Lecture 7: Feedback

Thursday, October 22, 2020 9:15 AM

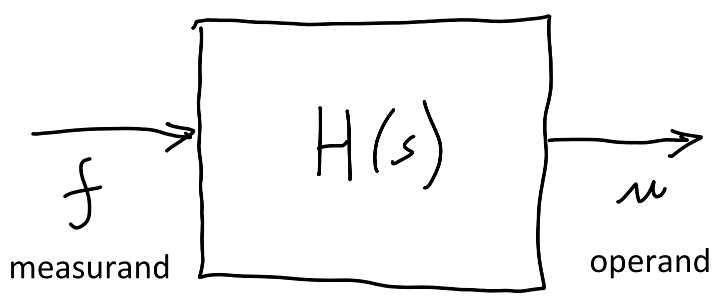
References:

Tranquillo JV. *Biomedical Signals and Systems*, Morgan & Claypool Publishers, Dec. 2013. Ch. 7 (Sec. 7.1 - 7.5).

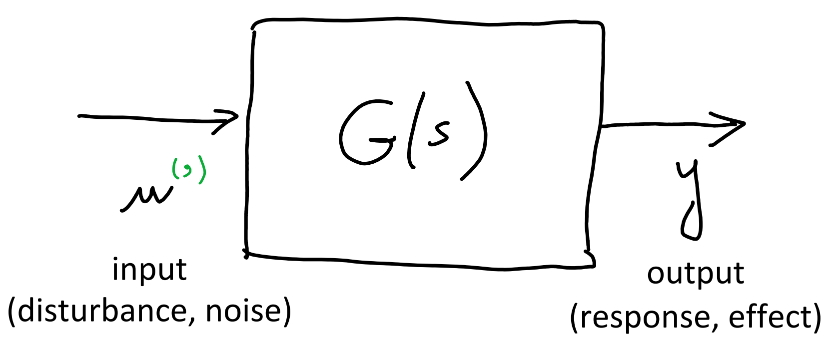
Ahmed R. *MATLAB Simulink Tutorial*, Udemy.

Open loop systems:

A "controller" biosystem (which we design, or which nature has designed/evolved):

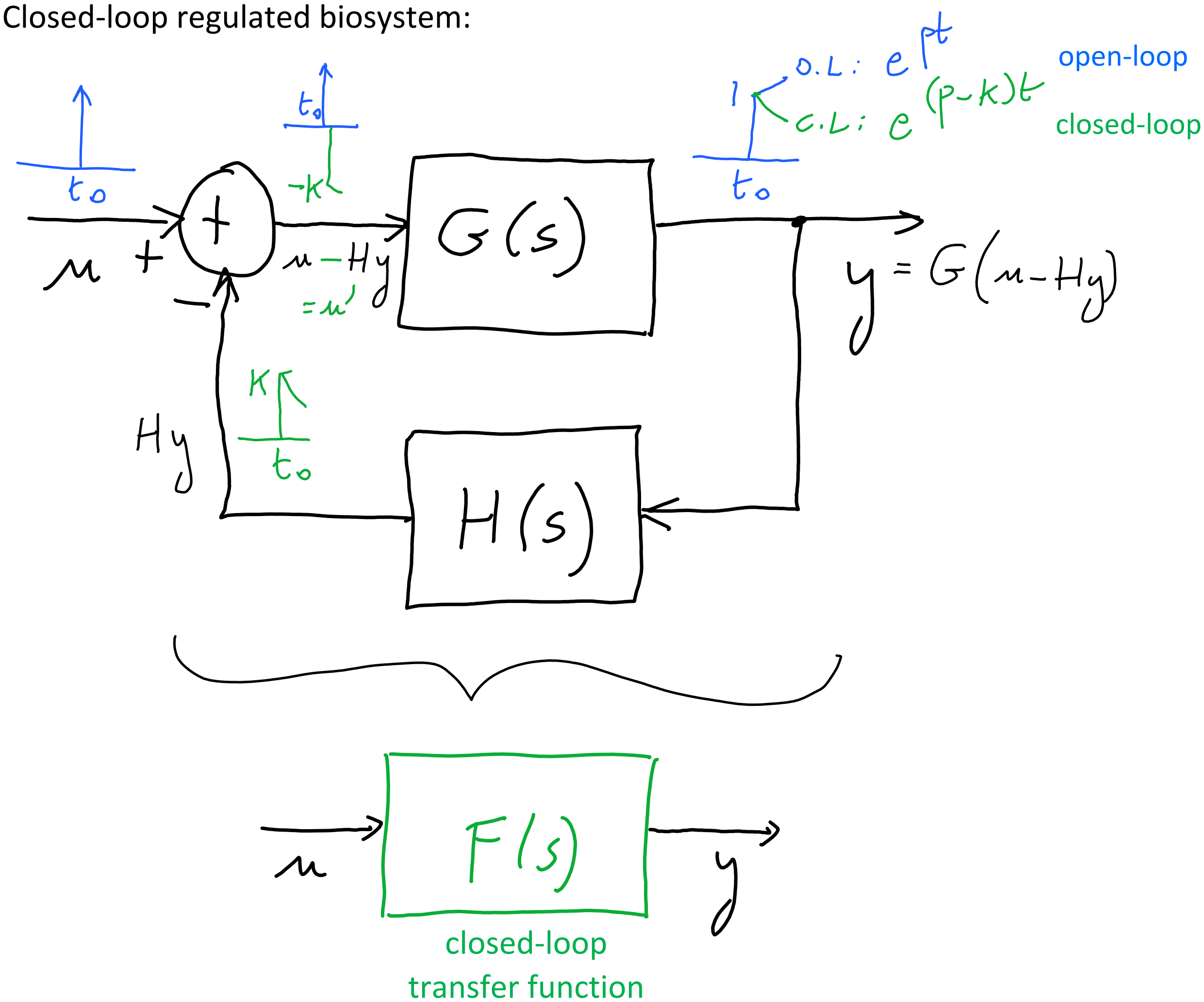


A "plant" biosystem (given by biology, possibly unstable, to be regulated/remediated):



Closing the loop with *negative feedback*:

Closed-loop regulated biosystem:



Closed-loop transfer function:

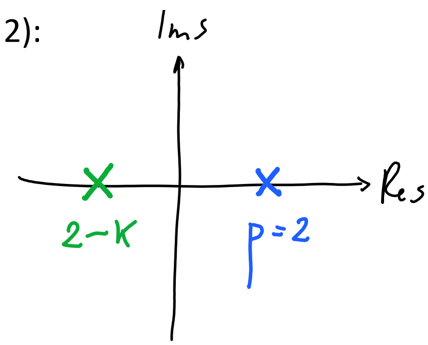
$$(1 + GH)\,y = G\,u$$

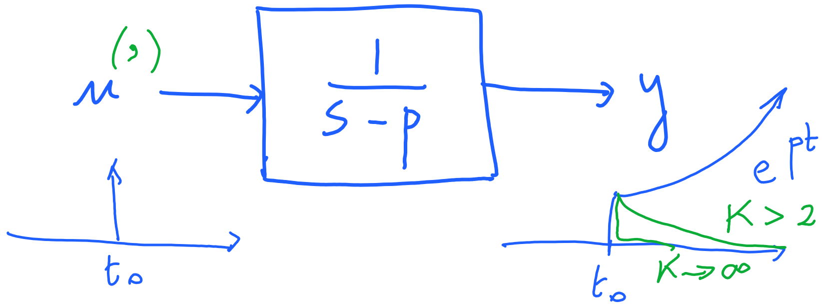
$$F(s) = \frac{y(s)}{u(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

Negative feedback by the controller *H*(*s*) regulates the biosystem *G*(*s*) by moving the locations of its poles, in order to make its (possibly unstable) dynamics (more) stable.

Example unstable system (single positive pole p = 2):

$$G(s) = \frac{1}{s - p}$$





Proportional control (constant scalar):

$$H(s) = K$$

$$\frac{d}{dt} y = p y + u^{(s)}$$

@ t_0: $2 \cdot 1 - K \cdot 1$

$K > 2 \Rightarrow$ STABLE

Feedback design (proportional control):

$$F(s) = \frac{G(s)}{1 + K\, G(s)} \rightarrow 0 \quad \text{as} \quad K \rightarrow \infty$$

Large feedback gain *K* suppresses the response of the regulated biosystem to perturbation or noise at the input: $y(t) = 0$ even when $u(t) \neq 0$.

More generally, the stability of the closed-loop response depends on the feedback gain, and on the poles *and* zeros of the biosystem:

$$G(s) = \frac{N(s)}{D(s)}$$

numerator: zeros

denominator: poles

$$F(s) = \frac{N(s)}{D(s) + K\,N(s)}$$

poles: $D(s) + K\,N(s) = 0$

$$s - 2 + K = 0$$

$$s = 2 - K$$

stable for $K > 2$

Negative feedback allows to move the poles based on the zeros, and proportionally to the feedback gain *K*.