BEENG 122A Fall 2025 HW #2

Due *Friday, October 17* at 11:59pm

This homework reiterates some of the problems in HW #1, now studying the system response with block diagrams and using simulation tools for analysis. You may find it helpful to use the MATLAB Simulink tools available to you.

1. [55 pts] Here we reconsider the second-order biosystem of Problem 1 in the previous homework, with state variables $u(t)$ and $v(t)$ driven by a force input $f(t)$:

$$\begin{aligned} \frac{du}{dt} &= v(t) \\ \frac{dv}{dt} &= -a\,v(t) - b\,\sinh(c\,u(t)) + d\,f(t) \end{aligned}$$

and with identical parameters $a = 1\ \mathrm{s}^{-1}$, $b = 1\ \mathrm{m\,s}^{-2}$, $c = 2\ \mathrm{m}^{-1}$, and $d = 2\ \mathrm{kg}^{-1}$.

(a) [10 pts] Sketch (or print) an equivalent block diagram of this nonlinear ODE system. *Hint*: use integrator blocks to implement the dynamics of the state variables, and represent each term in the derivatives tapping the input and state variables with corresponding gain blocks. You can represent the sinh function as a static nonlinear activation block with the transfer function $y = \sinh(x)$.

(b) [15 pts] Show the step response of the nonlinear system for the following three values of the step in force $\tilde{f}$ from the baseline $\overline{f} = 0.01$ N (where $f = \overline{f} + \tilde{f}$):

i. $\tilde{f} = 0.1$ N;
ii. $\tilde{f} = 1$ N; and
iii. $\tilde{f} = 10$ N.

Is the response linear in the step size? Explain what you observe.

(c) [15 pts] Now consider the linearized version of the system around its baseline input $\overline{f} = 0.01$ N. Write its transfer function in the Laplace domain. Show the step response of this linearized system, in comparison with the ones you obtained for the nonlinear system at different step sizes in (b). Explain your findings.

(d) [15 pts] For a step $\tilde{f} = 0.001$ N, estimate the rise time t_{rise} as the time to initially reach 90 % of the final value, and the settling time t_{settle} as the time to settle within 10 % of the final value.

2. [45 pts] Now we reconsider the simple model of regulation of glucose metabolism through insulin secretion. As studied in last week's Problem 3, the model is described by the following set of ODEs in the insulin concentration $C(t)$ and glucose concentration $G(t)$, driven by an input $I(t)$ of insulin injection:

$$\begin{aligned} \frac{dC}{dt} &= \alpha\,I(t) - \frac{1}{\tau}\,C(t) \\ \frac{dG}{dt} &= -k\,C(t)\,G(t) \end{aligned}$$

where $\alpha = 0.2\ \mathrm{L}^{-1}$, $\tau = 1$ min, and $k = 1$ L/s mmol.

(a) [10 pts] Sketch (or print) an equivalent block diagram of this nonlinear ODE system.

(b) [15 pts] A total of 10 μmol insulin is injected at once in the bloodstream, all at time $t = 0$. Show the waveforms of the resulting concentrations of insulin $C(t)$ and glucose $G(t)$, from initial conditions $G(0) = G_0 = 1$ mmol/L, and $C(0) = C_0 = 0$. *Hint*: this should look identical to what you got in Problem 3 (e) of last homework, and close to what you got for its linearized version in Problem 3 (d).

(c) [20 pts] Instead, insulin is now slowly released into the bloodstream, at a constant rate of 100 μmol/min. Show the waveforms of the resulting concentrations of insulin $C(t)$ and glucose $G(t)$, from the same initial conditions as in (b). How long does it take for the blood glucose level to be halved, down to 0.5 mmol/L? And what are the steady-state levels for the insulin and glucose concentrations in the bloodstream, assuming insulin injection is maintained at its constant rate?