Review 1: Practice Quiz 1

Quiz 1 covers all material, as covered in Lectures 1 through 7, and Homework 1 through 3. It is open book, open notes, and online, but web search is prohibited. ***No collaboration or communication in any form is allowed***, except for questions to the instructor and TAs.

Quiz 1 will be posted online, and is due over Canvas as scheduled. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.

References

Tranquillo JV. *Biomedical Signals and Systems*, Morgan & Claypool Publishers, Dec. 2013. Ch. 1 - Ch. 8.

PRACTICE QUIZ

Problem 1

Consider the following biochemical reaction taking place in an organ in the body:

$$A \underset{k_r}{\overset{k_d}{\rightleftharpoons}} B + C$$

where compound A decomposes into compounds B and C at rate k_d, and B and C recombine into A at rate k_r. Assume B and C are maintained at constant concentrations $[B]_0$ and $[C]_0$ inside the organ.

1. Write the ODE that describes the kinetics of the reaction in the concentration [A].

2. Find the equilibrium (*i.e.*, steady-state) concentration $[A]_0$ of compound A in the organ, and find the time constant of the reaction.

3. Now, assume compound A flows out of the volume V of the organ at a flow rate Q. Write the ODE in the concentration [A] that describes both the reaction kinetics and the flow of A. Find the time constant.

4. Use Laplace transforms to find the concentration [A] as a function of time, starting from zero initial condition $[A](0) = 0$, with the following parameters: $k_d = 0.5$ / min, $k_r = 0.1$ L / min mmol, $Q = 2$ L/min, $V = 4$ L, $[B]_0 = 10$ mmol/L, and $[C]_0 = 1$ mmol/L.

Problem 2

Consider the following set of ODEs describing the dynamics of a biomechanical system with mass m and stiffness k, with force $f(t)$ driving the input, and with velocity $v(t)$ at the output:

$$\frac{du}{dt} = v(t)$$

$$m\frac{dv}{dt} = -k\,u(t) + f(t)$$

1. Find the Laplace transform of velocity $v(s)$ as a function of the Laplace transform of the force $f(s)$, and initial conditions in velocity $v(0) = v_0$ and in position $u(0) = u_0$.

2. For zero force $f(t) = 0$, and for given initial conditions $v(0) = v_0$ and $u(0) = u_0$, find the velocity $v(t)$ as a function of time.

3. Find the transfer function $H(s) = v(s) / f(s)$ of the system, and find the poles and zeros.

4. Now consider closed-loop feedback, in which the force f(t) is given by

$$f(t) = f_{ext}(t) - K\,v(t)$$

where $f_{ext}(t)$ is the externally applied force, and K is the feedback gain. Draw the closed-loop system block diagram, and find the closed-loop transfer function $F(s) = v(s) / f_{ext}(s)$.

5. Find the value of the feedback gain K for which the closed-loop system is critically damped.

Problem 3

1. What is the resistance of a series junction of two vessels, each with resistance *R*? What is the compliance of a parallel junction of two vessels, each with compliance *C*?

2. Give some example of a nonlinearity in a bioengineering system, and (briefly) describe how it affects its operation or characteristics, and how to deal with it.

3. Explain differences between the Laplace transform and the Fourier transform.