Mathematical Model of Sexual Response

Kyubin Jang, Calysta McKinney,
Kylie Nicholson, Lorelai Schoch,
Felix Tajanko

The Human Sexual Response

Several theories currently exist about the cyclical process of the phases of sexual response.

- In some of these models, comparisons are made with a cusp catastrophe.

- **The Masters-Johnson Model**
  - Made up of four stages: excitement, plateau, climax, & resolution.
- **The Kaplan-Model**
  - Contrasting with Masters & Johnson, Kaplan simplifies the model to three phases.
    - The three phases are: desire, excitement, and climax.

All of these models define two inputs that increase arousal – physiological and psychological.

We assume an individual is receiving external stimuli with both physiological and psychological components. This causes the phases of sexual response to occur.

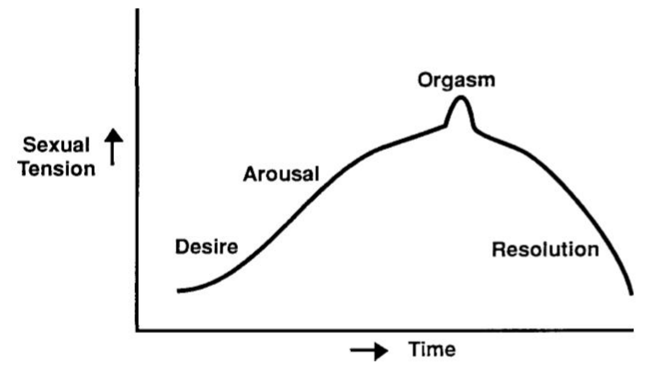


FIGURE 6. Masters/Johnson/Kaplan model of sex response.

The male sexual response cycle

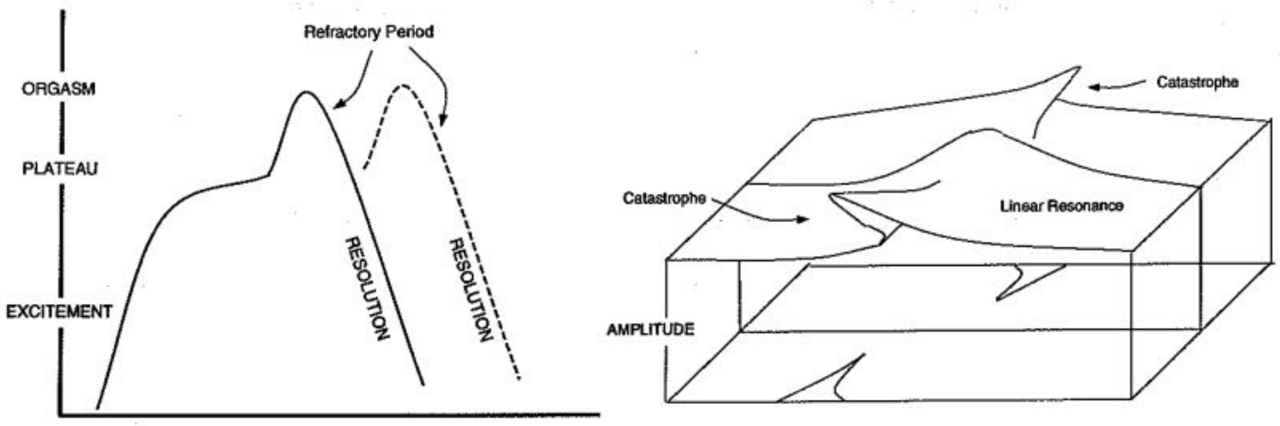


Figure I

Figure II

Figure I describes the male response based on the Masters and Johnson model.

Figure II depicts the cusp catastrophe model, which has the same cusp model but is not dependent on time. A catastrophe is determined to be an orgasm, which leads to resolution.

This may not be accurate for all orgasm cycles, so the equations from the cusp model are tweaked by Blyuss and Kyrychko

Hubey, H. M. "Dynamics and Geometry of the Human Sexual Response ." Dynamics-Hum REsponse, Montclair State University, 1997, www.montclair.edu/profilepages/media/344/user/dynamics-HumSexREsponse.pdf.

Ordinary Differential Equations

PHYSIOLOGICAL COMPONENT

$$\dot{u} = f(u) - v + E_u,$$

PSYCHOLOGICAL COMPONENT

$$\dot{v} = \epsilon\left[(E_v - E_{v_0}) + au - bv\right]$$

- Using these two equations, derived from the Masters and Johnson model we can derive a mathematical model of the system
- They relate to each other with inputs E_u and E_v, physiological and psychological stimulus respectively.

Blyuss, K. B., and Y. N. Kyrychko. "Sex, Ducks, and Rock 'n' Roll: Mathematical Model of Sexual Response." AIP Publishing, AIP Publishing, 3 Apr. 2023, pubs.aip.org/aip/cha/article/33/4/043106/2882751/Sex-ducks-and-rock-n-roll-Mathematical-model-of.

Components of the Equation

PHYSIOLOGICAL COMPONENT

$$\dot{u} = f(u) - v + E_u,$$

f(u) - N-shaped basal function

v - Psychological arousal

E_u - External physiological stimulus

PSYCHOLOGICAL COMPONENT

$$\dot{v} = \epsilon\left[(E_v - E_{v_0}) + au - bv\right]$$

ϵ - Physiological to psychological scaling factor

E_v - External psychological stimulus

E_{v0} - Baseline psychological arousal

a - physiological arousal gain

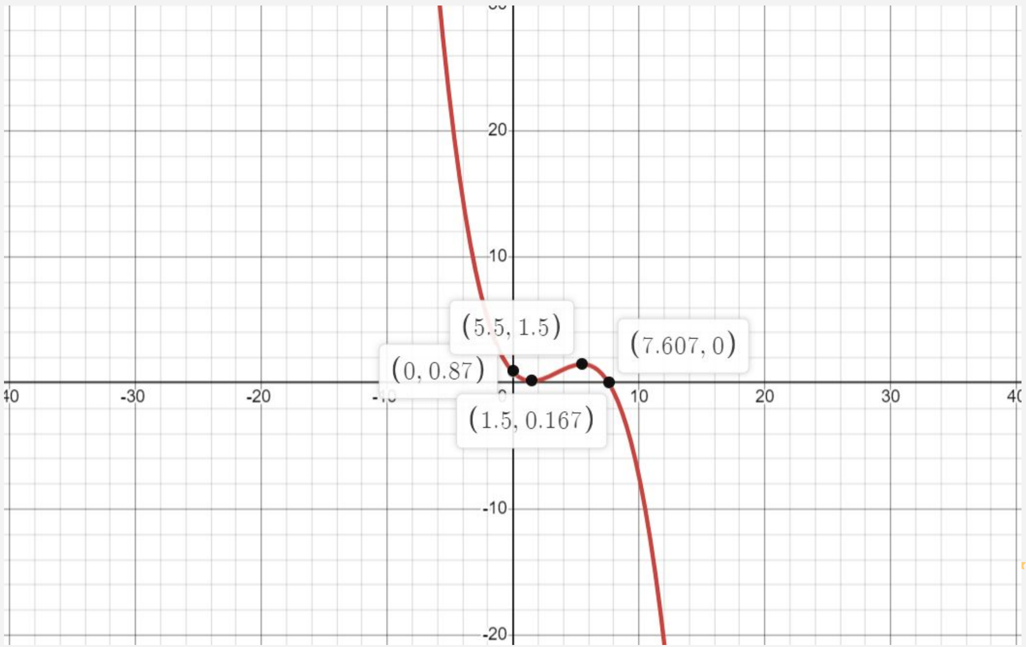
u - physiological arousal

b - homeostasis gain

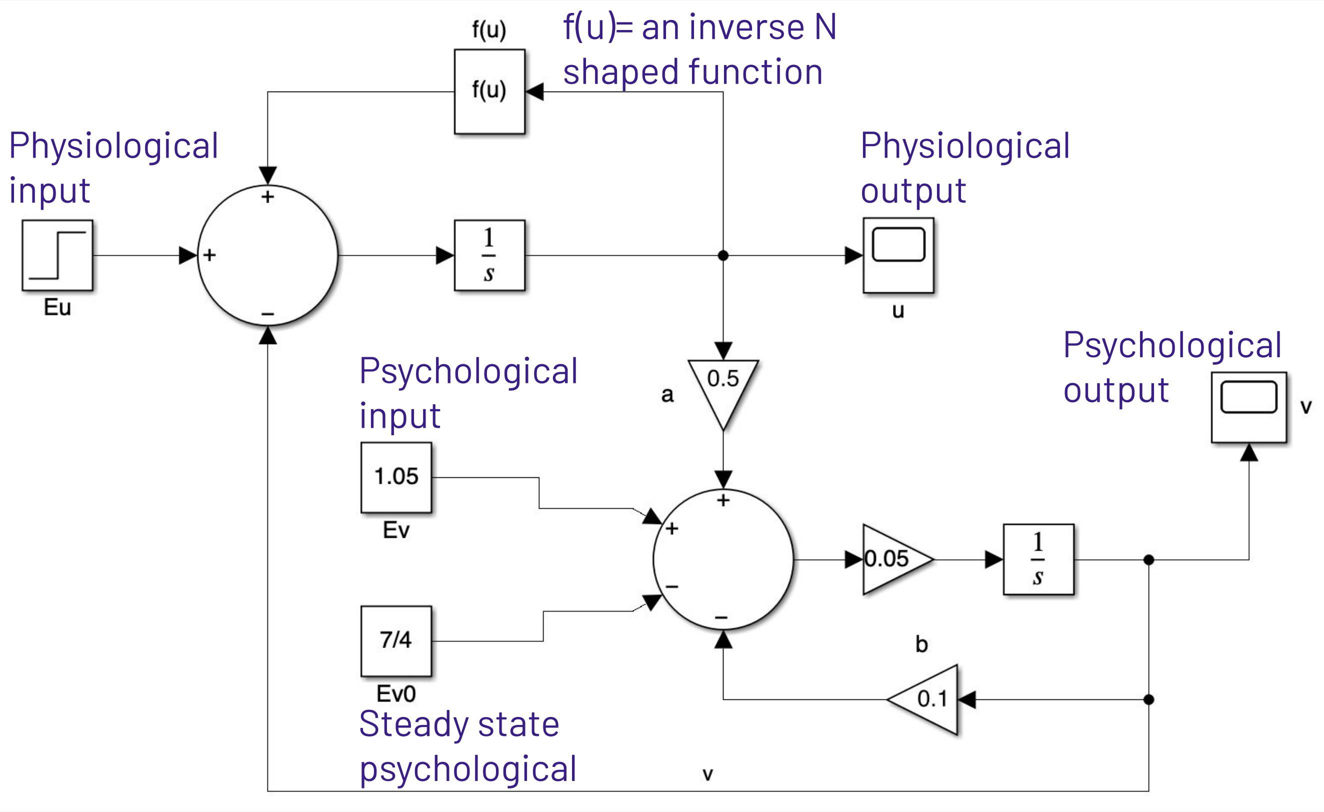
v - psychological arousal

Control Signal F(u)

- F(u) is an inverse N-shaped function as used in the paper by Blyuss & Kyrychko.
- This is used as a framework for the feedback loop to build upon.
- Used in the case of a typical functioning sexual response system. This can be replaced by a different function in the case of biological dysfunctions.



Block Diagram



This system involves the reproductive and nervous systems and physical input from an external force

The coefficients from Blyuss & Kyrychko's research are assumptions about the values of input for the sexual response to occur

Performance Goals

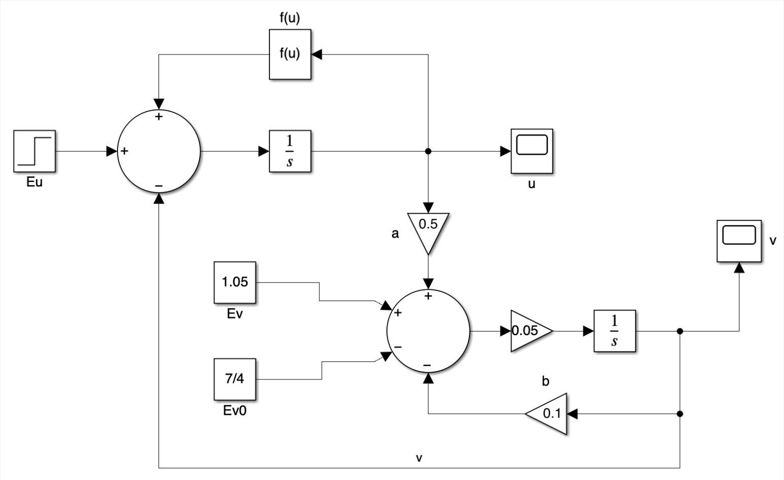
Describe the performance goals and any realistic operational constraints.

- Performance Goals: We aim to accurately model the human sexual response cycle based on the Masters-Johnson 4-phase model describing the excitation, plateau, orgasm, and resolution periods experienced during one sexual cycle in terms of the relationship between physiological and psychological input functions.

- Operational Constraints: The function we are trying to model is extremely complicated for both male and female responses and can have many confounding variables. Due to the relationship between both the psychological and physiological variables, the system is very complex and thus we have modeled the response as one system to the best of our ability.

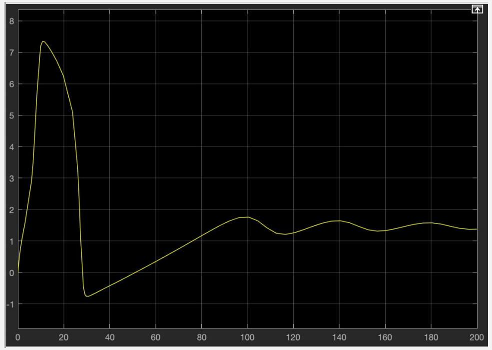
SIMULINK Model of the System

– We are assuming that other bodily systems are operating effectively to not interact with the physiological or psychological variables.
– External physical input for physiological component, constants for psychological

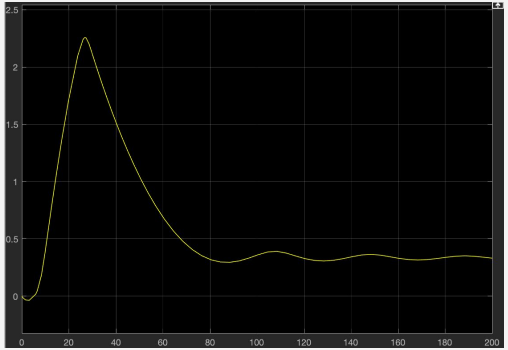


SIMULINK Model

$$f_1(u) = \frac{167}{192} - \frac{33}{32}u + \frac{7}{16}u^2 - \frac{1}{24}u^3,$$



Physiological Arousal (u)



Psychological Arousal (v)

Linearized ODEs

Steady state values:

Physiological:

$$\bar{u}_1 = 1.46698$$
$$\bar{u}_2 = 4.51232 + 10.53586i$$
$$\bar{u}_3 = 4.51232 - 10.53586i$$

External physiological stimulus:

$$\bar{E}_u = {}^{1}/_{6}$$

Psychological with the real $\bar{u}$:

$$\bar{v} = 0.333607$$

Using these values, we can write our linearized equations:

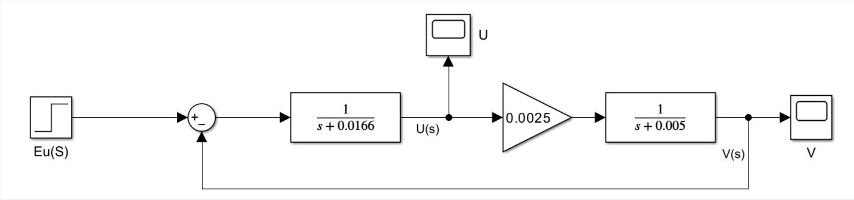
$$\frac{d\breve{u}}{dt} = \left(-\frac{33}{32} + \frac{7}{8}\bar{u} - \frac{1}{8}\bar{u}^2\right)\breve{u} - \breve{v} + \breve{E}_u$$
$$= (-0.0166)\breve{u} - \breve{v} + \breve{E}_u$$
$$\frac{d\breve{v}}{dt} = (\epsilon a)\breve{u} - (\epsilon b)\breve{v}$$

Laplace Transform

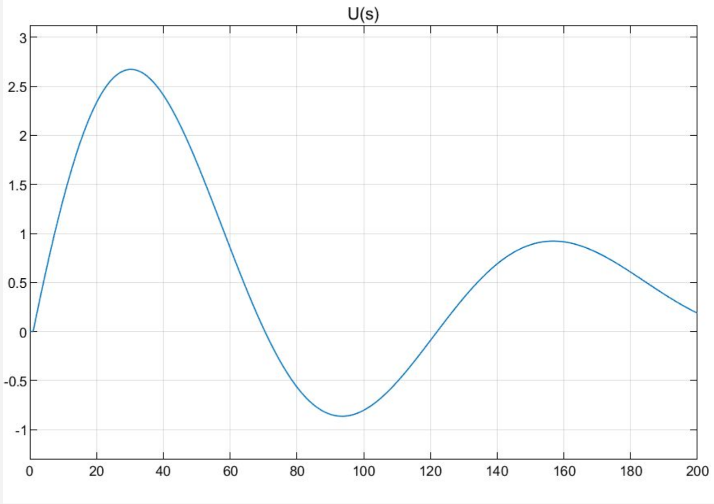
- Take the Laplace transform of all the elements and recast the block diagram in transfer function notation.

$$\frac{U(s)}{E_u(s)} = \frac{s + 0.005}{s^2 + 0.0216s + 0.0251}$$

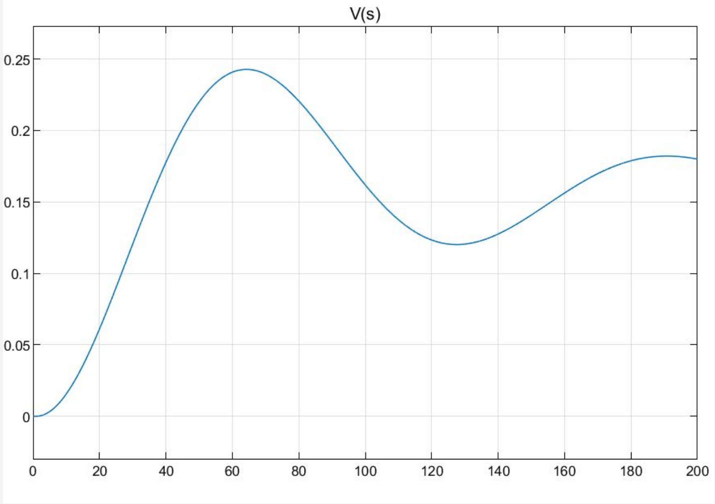
$$\frac{V(s)}{E_u(s)} = \frac{0.0025}{s^2 + 0.0216s + 0.0251}$$



Physiological:

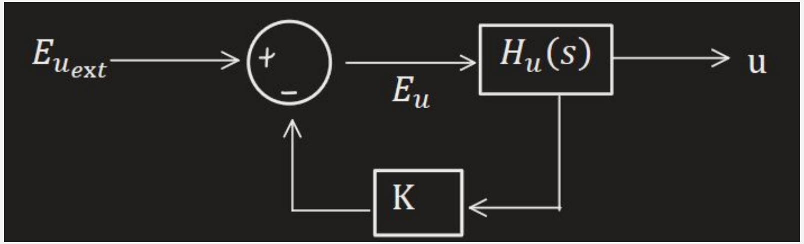


Psychological:



Overall Transfer Function

Physiological:



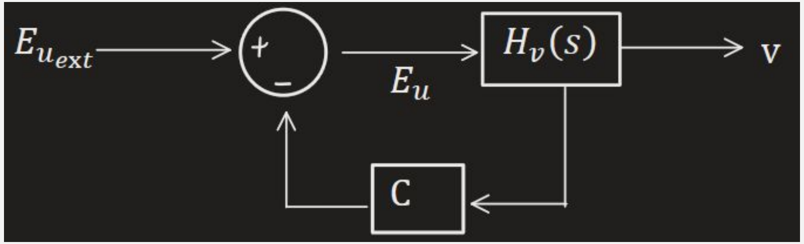
Closed-loop feedback: $E_u(t) = E_{u_{ext}}(t) - \mathrm{K}u(t)$

Where $E_{u_{ext}}(t)$ is the physiological applied external stimulus and K is feedback gain

Closed loop transfer function: $H_u(s) = \frac{u(s)}{E_{u_{ext}}(s)}$

$$\frac{U(s)}{E_{u_{ext}}(s)} = \frac{H_u(s)}{1 + H_u(s)K}$$

Psychological:



Closed-loop feedback: $E_u(t) = E_{u_{ext}}(t) - \mathrm{C}v(t)$

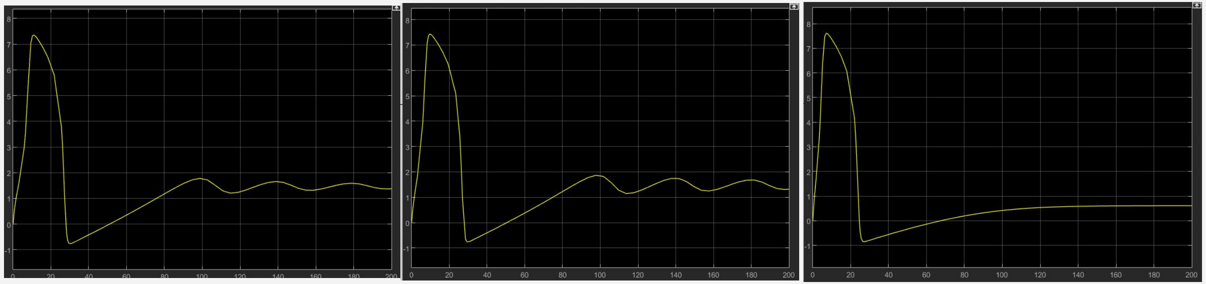
Where $E_{u_{ext}}(t)$ is the physiological applied external stimulus and C is feedback gain

Closed loop transfer function: $H_v(s) = \frac{v(s)}{E_{u_{ext}}(s)}$

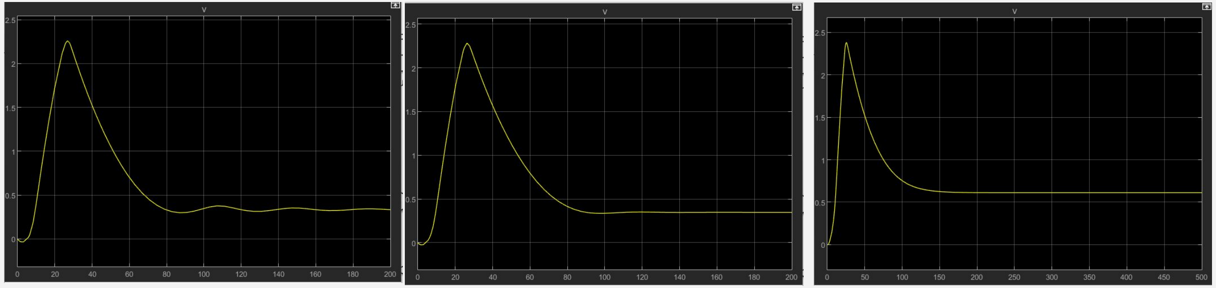
$$\frac{V(s)}{E_{u_{ext}}(s)} = \frac{H_v(s)}{1 + H_v(s)C}$$

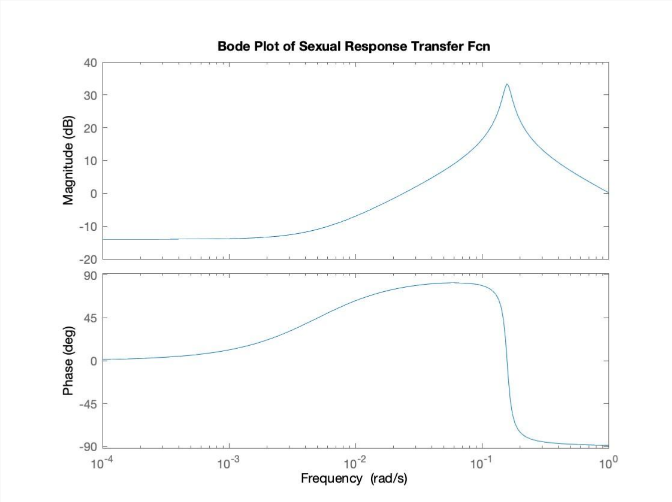
Sensitivity Analysis

- Changing the physical stimulation value E_u by small perturbations (0.01, 0.1, 0.5) caused the following changes in physical response u:



- Changing the psychological stimulation value E_v by small perturbations (0.01, 0.1, 0.5) caused the following changes in psychological response v:

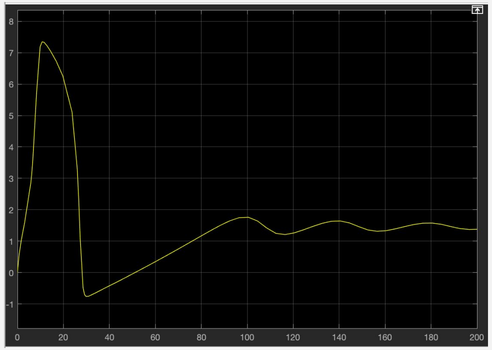




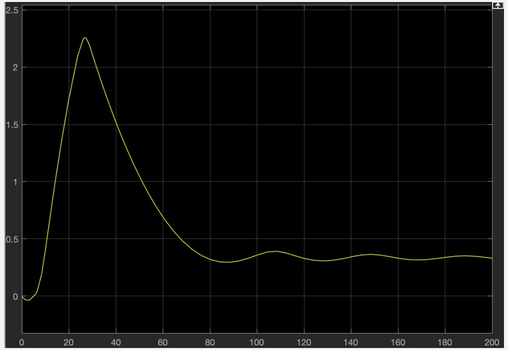
- Max. amplitude at 0.161/s
- One zero (+90° phase) and two poles (-180°) visualized in the phase diagram

Errors

- Steady state errors
  - Increases proportionally with gain. Visible in the system since the steady state value of u and v do not settle at 0 despite it being the normal value.
- Linearization errors
  - Becomes more significant as the input values deviate from the baseline values at which the system was linearized
- Parametric functions and constants in the system
  - The system is a general model that does not account for variances between individuals – errors for clinical use

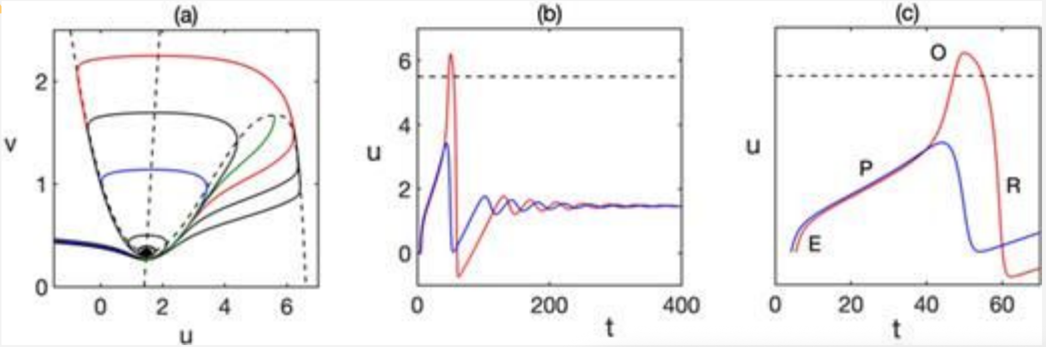


Physiological Arousal



Psychological Arousal

Physiological Observations



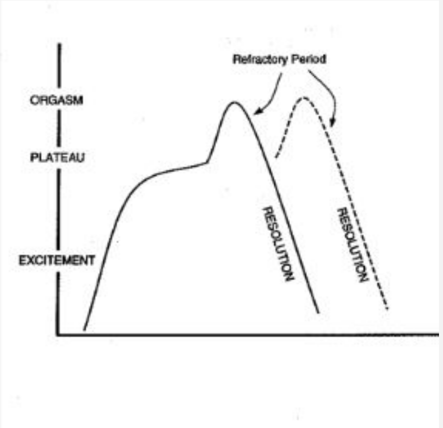
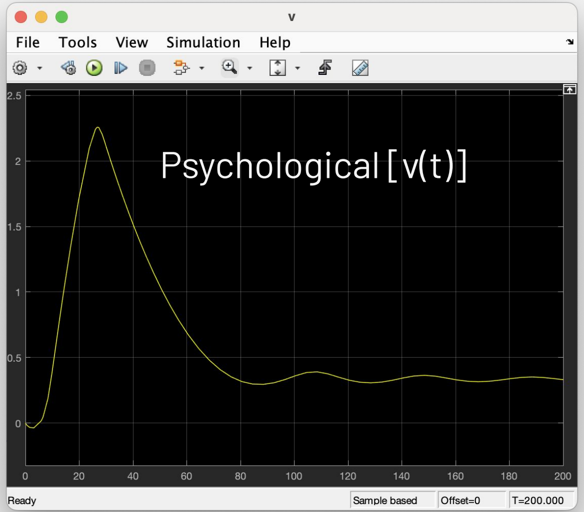
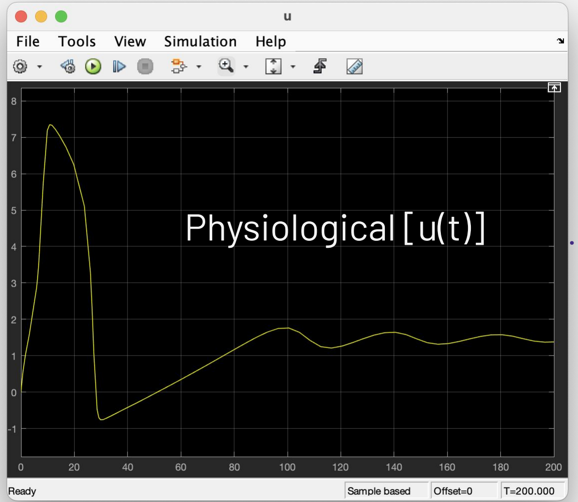


Figure I

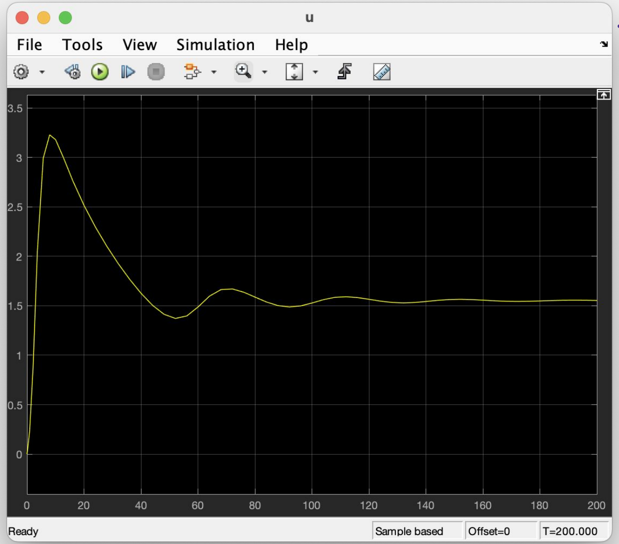




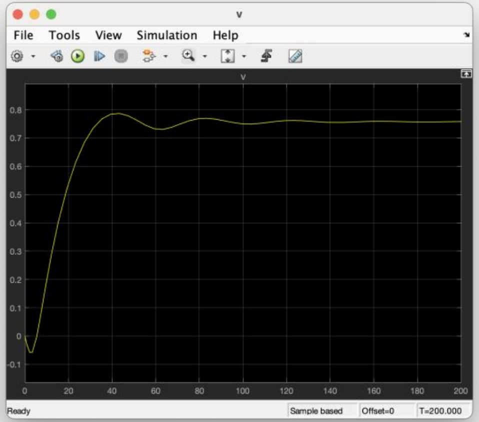
- Matches projected u(t)
- u(t) follows physiological pattern (Master's/Johnson model)
- v(t) makes sense conceptually, with psychological arousal increasing with physical arousal, and still outlasting though remission
- Decreases as a negative exponential

Clinical Syndrome

- A clinical syndrome that would cause for a modified version of our system would be anorgasma.
- An individual may have desire for sex or an orgasm, and yet physiologically their body cannot orgasm, or not easily.
- This means that while the psychological system is working, the physiological system is incapable of breaching the level necessary for climax.
- Mathematically this would be represented as the physiological arousal reaching a horizontal asymptote.
- Compared to the standard response, the physiological arousal doesn't reach the magnitude needed for climax and thus doesn't display the resolution behavior associated with it.



f(u)=(0.69*sin(u))*exp((-0.1*u))



Simulation as an Alternative

Advantages

- In a perfect system it would model the Master's / Johnson cycle fairly well.
- This model could be used to reproduce the arousal cycle without using a patient in a potentially embarrassing way.
- This model can also be used to test the effects of various sexual dysfunctions without needing to seek out patients with potentially rare disorders.

Limitations

- This model cannot be representative of individuals who have atypical psychological arousal behavior
- Psychological arousal can depend of a number of factors outside of physiological arousal.
- There are assumptions made about the coefficients within the system, and therefore a model will never be as accurate as a physiological experiment.

Augmenting the Model for Women

- As this model is based in the mechanisms of the male sexual response cycle – it may be beneficial to further develop this model for characterizing the female response cycle.
- The female response cycle has received comparatively little attention in the literature due to the historic academic landscape.
- The female response cycle is notably more complex than that of male's due to a greatly diminished refractory period, allowing for multiple climaxes in one response cycle.
- Similarly to the male response, the female response is a mixture of physiological and psychological stimulations combined to complete the response cycle

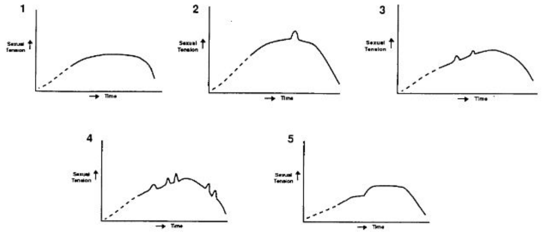


FIGURE 7. Common curves depicting women's sexual arousal ± orgasmic release.

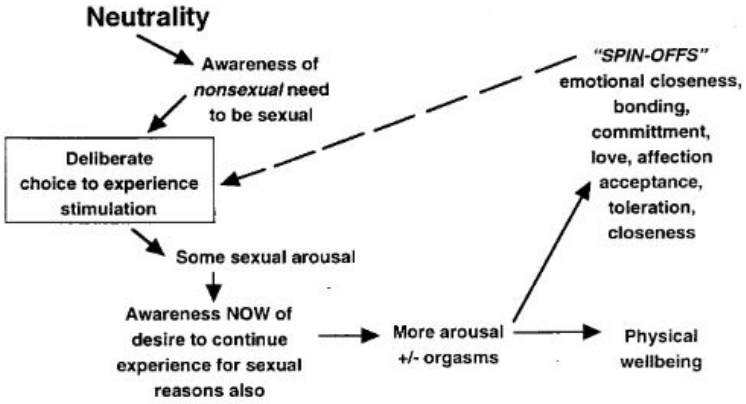


FIGURE 1. A women's sex response cycle.

Basson, Rosemary. "The Female Sexual Response: A Different Mode." Taylor and Francis Online, 2000, www.tandfonline.com/doi/epdf/10.1080/009262300278641.

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- https://www.montclair.edu/profilepages/media/344/user/dynamics-HumSexREsponse.pdf

Thanks!

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