

# Review 1: Practice Quiz 1

Quiz 1 covers all material, as covered in Lectures 1 through 7, and Homework 1 through 3. It is open book, open notes, and online, but web search is prohibited. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.

Quiz 1 will be posted online, and is due over Canvas as scheduled. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.

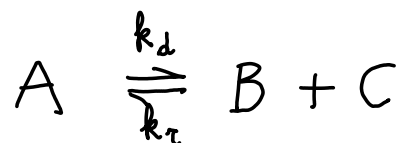
## References

Tranquillo JV. *Biomedical Signals and Systems*, Morgan & Claypool Publishers, Dec. 2013. Ch. 1 - Ch. 8.

## PRACTICE QUIZ

### Problem 1

Consider the following biochemical reaction taking place in an organ in the body:



where compound A decomposes into compounds B and C at rate  $k_d$ , and B and C recombine into A at rate  $k_r$ . Assume B and C are maintained at constant concentrations  $[B]_0$  and  $[C]_0$  inside the organ.

1. Write the ODE that describes the kinetics of the reaction in the concentration  $[A]$ .
2. Find the equilibrium (*i.e.*, steady-state) concentration  $[A]_0$  of compound A in the organ, and find the time constant of the reaction.
3. Now, assume compound A flows out of the volume  $V$  of the organ at a flow rate  $Q$ . Write the ODE in the concentration  $[A]$  that describes both the reaction kinetics and the flow of A. Find the time constant.
4. Use Laplace transforms to find the concentration  $[A]$  as a function of time, starting from zero initial condition  $[A](0) = 0$ , with the following parameters:  $k_d = 0.5 / \text{min}$ ,  $k_r = 0.1 \text{ L} / \text{min mmol}$ ,  $Q = 2 \text{ L/min}$ ,  $V = 4 \text{ L}$ ,  $[B]_0 = 10 \text{ mmol/L}$ , and  $[C]_0 = 1 \text{ mmol/L}$ .

## Problem 2

Consider the following set of ODEs describing the dynamics of a biomechanical system with mass  $m$  and stiffness  $k$ , with force  $f(t)$  driving the input, and with velocity  $v(t)$  at the output:

$$\frac{du}{dt} = v(t)$$

$$m \frac{dv}{dt} = -k u(t) + f(t)$$

1. Find the Laplace transform of velocity  $v(s)$  as a function of the Laplace transform of the force  $f(s)$ , and initial conditions in velocity  $v(0) = v_0$  and in position  $u(0) = u_0$ .
2. For zero force  $f(t) = 0$ , and for given initial conditions  $v(0) = v_0$  and  $u(0) = u_0$ , find the velocity  $v(t)$  as a function of time.

3. Find the transfer function  $H(s) = v(s) / f(s)$  of the system, and find the poles and zeros.
4. Now consider closed-loop feedback, in which the force  $f(t)$  is given by

$$f(t) = f_{\text{ext}}(t) - K v(t)$$

where  $f_{\text{ext}}(t)$  is the externally applied force, and  $K$  is the feedback gain. Draw the closed-loop system block diagram, and find the closed-loop transfer function  $F(s) = v(s) / f_{\text{ext}}(s)$ .

5. Find the value of the feedback gain  $K$  for which the closed-loop system is critically damped.

### Problem 3

1. What is the resistance of a series junction of two vessels, each with resistance  $R$ ? What is the compliance of a parallel junction of two vessels, each with compliance  $C$ ?
2. Give some example of a nonlinearity in a bioengineering system, and (briefly) describe how it affects its operation or characteristics, and how to deal with it.
3. Explain differences between the Laplace transform and the Fourier transform.