Designing and Modeling a Hypertension Control System During Surgery

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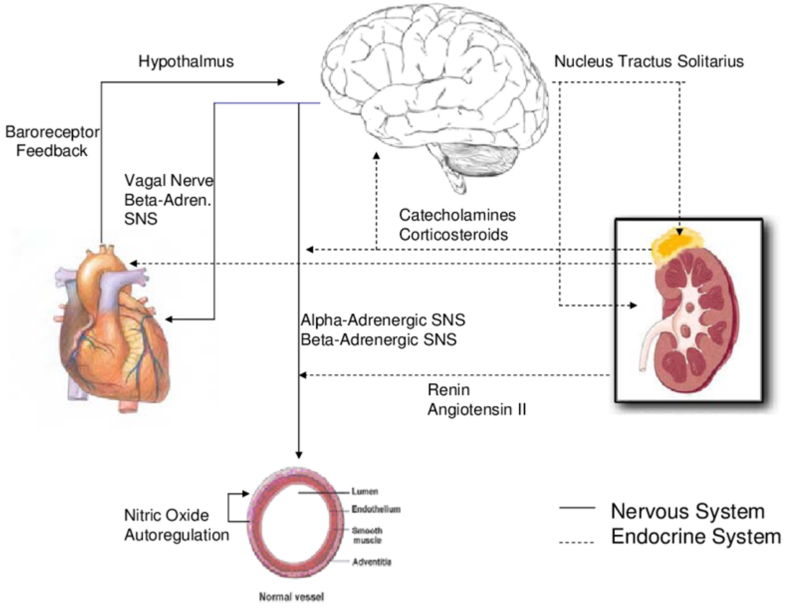
Introduction

- Blood pressure (BP) regulation can be modeled as a homeostatic control system.
- Monitoring and controlling **hypertension** during surgery is **pivotal** to successful outcomes.[1]
    - “...hypertension increased perioperative cardiovascular complications by 35%.” [2]
    - “As much as 25% of patients having major non-cardiac surgery have perioperative hypertension.” [3]
    - “As much as 80% of patients having cardiac surgery have perioperative hypertension.” [4,5]



Homeostatic BP regulation.

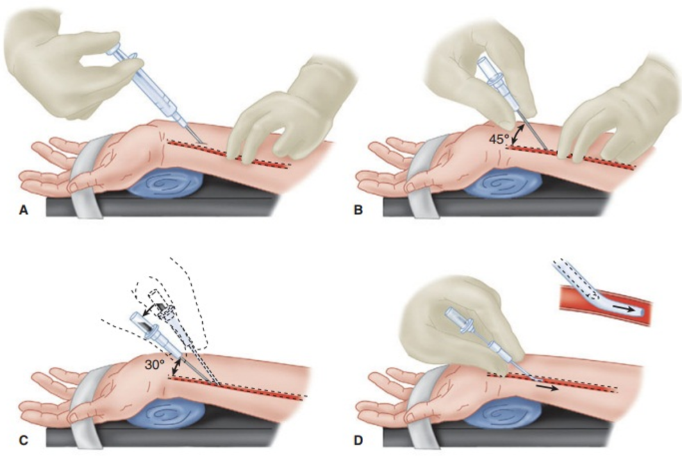
Gantt, M. (2021). The Effect Of Combat Exercises On Cardiovascular Response: An Exploratory Study.

Problem Statement:

Design a control system that **monitors and controls** BP for patients experiencing **acute hypertension** during **surgery**.

Proposed Solution

- Design and model PID control system that monitors and regulates BP using an ideal invasive BP sensor and intravenous drug injector to treat hypertension during surgery.
- Sodium nitroprusside (SNP) will be dynamically and intravenously injected since it is well-characterized and commonly used as a vasodilator to treat hypertension during surgery.[6]



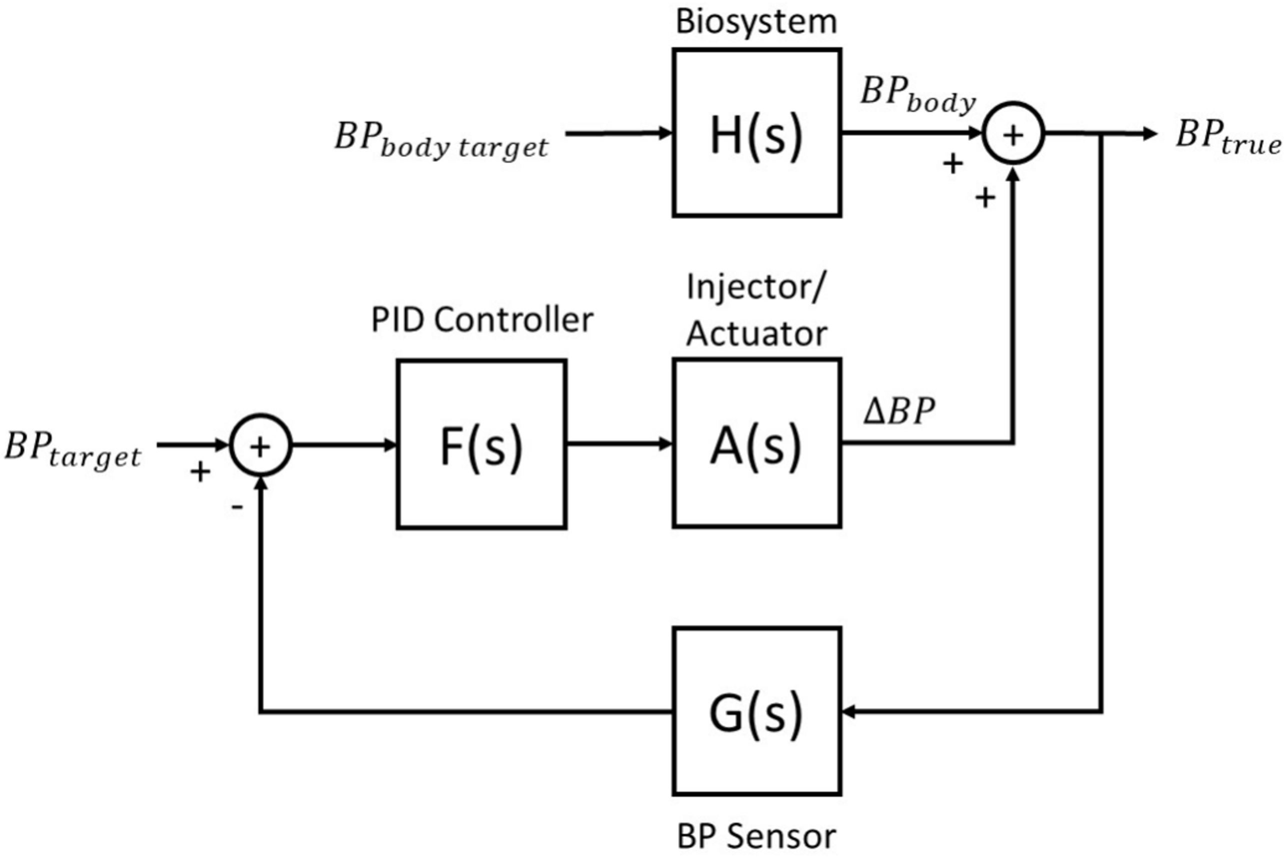
John Butterworth, J., & Monitoring, I. (2021). Invasive Arterial Blood Pressure Monitoring.

Sodium Nitroprusside Intravenous: Uses, Side Effects, Interactions, Pictures, Warnings & Dosing - WebMD. (2021).

Assumptions

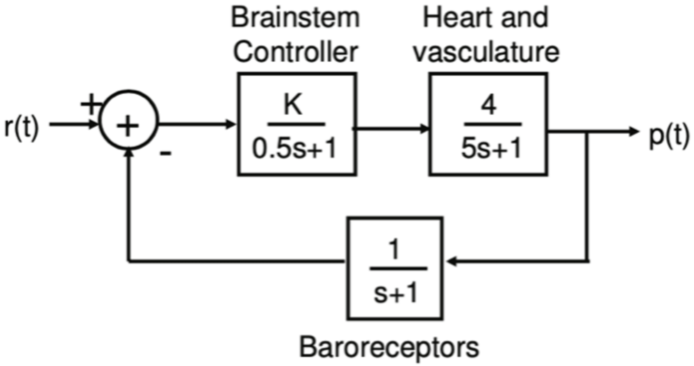
- The patient has a constant high blood pressure setpoint during surgery i.e. hypertension, $BP_{body\ target}$ = (constant).
- BP sensor is capable of ideal measurement i.e. $G(s) = 1$.
- Operation is under a small enough time frame such that the body does not develop a resistance to SNP.
- Gain of control (K) within biosystem is equivalent to patient sensitivity.

Block Diagram



Transfer Function: Biosystem

Biosystem for BP regulation is a closed loop system[7]



Transfer function

$$H(s) = \frac{\frac{K}{(0.5s+1)} * \frac{4}{(5s+1)} * \frac{1}{(s+1)}}{1+\frac{K}{(0.5s+1)} * \frac{4}{(5s+1)} * \frac{1}{(s+1)}} = \frac{\frac{K}{(0.5s+1)} * \frac{4}{(5s+1)} * \frac{1}{(s+1)}}{\frac{(0.5s+1)*(5s+1)*(s+1)+4K}{(0.5s+1)*(5s+1)*(s+1)}} = \frac{4K}{(0.5s+1)*(5s+1)*(s+1)+4K}$$

where K represents the gain of the controller dependent on each patient. A range of values for K still needs to be found.

Transfer Function: Actuator

Transfer function relating change in blood pressure and rate of nitroprusside infusion[8]

$$A(s) = \frac{\Delta P_d(s)}{I(s)} = \frac{k e^{-T_i s}(1 + e^{-T_c s})}{\tau s + 1}$$

where k represents sensitivity of the patient, T_i represents transport time delay, T_c represents recirculation transport time delay, and τ represents the system's time constant.

Transfer Function Parameters for Patient Types

Parameter	Sensitive	Nominal	Insensitive
k	-9	-0.7143	-0.1786
α	0	0.4	0.4
T_i	20	30	60
T_c	30	45	75
τ	30	40	60

Transfer Function: PID Controller

Fractional-order transfer function for PID controller[8]

$$F(s) = K_p + \frac{K_i}{s^\lambda} + K_d s^\mu$$

where K_p represents proportional gain, K_i represents integral gain, and K_d represents derivative gain. λ and μ can be anywhere between 0 and 2 and will be both set to 1 for ease of computation.

Transfer Function: PID Controller (cont.)

Values for Kp Ki plane[8]

$$K_p = -K_d \omega^{\mu} \frac{\sin\left(\frac{\pi}{2}(\lambda+\mu)\right)}{\sin\left(\frac{\pi}{2}\lambda\right)} - \frac{R(\omega) - \cos\left(\frac{\pi}{2}\lambda\right) I(\omega)}{R^2(\omega) + I^2(\omega)}$$

$$K_i = K_d \omega^{\mu+\lambda} \frac{\sin\left(\frac{\pi}{2}\mu\right)}{\sin\left(\frac{\pi}{2}\lambda\right)} - \frac{\omega^2 I(\omega)}{\sin\left(\frac{\pi}{2}\lambda\right) + \left(R^2(\omega) + I^2(\omega)\right)}$$

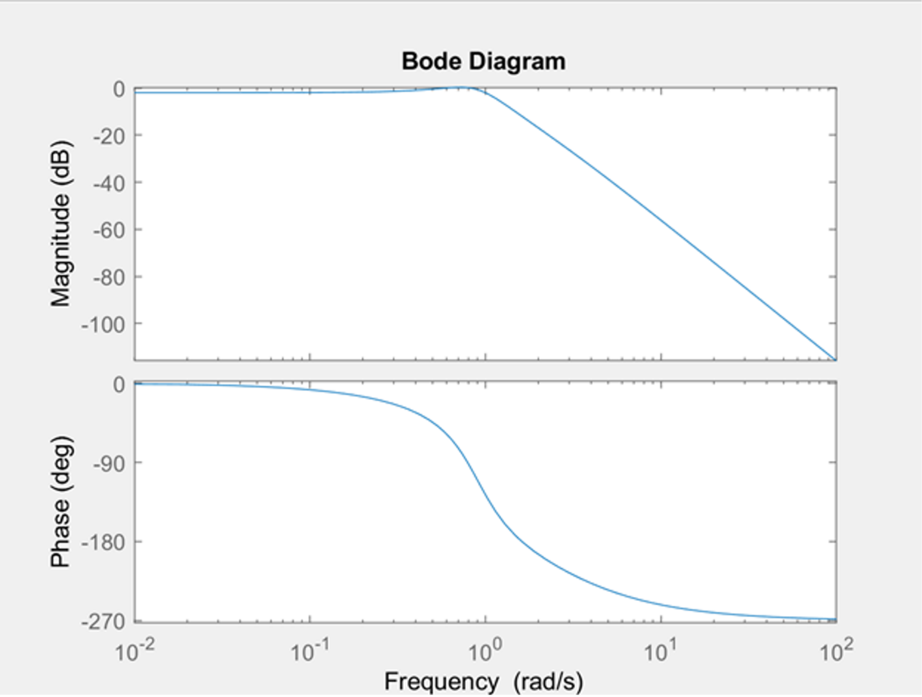
Values for Kp Kd plane[8]

$$K_i = -K_p \frac{\omega^{\lambda} \sin\left(\frac{\pi}{2}(\mu)\right)}{\sin\left(\frac{\pi}{2}(\lambda+\mu)\right)} - \frac{\omega^{\lambda}\left(\sin\left(\frac{\pi}{2}\mu\right) R(\omega) + \cos\left(\frac{\pi}{2}\mu\right) I(\omega)\right)}{\sin\left(\frac{\pi}{2}(\lambda+\mu)\right)\left(R^2(\omega) + I^2(\omega)\right)}$$

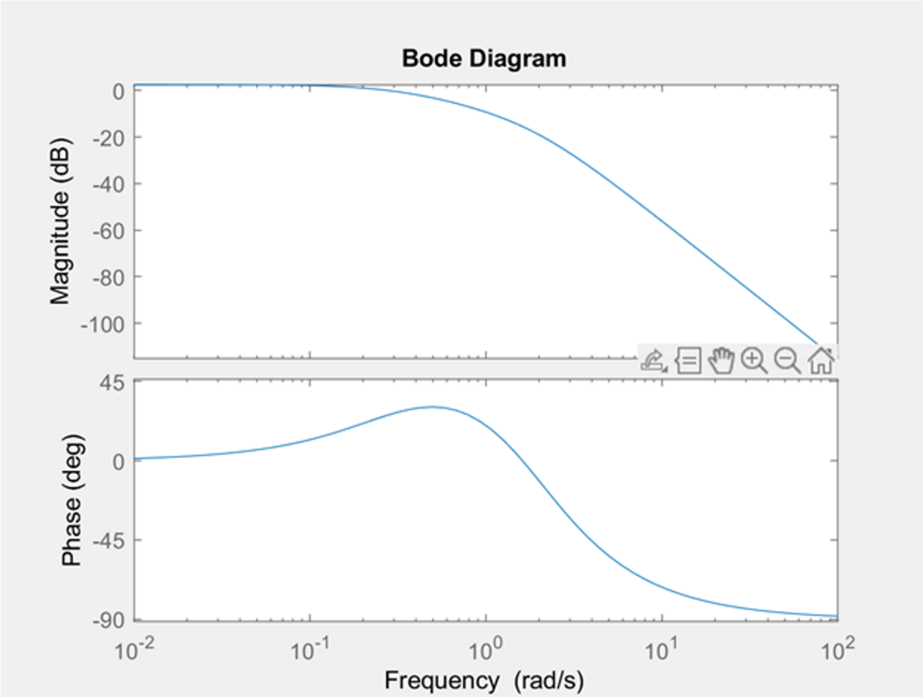
$$K_d = \left(\frac{K_p}{\omega^{\mu}}\right) \frac{\sin\left(\frac{\pi}{2}(\mu)\right)}{\sin\left(\frac{\pi}{2}(\lambda+\mu)\right)} - \frac{\sin\left(\frac{\pi}{2}\mu\right) R(\omega) - \cos\left(\frac{\pi}{2}\mu\right) I(\omega)}{\omega^{\mu} \sin\left(\frac{\pi}{2}(\lambda+\mu)\right)\left(R^2(\omega) + I^2(\omega)\right)}$$

Biosystem Bode Plot

For when K=1



For when K= -1



Discussion

- Assumptions may not be grounded in reality:
  - Patients' BP target is dynamic and may change over the course of surgery. We assume the worst case by setting the patient BP target to a higher end value found during surgery.
  - BP sensors are not perfect i.e. $G(s) < 1$.
- All patients are different, so the system has to be able to respond to many different variables.
- When translating to physical product, we must consider the harmful effects of injecting too much compound over a period of time (physical limiting factor).

Future Work

- Find range of values for K for biosystem, H(s), by analyzing real patient data and approximating extreme case values on low and high ends.
- Simulate system with different values of proportional, integral, and derivative gains.
- Find PID controller values optimized for all patient types.
- Analyze system response to blood pressure change.
- Potentially consider adding a different compound injector that raise BP (hypotension treatment).

References

1. Aronow, W. (2017). Management of hypertension in patients undergoing surgery. Annals Of Translational Medicine, 5(10), 227-227. doi: 10.21037/atm.2017.03.54
2. Howell SJ, Sear JW, Foëx P. Hypertension, hypertensive heart disease and perioperative cardiac risk. Br J Anaesth 2004;92:570-83. 10.1093/bja/aeh091
3. Dix P, Howell S. Survey of cancellation rate of hypertensive patients undergoing anaesthesia and elective surgery. Br J Anaesth 2001;86:789-93. 10.1093/bja/86.6.789
4. Cheung AT. Exploring an optimum intra/postoperative management strategy for acute hypertension in the cardiac surgery patient. J Card Surg 2006;21 Suppl 1:S8-S14. 10.1111/j.1540-8191.2006.00214.x
5. Haas CE, LeBlanc JM. Acute postoperative hypertension: a review of therapeutic options. Am J Health Syst Pharm 2004;61:1661-73; quiz 1674-5.
6. "Sodium Nitroprusside". The American Society of Health-System Pharmacists.
7. Tranquillo, Joseph Vincent. Biomedical Signals and Systems. Morgan & Claypool, 2014.
8. Urooj, S., & Singh, B. (2019). Fractional-Order PID Control for Postoperative Mean Arterial Blood Pressure Control Scheme. Procedia Computer Science, 152, 380-389. doi: 10.1016/j.procs.2019.05.002

Questions?

Differential Model

$$F_c(s) = G_c\left(1 + \frac{1}{\tau_c s}\right)$$

$$\tau_c = \tau$$

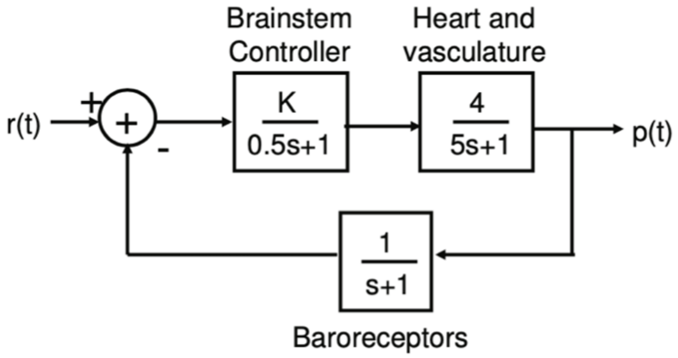
$$G_c = \frac{\tau}{G[\lambda + (1 + \alpha)\, T_i + \alpha T_c]}$$

$$T(s) = \frac{1}{1 + \alpha} \frac{e^{-T_i s}(1 + \alpha e^{-T_c s})}{\dfrac{\lambda}{1 + \alpha} S + 1}$$

Fc(s) is the controller transfer function for the closed loop transfer function T(s)

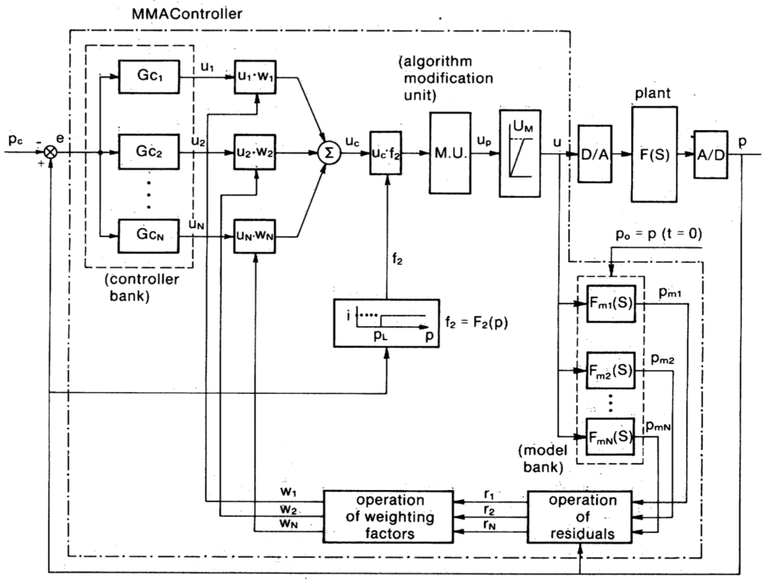
$$F_c(s) = \frac{1}{F(S)} \frac{T(s)}{1 - T(s)}$$

Block Diagram



The biosystem is represented by this block diagram.

Block diagram to the right is for the multiple model adaptive controller which is simplified to the block diagram below



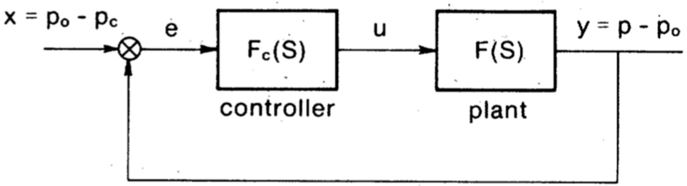


Fig. 2. An equivalent continuous system.

Fig. 1. MMAC system structure.