Biosystem:

Input (driving force)

\[ x'(t) = [f(t), u(t)]^T \]

\[ \text{ODEs} \]

Output (system response)

Linear Time-Invariant (assumption; simplifies analysis using tools of Laplace/Fourier transforms)
\begin{align*}
\frac{d\mathbf{u}}{dt} &= \ldots \left( f, u, v \right) \\
\frac{d\mathbf{u}}{dt} &= \ldots \left( f, u, v \right) \\
\frac{d\mathbf{x}}{dt} &= \mathbf{F} \left( f, \mathbf{x} \right)
\end{align*}

Dynamics of the state variables of the system described by a set of ordinary differential equations (ODEs)

Same, in vectorial notation

Examples in bioengineering:

<table>
<thead>
<tr>
<th>Input</th>
<th>Biosystem</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>(driving force)</td>
<td>(internal state dynamics)</td>
<td>(system response)</td>
</tr>
<tr>
<td>insulin</td>
<td>blood</td>
<td>glucose level</td>
</tr>
<tr>
<td>force</td>
<td>limb</td>
<td>motion</td>
</tr>
<tr>
<td>parameter signal</td>
<td>heart</td>
<td>ECG rhythm</td>
</tr>
</tbody>
</table>
Outline of the course:

1. **[Weeks 1-3]** Model biosystems to characterize the dynamics of input-output relations, and analyze their stability using tools of linear systems analysis. To bioengineers, this provides understanding of what factors affect health and lead to disease.

2. **[Weeks 5-7]** Design control systems that drive biosystems towards desirable states with stable dynamics. To bioengineers, this provides systematic means to regulate biosystems to improve health and remediate disease.

3. **[Weeks 8-10]** Case studies in bioengineering control systems design, including emerging topics in bioinspired and neuromorphic systems engineering. Final project.
Feedback and control in a nutshell:
(a teaser and motivator; more in Weeks 3-7)

\[ \mu = F \cdot H(t - \mu) \]

\[ (1 + FH) \mu = FH \cdot t \]

\[ \mu = \frac{FH}{1 + FH} \cdot t = \frac{1}{1 + \frac{1}{FH}} \cdot t \]

\[ \Rightarrow FH \to \infty \quad \text{in order for} \quad \mu \to t \]

\[ 1,000 \quad 0.399 \quad 1 \]
Problem with high-gain feedback control: stability under influence of system delays:

Effect of perturbation through loop gain

-100
\[ \Delta m = 0.1 \]
\[ \Delta t = 0 \]

Stable if sufficiently fast

Small perturbation around steady-state

Amplifying effect of large system loop gain under long delays

Squashing effect under short delays

Fast negative feedback (rapid decay)

Undershoot (due to delay)

Delayed negative feedback leads to instability (oscillation)

Positive feedback leads to instability (exponential run-away)

Target (control objective)

High-gain negative feedback

Positive feedback leads to instability

Small perturbation around steady-state

Lecture 1: Introduction

Thursday, October 1, 2020

9:14 AM
Takeaway: dynamical systems are complex, and control is hard! Before we may attempt the systematic design of effective control systems, we need a fundamental understanding of the dynamic response and stability of linear systems, using tools of linear analysis and linear transforms (Weeks 1 & 2).