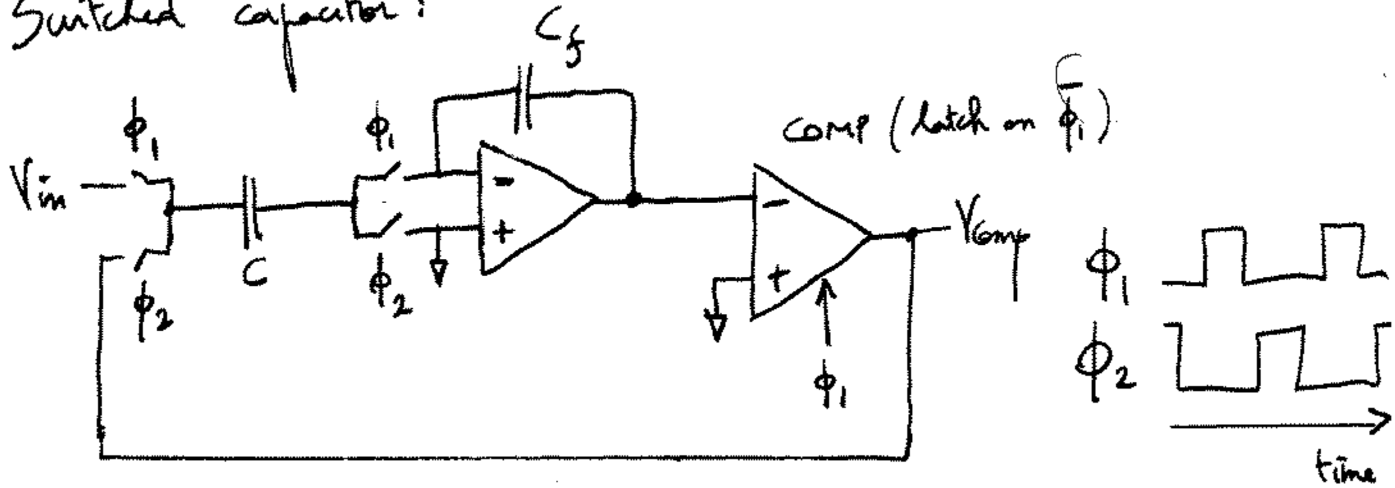


⇒ Quantization noise in signal band DECREASES STRONGLY with L (order of modulation loop)

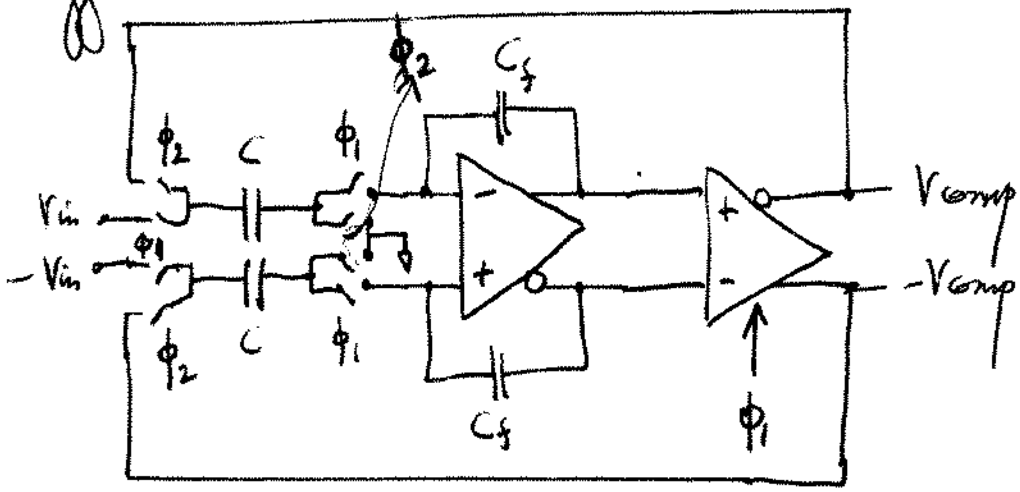


CMOS-C IMPLEMENTATION : (1st ORDER Δ-Σ)

• Switched capacitor :



differential :



• Also :

- transconductance continuous-time
- current mode continuous-time
- switched-current

etc...