BENG 122A Fall 2025 HW #5

Due *Friday, November 14* at 11:59pm

This final homework combines all methods learned in the class, now including Bode design and stability analysis, and sets the stage for the final project. As before, you may find it helpful to use MATLAB or Simulink to generate and graph numerical results.

1. [20 pts] In preparation of the *final project*, we ask that each of you identify one (any!) problem in bioengineering that calls for a biosystem control solution that you are now able to formulate, applying the design principles and analysis tools that you have learned in this course. Choose a project title that defines the scope of your proposed project, and write a brief (one paragraph) description of the problem statement, approach, and anticipated outcomes. You are welcome (and encouraged!) to do this with your peers in a group of three (minumum) to five (preferred); if so, list all names of your group (and it is understood that all members of the group submit the same proposal). If you don't yet have a group, you will still have the opportunity to team based on shared interests across other project proposals from your peers, which will be posted on Canvas. Submit your one-page (max) proposal, with title, name(s), and description, as part of your homework 5 submission over Gradescope.

2. [35 pts] Consider a general second-order section biosystem with two poles:

$$H(s) = \frac{A}{s^2 + 2\zeta a\, s + a^2} \tag{1}$$

with DC gain $A > 0$, natural frequency $a > 0$, and damping coefficient $\zeta > 0$.

   (a) [10 pts] Show that the Bode plot of $H(j\omega)$ has:

      i. amplitude $A \,/\, a^2$ (or $20 \log_{10} A - 40 \log_{10} a$ dB) and zero phase at low frequencies $\omega \ll a$;

      ii. amplitude $A \,/\, 2\zeta a^2$ (or $-20 \log_{10} \zeta - 6$ dB offset) and -90^o phase at the natural frequency $\omega = a$; and

      iii. amplitude sloping downward by -40 dB per decade and -180^o phase at high frequencies $\omega \gg a$.

   (b) [10 pts] Show that in case of overdamping ($\zeta > 1$) and critical damping ($\zeta = 1$) this is consistent with the Bode plot contributions anticipated for the corresponding two real negative poles.

   (c) [5 pts] Show that in the underdamped case ($\zeta < 1$) the amplitude exhibits resonance, peaking at the natural frequency, and reaches infinity in the undamped ($\zeta = 0$) case.

   (d) [10 pts] Use the Matlab transfer function (`tf`) model and Bode (`bode`) tool to graph the Bode plot of the transfer function $H(s) = \tilde{u}(s) \,/\, \tilde{f}(s)$ in HW #4 Problem 1. Verify, on the graph, that the above observations are correct.

3. [45 pts] Finally, we reconsider the closed-loop feedback proportional-integral-derivative (PID) control for regulation of glucose metabolism through insulin[1] secretion, as studied in HW #1 through HW #4 (Problem 2), but now through Fourier frequency analysis and Bode design of the open-loop

[1] Or, more generally, complementary insulin and glucagon secretion for down and up regulation of glucose as in HW #4.

system frequency response $\text{OL}(j\omega) = \text{PID}(j\omega)\,\text{Meas}(j\omega)\,\text{Bio}(j\omega)$ for stability of the closed-loop system response $\text{CL}(j\omega) = \text{OL}(j\omega)\,/\,(1+\text{OL}(j\omega))$. Remember for each of these transfer functions that they are identical in the Laplace domain and in the Fourier domain, simply substituting $s = j\omega$. As in the previous homework, for small variations ($\tilde{I}$, $\tilde{C}$, $\tilde{G}$, $\tilde{G}_{\text{meas}}$, and $\tilde{T}$) in the signals (I, C, G, G_{meas}, and T) around their steady-state values ($\overline{I}$, $\overline{C}$, $\overline{G}$, $\overline{G}_{\text{meas}}$, and $\overline{T}$) these are:

$$\text{Insulin-glucose biosystem:} \quad \text{Bio}(s) \stackrel{\text{def}}{=} \frac{\tilde{G}(s)}{\tilde{I}(s)} = -\frac{\alpha k \overline{G}}{(s+\frac{1}{\tau})\,(s+k\overline{C})} \tag{2}$$

$$\text{Glucose measurement system:} \quad \text{Meas}(s) \stackrel{\text{def}}{=} \frac{\tilde{G}_{\text{meas}}(s)}{\tilde{G}(s)} = \frac{1}{1+\tau_{\text{meas}}\,s} \tag{3}$$

$$\text{PID control system:} \quad \text{PID}(s) \stackrel{\text{def}}{=} \frac{\tilde{I}(s)}{\tilde{T}(s)-\tilde{G}_{\text{meas}}(s)} = -\frac{K_d\,s^2 + K_p\,s + K_i}{s} \tag{4}$$

where we further make the simplifying observation that the system operates at zero steady-state insulin concentration $\overline{C} = 0$ and at target steady-state glucose concentration $\overline{G} = \overline{G}_{\text{meas}} = \overline{T}$, since $\overline{I} = 0$. As before, $\overline{T} = 1$ mmol/L, $\alpha = 0.2\ \text{L}^{-1}$, $\tau = 1$ min, $k = 1$ L/s mmol, and $\tau_{\text{meas}} = 2$ min.

(a) [10 pts] Find the poles and the zeros of the open-loop system $\text{OL}(s)$, for purely proportional control with $K_p = 0.01$ L/min. Sketch (or graph) the Bode plot for the open-loop system frequency response $\text{OL}(j\omega)$, and find its phase margin. What does this phase margin predict about the stability of the closed-loop system? Check the consistency of this prediction with what you observed for the transient simulation in HW #4 Problem 2 (b) i.

(b) [10 pts] Repeat the above in (a), but now with additional derivative control $K_d = 5$ L, keeping proportional control at $K_p = 0.01$ L/min. Again, predict the stability of the closed-loop system from the phase margin, and check the consistency of this prediction with your observations in HW #4 Problem 2 (b) ii.

(c) [10 pts] Repeat again, but now adding integral control with $K_i = 0.01$ L/min^2 while keeping $K_p = 0.01$ L/min and $K_d = 5$ L for full PID control, and checking the consistency of the stability predicted from the phase margin with your observations in HW #4 Problem 2 (b) iii.

(d) [15 pts] Finally, we design an optimal PID controller to give the closed-loop system a stable response with highest possible accuracy and bandwidth, by determining the PID coefficients to give the open-loop system a phase margin of (approximately) 45 degrees. Explain the role of the two zeros in this optimal PID controller to cancel select poles in the biosystem and measurement system. For these ideal PID parameters, graph the Bode plot to verify the phase margin, and rerun your transient simulation in HW #4 Problem 2 (b) to verify the stability, accuracy, and bandwidth of the closed-loop system.

Reaching this milestone, you're now experts in biosystem control design! On to your final projects!