Modeling of a Residual Limb-Air Bladder System for Use in Adaptable Prosthetic Socket Development

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Abstract— **Every year hundreds of thousands people around the world get amputated. In the first two years their amputation changes rapidly in shape and size, requiring many remakes to maintain proper fitting. After the first two years daily fluctuations are still seen, but at a much smaller scale. These changes lead to a poorly fitted prosthetic device, requiring high proprioception to understand the changes that need to be made. As the necessity of prosthetics and their relative accessibility grows, a paramount concern has become the quality of prosthetics available to patients. Around 57 percent of lower limb amputees report considerable discomfort while about 90 percent of those uncomfortable report severe pain associated with their prosthetics. One potential solution is an adjustable prosthetic socket where the circuit design reflects a system that adjusts the socket shape to the residual limb that it is fit on [1]. A closed loop feedback system with force sensors (FSR) and a PID controller is utilized to adjust the inflation or vacuum to alter the internal socket volume. The results will be a vacuum and air pump that can be turned on/off when the threshold value is not met to allow for the greatest fit of the prosthetic.**

Clinical Relevance— **The development of a prosthetic design that adjusts to a residual limb it fits can decrease the pistoning effect and other signs of discomfort, reducing clinical cases of reamputation [2].**

I. Introduction

Amputees are far from an anomalous demographic with 1 in 190 American currently being amputated. With the rise of diabetes and other related diseases such as peripheral arterial disease (PVD), a plurality of the 1.7 million amputees, and the 28 million at risk of amputation in the United States are at risk due to these diseases over trauma [3]. Patients with PVD are particularly affected by developments in prosthetics, because of their residual limbs' relative sensitivity to prosthetic fat. Because their venous flow often becomes occluded, there are limbs daily volume changes are often drastic enough to render a previously well-fitting prosthetic as ill-fitting. Additionally long-term residual limb volume is greatly variable among PVD patients who see their limb volume decrease by 9% for 160 days after amputation [4]. Though clinicians recommend waiting until this volume change occurs for PVD patients to proceed with prosthetic fittings, the wait time leads to less muscle memory and greatly diminishes the success of an amputee's post-prosthetic physical activity.

The amount of amputees is expected to double by the year 2050, yet many amputees remain dissatisfied with prosthetic options that they have and that others in the future may be presented with [5]. Among the factors that lead to this dissatisfaction are wound healing time, residual limb volume fluctuation and prosthetic irritants [6]. Residual limb volume is especially important in considerations for developing prosthetics for diabetic amputees. These patients often have to go through re-amputation on their lower extremities due to all fitting prosthetics because of the lack of emphasis on volumetric changes when creating a prosthetic [7,8].

Volumetric concerns however, are not limited to PVD and diabetes amputees. The first stage is the first few months after the amputation. Here the residual limb decreases in volume between 17 and 35% due to muscle atrophy, edema and other biological readjustments post operation [9,10]. After maturing, the limb's volume will continue to fluctuate 5% for up to an additional 1.5 years. This maturing phase is targeted through prosthetic developments like the one explored here.During this phase, volumetric changes caused by the condition of the amputee's body are thought to occur because of these 3 main factors: pooling of blood in the venous compartment, arterial vasodilation, and changes in the interstitial fluid volume. Given that infection from unwanted, often volume-related socket pressure affects between 13% and 48% of amputees, prosthetic developments addressing volumetric measurements remain imperative.

While many factors require a simple change such as the use of a different material that does not cause irritation on an amputee's residual limb, volume fluctuation is harder to account for. It is a physical component that is highly variable between patients and also per patient based on their daily activity, health, salt intake and numerous other environmental and biological factors.Yet, this is the factor that remains the leading cause of discomfort among prosthetic users [11]. If the prosthetic socket is too tight, it may put pressure on unwanted areas on the residual limb. If it is too loose, it may cause the pistoning effect which results from a residual limb constantly shifting in a loose, ill-fitting socket.

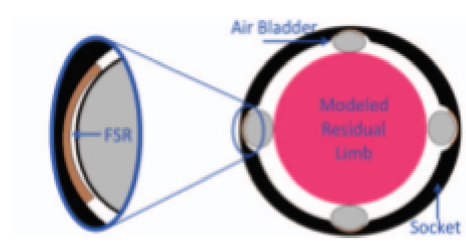

Figure 1. Shows layout of air bladders, FSRs,socket and the limb. FSR are force sensing resistors [14].

Currently, a study by McLean et al. has shown the merits of a mobile device-controlled adjustable socket that releases every couple minutes during ambulation. However, this control system could be improved upon with the addition of a pressure and shear sensing liner system controlling a feedback loop. One study used a PID controller to manage the tightness of a socket via a motor that controlled the tightness of cables that then adjusted the size of the socket [12]. An alternative to the cables is a system of air bladders

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that can surround the socket and inflate or deflate as needed by the amputee to fit them for extended periods of time through volumetric changes. This approach paired with a multi-sensor liner allows for the great flexibility in creating a prosthetic that is dependent on volume changes that then inform the fit of the prosthetic socket. This sensing system paired with an element of user input and calibration based on their personal comfort levels with tightness can provide an increasingly successful prosthetic experience for users without the impending probability of re-amputation occurrence.

Overall, the need for a volumetric sensing socket design is an ever important one that can be met with a system that implements sensing capabilities and flexible structures such as air bladders.

II. ASSUMPTIONS

In modeling the air bladder system and its relationship to a patient's target residual limb several key assumptions were made to simplify the calculations associated with the residual limb and the prosthetic socket. It was assumed that skin and muscle are deformable quantities that could be modeled as a spring damper system until bone was reached. The skin and muscle were unified as one quantity and one system though they would ideally be modeled as two separate entities themselves, acting upon each other. Since they were combined as the singular deformable element here though, this separation was not made. Meanwhile bone was modeled as a rigid, non-deformable quantity. Additionally, the modeled relations between the combined deformable anatomical structures and air bladder are ideal and linear with the air bladder being modeled as a piston, rather than a balloon to allow for a simplified spring damper system. This airbladder system is also overdampened as it fails to even complete one oscillation; the eventual relevance of this is seen in the calculation of the bladder's damping coefficient. leading to the event In the system, the only degree of freedom existed in the direction of the air bladder. The dampers were treated like resistors while the springs were treated as capacitors.

Apart from assumptions regarding the spring damper system, another was made regarding the FSR which were treated as perfect measurement tools with no error and therefore, no time delay to account for such error.

The last set of assumptions were made specifically concerning some numerical quantities regarding the weight of a below-knee prosthetic leg and the desired force throughout a below-knee prosthetic socket where desired pressure often ranges from 5.0 kPa to 40.7 kPa.

TABLE 1: ASSUMPTIONS OF QUANTITIES RELATED TO BELOW-KNEE PROSTHETICS FROM ACCURATE MODELING OF THE SYSTEM.

Average weight of below knee prosthetic leg	4 lbs
Desired force of prosthetic socket	40 kPa

III. BIOSYSTEM

The biosystem was modeled as two spring damper systems connected in series to account for the combined skin and muscle flanking the bone on two sides with the air bladder acting as in series with the linear model,

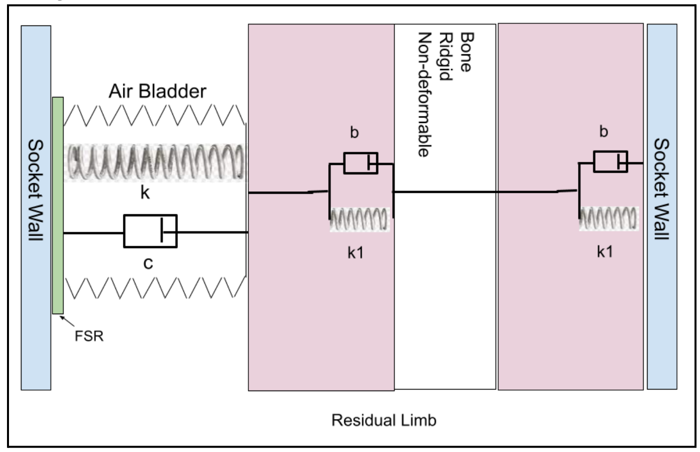

Figure 2: The muscle and airbag modeled as they would appear in a prosthetic socket.

All three spring-damper systems are set in parallel within each of the three systems, but set in series in relation to the other two systems as seen in Figure 2. The spring was modeled as a capacitor with the spring constant k equaling 1 over capacitance (C).

$$k = \frac{1}{C}$$

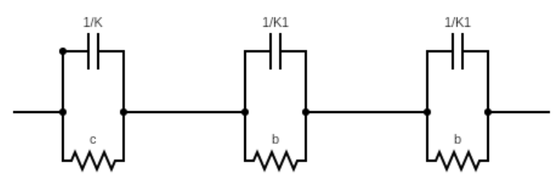

Figure 3: System from Figure 1 modeled as a circuit based on known capacitor-resistor relationships. K is the spring coefficient of the air bladder, K1 is for the skin; c is the damping of the air bladder and b is for the skin.

Here, the calculated impedances of the capacitors (Z_C) and resistors (Z_R) are as follows:

$$Z_C = \frac{1}{jwC} = \frac{1}{sC}$$

$$Z_R = R$$

Where,

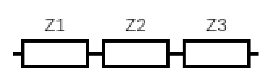

Figure 4: Diagram of impedances of circuit

$$Z_1 = \frac{1}{c} + \frac{1}{\frac{k}{s}} = \frac{1}{c} + \frac{s}{k}$$

$$Z_2 = \frac{1}{b} + \frac{1}{\frac{k1}{s}} = \frac{1}{b} + \frac{s}{K_1}$$

Figure 5: Diagram of impedance following simplification

This set of impedances can be further simplified into a total impedance,

$$Z_T = Z_1 + Z_2 + Z_3 =$$

$$\left(\frac{1}{c} + \frac{s}{K}\right) + \left(\frac{1}{b} + \frac{s}{K_1}\right) + \left(\frac{1}{b} + \frac{s}{K_1}\right) =$$

$$\frac{KbK_1 + scbK_1 + 2KK_1c + 2sKcb}{cKbK_1}$$

$$Z_T = \frac{b+2c}{cb} + \frac{K_1+2K}{KK_1}s = a + K_t s$$

$$new\ spring\colon\ K_t = \frac{KK_1}{K_1+2K} \quad new\ damper\colon a = \frac{b+2c}{cb}$$

which can then be used to find spring and damper coefficients allowing the entire system to be modeled as a single comprehensive spring-damper system as shown in Figure 5.

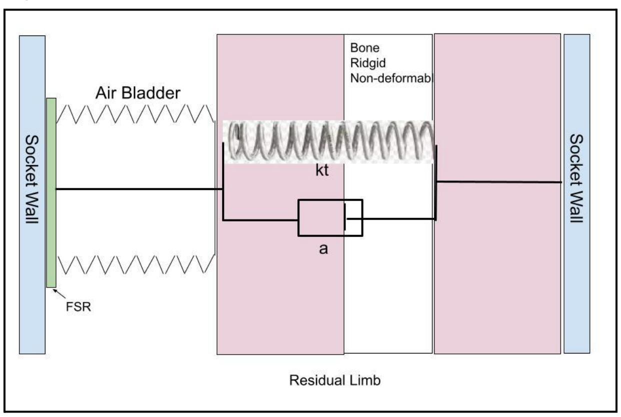

Figure 6: Biosystem modeled as a single spring-damper system.

IV. METHODS

A. Determining the spring (k) and damper (c) coefficients

To determine the coefficients of the soft tissues, previous literature on the coefficients for a deltoid muscle was utilized [13]. In the referenced article, the coefficients were found by striking a pendulum on the deltoids of subjects and an infrared camera was used to measure the change in distance of the weight upon contact. Utilizing a spring damper mass model, accurate spring and damping coefficients were found. These values averaged were then used as the spring-damper coefficients for the combined skin and muscle.

TABLE 2: SOFT TISSUE COEFFICIENTS

Spring coefficient (K_1)	18470 N/m
Damping coefficient (b)	1824 Ns/m

To determine another set of coefficients for the air bladder, a small experiment was conducted using an air bladder on hand as seen in Figure 6 during experimentation.

Figure 7: Air bladder with known weight placed

To find the spring and damping coefficients, a known weight was suspended from the posterior end of the air bladder as shown in Figure 6. Following this, the displacement of the bladder was measured. Then, the spring coefficient was determined via Hooke's Law,

$$F = -kx$$

The experiment gave the values shown in Table 3.

TABLE 3: COEFFICIENT DETERMINATION INTERMEDIATES FROM AIR BLADDER EXPERIMENT

Height of air bladder without mass attached	29/32 inches
Weights attached to the bottom	11.2 oz
Height of air bladder with mass attached	33/32 inches
Change in height (x)	⅛ inches
Mass of weight (m)	3.120N

Using Hooke's Law, the spring constant shown in Table 4, k, was determined.

From this, the damping ratio equation (1) was then used to calculate the damping coefficient.

$$\zeta = \frac{c}{2\sqrt{mk}} \tag{1}$$

With the understanding that the damping ratio would have to reflect an overdamped system, an estimated ratio of 20 was used in the equation above when calculating the damping coefficient.

TABLE 4: AIR BLADDER COEFFICIENTS

Spring coefficient (k)	982 N/m
Damping coefficient (c)	700 Ns/m

B. Mathematical Modeling

Using the system shown in Figure 5, the forces were summed to find based on a study by Moreira et al. in (2),

(2)

$$m\frac{d^2x}{dt^2} + a\frac{dx}{dt} + k_t x(t) = f(t)$$

where m was the mass, k was the spring constant and c was the damping constant [15]. Since (2) was linear, the Laplace was directly taken,

(3)

$$\mathcal{L}(f(t)) = F(s) = ms^2x(s) + asx(s) + k_t x(s)$$

which allowed the subsequent discovery of the transfer functions of the whole system including the air bladder and residual limb,

(4)

$$\frac{F(s)}{X(s)} = ms^2 + as + k_t$$

where the input was the change in distance and the output was the force.

Algebraically, this computation was done as shown below:

(5)

$$F(t) = \beta x(t) + \alpha\dot{x}(t) - \gamma\dot{F}(t)$$

(6)

$$\alpha = b\frac{k_2}{k_1+k_2}, \beta = \frac{k_1k_2}{k_1+k_2}, \gamma = \frac{b}{k_1+k_2}$$

(7)

$$\mathcal{L}(F(t)) = F(s) = \beta x(s) + s\alpha X(s) - \gamma F(s)s$$

(8)

$$F(s) + \gamma sF(s) = \beta X(s) + s\alpha X(s)$$

(9)

$$\frac{F(s)}{X(s)} = \frac{\beta+\alpha s}{1+\gamma s}$$

After solving the spring and damping coefficients into the simplified model equations, k_t and a were determined.

TABLE 5: VALUES DETERMINED FROM MATHEMATICAL MODELING

k_t	9235 N/m
a	0.00109 Ns/m

V. SIMULINK MODEL

Following the mathematical modeling of the biosystem, the acquired values were then implemented into a feedback system that reflected the desired feedback where a PID control system would be implemented to account for volumetric changes of residual limb, pressure on the force sensing resistors along the socket, and subsequent volumetric changes in socket air bladder.

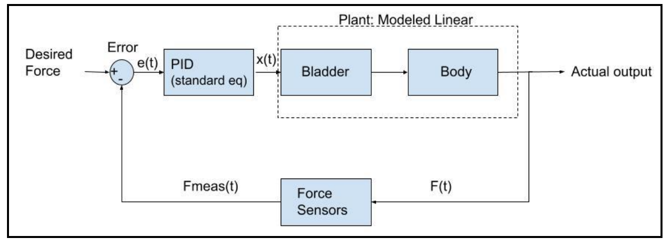

Figure 8: The Desired Feedback

In Simulink, the PID controller was modeled as a PI controller because it was a simple way to get a robust and fast response to a closed loop control while also eliminating any long term error in the system.

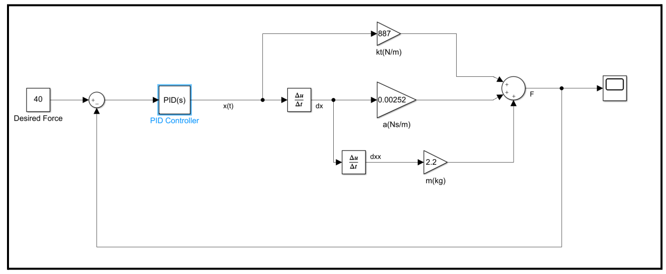

Figure 9: Simulink Block Diagram

The values used in the model were the desired force, 40 kPa, the k_t, 9235N/m, a, 0.00109 Ns/m and the mass of the modeled prosthetic leg, 2.2 kg.

VI. RESULTS

Using the PID tuner built into Simulink, it was determined that a 1:1 ratio between P and I consistently yielded the most promising results with a steady rise time of 4 seconds to reach the desired force of 40 kPa after which it swiftly leveled out as seen in Figure 9.

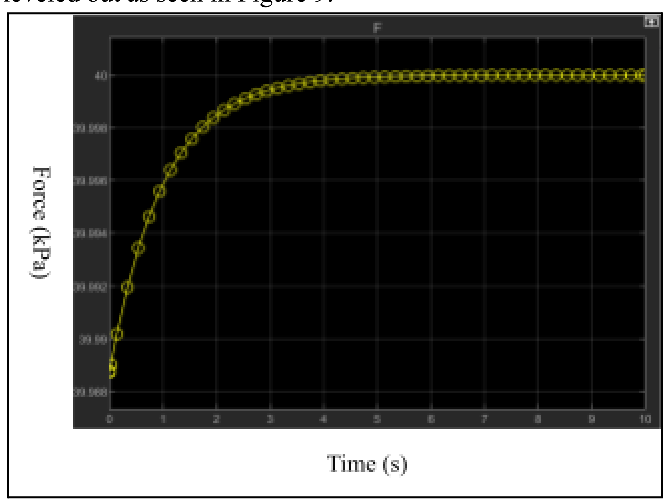

Figure 10: The Simulink output of the system's force response

VII. CONCLUSION

This model was simple, utilized a real skin response, was easy to tune, had a fast response and was very robust with quite promising results. However there are several improvements that can be made. In this model the applied force would be constant, however in real world applications this would not be true due to the changing volume and activity a prosthetic socket faces. Additionally, the force sensors were modeled as ideal sensors with no error between the actual force and measured force. However, this ideal condition is unlikely in actuality and would result in an error which can be modeled as a time delay:

$$\frac{dF_{meas}}{dt} = \frac{1}{\tau_{meas}}\left(F(t) - F_{meas}(t)\right) \tag{10}$$

$$\mathcal{L}\left(\frac{dF_{meas}}{dt}\right) = sF_{meas} = \frac{1}{\tau_{meas}}\left(F(s) - F_{meas}(s)\right) \tag{11}$$

$$\frac{F_{meas}(s)}{F(s)} = \frac{1}{\tau_{meas}s+1} \tag{12}$$

In the model the air bladder was acting in unison and changing in distance as dictated by the PID controller and outputting a force. The air bladder and residual limb would be two separate models acting upon each other rather than in unison. To model this, they have to be modeled separately. A true damping coefficient via experimentation of the air bladder would also be required to get a more accurate model. Another disadvantage to the current model is that the air bladder is not modeled as an air bladder at all, but rather a piston. To make a more accurate model the model should be revised to utilize the ideal gas law to accurately represent the air bladder as a compressible entity where the force applied, and therefore the change in distance, is dictated by the volume of air input into the bladder by air pumps.

This would model more accurately the real outcome to the system for eventual implementation into a prosthetic socket.

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