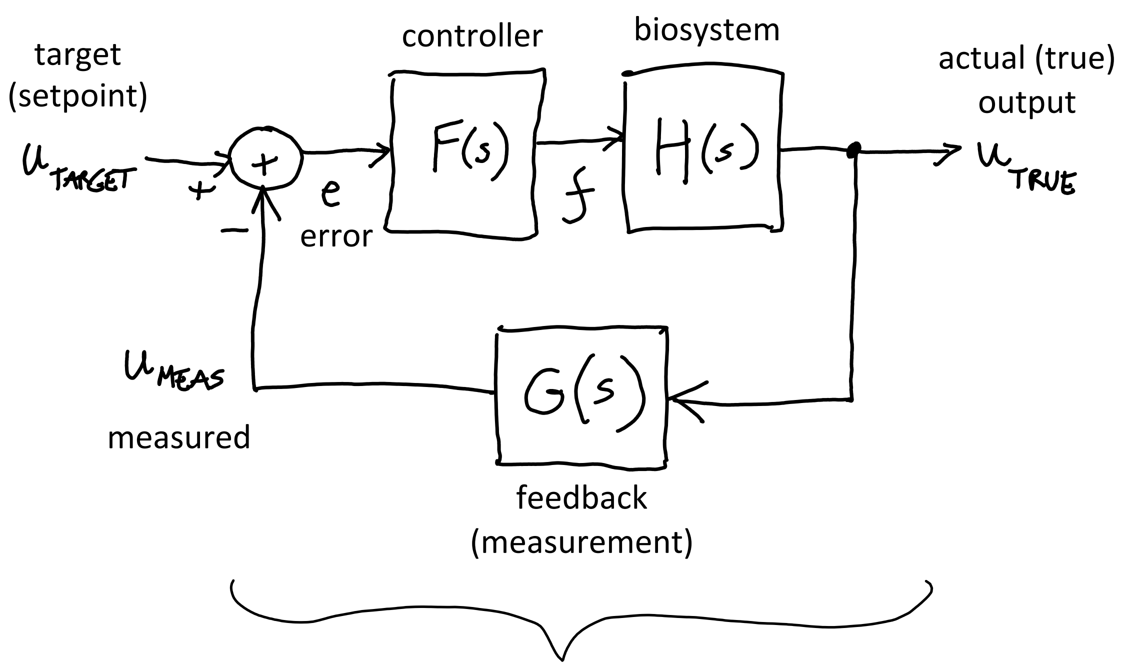
Lecture 9: Proportional, Integral, Derivative Control

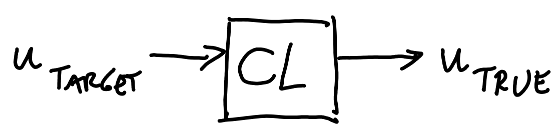
Thursday, November 5, 2020 8:43 AM

References:

Tranquillo JV. *Biomedical Signals and Systems*, Morgan & Claypool Publishers, Dec. 2013. Ch. 9 (Sec. 9.4 - 9.5).

General control setting (Lecture 8):





$$\frac{H(s) \cdot F(s)}{1 + G(s) \cdot H(s) \cdot F(s)}$$

closed-loop
transfer function

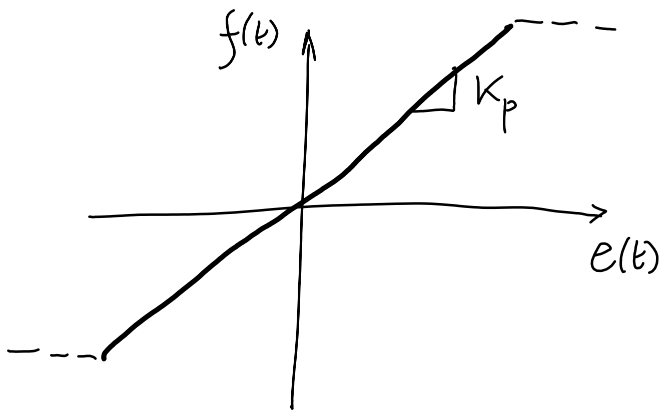
Our task is to design the controller *F*(*s*), based on a model of the biosystem H(s), and the measurement feedback system *G*(*s*).

We consider a mixture of three control strategies for F(s).

Proportional (P) control:

$$F = K_p$$

$$f(t) = K_p \cdot e(t) = K_p \left(u_{TARGET} - u_{MEAS}\right)$$



The effect of this design choice for controller *F*(*s*) on the closed-loop dynamics CL(*s*) depends on the models for the biosystem *H*(*s*) and measurement *G*(*s*).

Consider ideal measurement:

$$G(s) = 1$$

and two cases of an example biosystem with second-order dynamics:

$$H(s) = \frac{1}{ms^2 + \gamma s + k}$$

e.g., position of force-driven damped spring-mass system (second-order *lowpass*)

$$H(s) = \frac{s}{ms^2 + \gamma s + k}$$

e.g., velocity of force-driven damped spring-mass system (second-order *bandpass*)

Resulting closed-loop transfer function, in both cases:

$$CL(s) = \frac{\dfrac{s\,F(s)}{ms^2+\gamma s+k}}{1+\dfrac{s\,F(s)}{ms^2+\gamma s+k}} = \frac{s\,F(s)}{ms^2+\gamma s+k+s\,F(s)}$$

which for proportional control reduce to:

$$CL(s) = \frac{K_p\,s}{ms^2+\gamma s+k+K_p\,s}$$

K_p increases *speed* of the low-pass response, but *with steady-state error,* and *ringing (under-damped oscillations).*

Steady-state gain (s = 0):

$$\frac{K_p}{k+K_p} < 1$$

K_p increases *damping* of the low-pass response, but *without steady-state response,* and with *slow (over-damped) settling.*

Steady-state gain (s = 0):

$$0$$

These *steady-state errors* are remediated by adding *integral* control.

The dynamics are further improved by adding *derivative control*.

Proportional and integral (PI) control:

$$F(s) = K_p + K_i \frac{1}{s} = \frac{K_p s + K_i}{s}$$

$$f(t) = K_p\, e(t) + K_i \int_{-\infty}^{t} e(t)\, dt$$

$$\mathcal{L}(s) = \frac{s F(s)}{m s^2 + \gamma s + k + s F(s)} = \frac{\cancel{s}\,\frac{K_p s + K_i}{\cancel{s}}}{m s^2 + \gamma s + k + \cancel{s}\,\frac{K_p s + K_i}{\cancel{s}}}$$

$$= \frac{K_p s + K_i}{m s^3 + \gamma s^2 + (k + K_p) s + K_i} \qquad = \frac{K_p s + K_i}{m s^2 + (\gamma + K_p) s + k + K_i}$$

Steady-state gain (s = 0):

$$= 1$$

Steady-state gain (s = 0):

$$\frac{K_i}{k + K_i} < 1$$

Adding *derivative* control further improves on the *high-frequency response*.

Proportional, integral and derivative (PID) control:

$$F(s) = K_p + K_i \frac{1}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s}$$

$$f(t) = K_p e(t) + K_i \int_{-\infty}^{t} e(t)\,dt + K_d \frac{d}{dt} e(t)$$

$$CL(s) = \frac{s F(s)}{m s^2 + \gamma s + k + s F(s)} = \frac{\cancel{s}\,\dfrac{K_d s^2 + K_p s + K_i}{\cancel{s}}}{m s^2 + \gamma s + k + \cancel{s}\,\dfrac{K_d s^2 + K_p s + K_i}{\cancel{s}}}$$

$$= \frac{K_d s^2 + K_p s + K_i}{m s^3 + (\gamma + K_d) s^2 + (k + K_p) s + K_i} \qquad = \frac{K_d s^2 + K_p s + K_i}{(m + K_d) s^2 + (\gamma + K_p) s + k + K_i}$$

at low frequencies ($s = 0$): $= 1 \quad (K_i \neq 0)$

at high frequencies ($s \to j\infty$): $\dfrac{K_d}{m s}$

at low frequencies ($s = 0$): $\dfrac{K_i}{k + K_i} < 1$

at high frequencies ($s \to j\infty$): $\dfrac{K_d}{m + K_d} < 1$

Derivative control K_d improves *high-frequency* response (lowers the rise time)

Integral control K_i improves *low-frequency* response (reduces steady-state error)

Proportional control K_p allows to improve *mid-frequency* response (improve settling by critically damping the response)