Mathematical Model of Sexual Response

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***Abstract* — For much of modern medicine, the stages the body endures during sexual arousal, stimulation, and climax have not been quantified. Works by Masters & Johnson, Kaplan, and finally Blyuss & Kyrychko have progressed towards capturing this phenomena in more accurate and predictable manners. Recently, the sexual response cycle has been modeled in terms of ordinary differential equations representing a control system. Based on the work "Sex, ducks, and rock n' roll: Mathematical model of sexual response" by Blyuss & Kyrychko, we have modeled these equations and show the levels of physiological and psychological excitation over time throughout one cycle.**

***Clinical Relevance* — Sexual response and its mathematical interpretations continue to be an understudied topic. Applications of this research could potentially include identification of sexual disorders based on observed deviations from typical progression through the sexual response cycle.**

I. INTRODUCTION

A. Sexual Model

The sexual response system is an excitable response to external stimuli. There are several existing theories about the cyclical process of the phases of sexual response. In some cases, the model can be explained by comparison with the cusp catastrophe. In order to create a control model, there needed to be relations of variable inputs within the system, controlled by feedback loops and other controllers. In a sexual experience, there are two inputs to increase arousal, physiological and psychological. These influences are needed in order for an individual to be able to complete the response system by reaching climax. In this demonstration, an individual is receiving external stimuli with both physiological and psychological components. This causes the phases of sexual response to occur.

B. Control Model

This control model is built in order to design a system to predict the stages of sexual response in reaction to input variables or external stimuli. The goal is to be able to model the system appropriately and measure outputs.

II. RESEARCH

A. Masters-Johnson Model

In 1966, Masters & Johnson published "Human Sexual Response" [1] which was one of the first comprehensive models of the human sexual response. Their model consists of four stages, which together make up the Masters-Johnson sexual response cycle. The first phase is the excitement phase, which is marked by physical and psychological changes such as increased heart rate, muscle tension, and heightened awareness of sexual stimuli. The plateau phase is then a continuation and intensification of the physiological changes initiated in the excitement phase. During this phase, sexual arousal reaches a peak, further increasing the muscle tension, heart rate, and other bodily responses. This phase serves to prepare the body for climax. The orgasmic phase is the climax of the sexual response cycle coinciding in a release of sexual tension. Physiological responses occur, such as rhythmic contractions of the pelvic muscles, and in the case of males, ejaculation. The resolution phase serves to gradually return the body to its pre-arousal, or baseline, state. The physiological responses associated with this phase are decreases in heart rate and muscle relaxation. This phase may also serve as a refractory period during which the body is less responsive, or even rejective of further sexual stimulation. While the Masters-Johnson model did not incorporate more advanced statistical and mathematical analyses, it did serve as a foundation for which further research could build off of with its observations determined from empirical data.

B. Kaplan Model

In Kaplan's book, "The New Sex Therapy: Active Treatment of Sexual Dysfunctions" [2] published in 1974, she introduces the concept of the "sexual triad" – contrasting with the four phases set forth by Masters & Johnson. Kaplan's model differed from Masters & Johnson's in its emphasis on the role of sexual desire as a distinct and important component of the sexual experience. She also highlighted the interplay between psychological and physiological factors in shaping sexual response and dysfunction. The 3 interconnected phases that Kaplan proposed include the desire phase, the excitement phase, and the orgasm phase. In the desire phase, Kaplan emphasized the importance of sexual desire or libido as a distinct aspect of the sexual response – with desire being influenced by psychological, emotional, and interpersonal factors. Similar to the Masters-Johnson model, Kaplan included an excitement phase which acknowledged the physiological arousal that occurs in response to sexual stimuli. Contrary to Masters & Johnson, Kaplan considered orgasm as the culmination of the sexual response cycle, marked by intense pleasure and release of sexual tension.

C. Blyuss & Kyrychko

In their paper "Sex, ducks, and rock "n" roll: Mathematical model of sexual response" [3], Blyuss & Kyrychko derive a phenomenological mathematical model of a sexual response that contains variable representation of levels of physiological and psychological arousal. This model was aimed at reproducing and explaining salient features of the Masters–Johnson model in human males.

D. The Cusp Catastrophe

A sexual response cycle is a periodic trajectory, and there is a cusp catastrophe in the model due to the presence of sudden jumps between upper and lower branches on the amplitude response curve. A cusp catastrophe is a model that can be used to describe the relationship between physiological arousal and performance. In this model, performance is highest in a middle range of arousal where neither boredom nor anxiety are being experienced due to too high or low levels of psychological arousal [4]. However, this is not a completely accurate model. Upon further research in the Blyuss & Kyrychko paper, there is no cusp catastrophe associated with the amplitude of the response curve, because while a transition between branches occur, there are oscillations with only one value of the amplitude, and there is no change in the number of steady states of the model.

III. METHODS

A. Control Model Design

The paper by Blyuss & Kyrychko models the sexual response of human males with the system of equations below:

$$\dot{u} = f(u) - v + E_u, \tag{1}$$

Where **u** is physiological arousal, **f(u)** is an inverse-N shaped function of **u**, $\mathbf{E_u}$ is the physical stimulation and the **-v** represents the decrease of physiological arousal as psychological arousal increases. This is based on the cusp-catastrophe-like phenomena that occurs when physiological arousal decreases if psychological arousal is at a too high or too low level that could cause boredom or anxiety, which interfere with the success of a response cycle.

The function **f(u)** serves as a framework around which the system is built. The function is used to model a typical (healthy) sexual response – therefore, the function **f(u)** can be modified or replaced to model sexual disorders.

The variable **-v** may seem counter-intuitive, because it appears to suggest that the growth of physiological arousal decreases with increasing psychological arousal. This is justified by the findings that reaching an orgasm can be characterized as psychologically "letting go," [5] so we include the negative sign which decreases the overall physiological arousal as psychological arousal increases. A negative sign is also included to limit mental arousal, which can have lows we would associate with boredom/apathy and high levels which could result in anxiety similar to the cusp catastrophe model.

$$\dot{v} = \epsilon\left[(E_v - E_{v_0}) + au - bv\right] \tag{2}$$

where $\mathbf{E_{v0}}$ is a baseline level of psychological arousal, $\mathbf{E_v}$ is external psychological stimulation, also physiological arousal is assumed to increase psychological arousal, as represented by the term au where the coefficient a is a constant of proportionality that relates the rate of **v** and **u** . Justification for the last term (**-bv**) has the effect of returning the body to psychological homeostasis in line with the behavior associated with other pleasures. Finally, whereas physiological responses are rather quick and are controlled by the autonomic nervous system, processing of psychological stimuli is (much) slower; hence, the scaling parameter ϵ .

The variable **a** is a proportionality constant of psychological response due to the influence of the physiological component (in this case, we assume the growth of the physiological response increases by 0.5 times the magnitude of the psychological component).

The variable **b** is the magnification component of physiological response due to influence of psychological component. It must also be sufficiently small relative to the variable a in order to preserve steady states. In this project we used a value of 0.1 for **b**.

IV. RESULTS

A. SIMULINK

The block diagram for simulating the system was constructed as below:

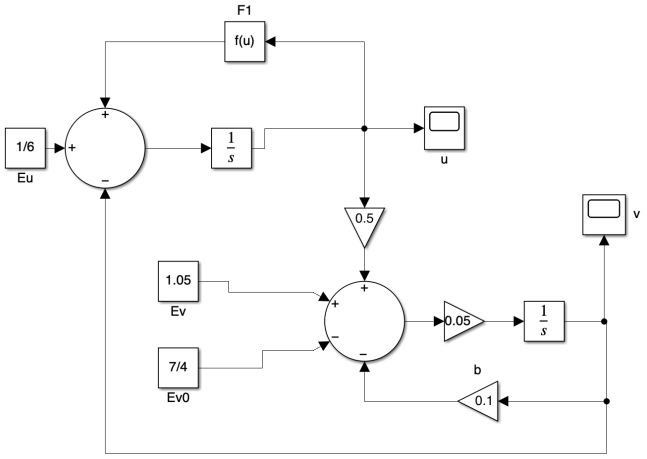

Figure 1. Block diagram of the sexual response system.

Running the model in SIMULINK provides graphs of physiological and psychological arousal, **u(t)** and **v(t)** respectively, as seen below.

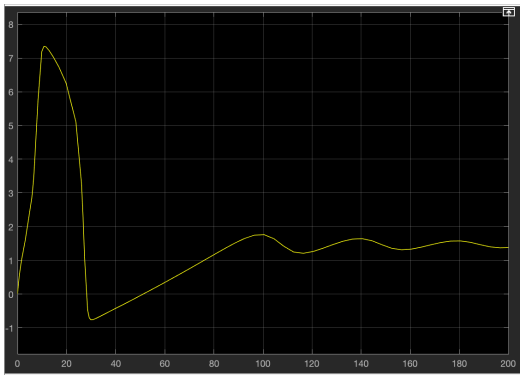

*Figure 2. Physiological Arousal (**u**)*

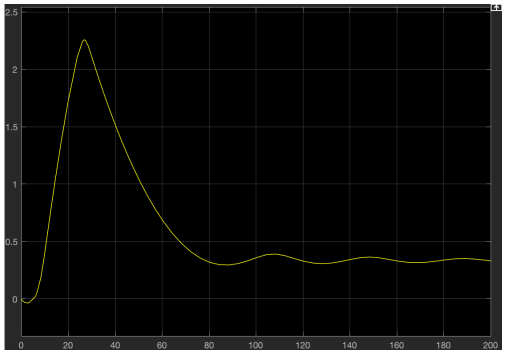

Figure 3. Psychological Arousal ***(v)***

This created model of physiological arousal (Figure 2) matches the u(t) projected by the Masters–Johnson model as seen below in (Figure 4b).

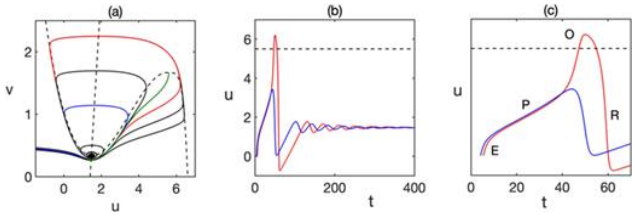

Figure 4. The Projected Model

Also, **v(t)** or psychological arousal makes sense conceptually (Figure 3), with psychological arousal increasing with physical arousal, and still outlasting through remission.

B. Linearization

Physiological steady state values of this system were calculated to be **u** = 1.467, 4.51232 + 10.535i, 4.51232 - 10.535i, but only the real value of **u** = 1.467 was used to compute linearization and to calculate the psychological steady state value of **v** = 0.334. Linearization equations for physiological (3) and psychological (4) arousal are as follows:

$$\frac{du}{dt} = \left(-\frac{33}{32} + \frac{7}{8}\bar{u} - \frac{1}{8}\bar{u}^2\right)u - v + E_u \tag{3a}$$

$$= (-0.0166)u - v + E_u \tag{3b}$$

$$\frac{dv}{dt} = (\epsilon a)u - (\epsilon b)v \tag{4}$$

C. Laplace Transform

The Laplace Transforms for the physiological (5) and psychological (6) arousal are as follows:

$$s \cdot U(s) = -0.0166U(s) - V(s) + E_u(s) \tag{5}$$

$$s \cdot V(s) = \epsilon a U(s) - \epsilon b V(s) \tag{6}$$

D. Overall Transfer Function

The overall transfer functions for the physiological (7) and psychological (8) arousal are as follows:

$$\frac{U(s)}{E_u(s)} = \frac{s + 0.005}{s^2 + 0.0216s + 0.0251} \tag{7}$$

$$\frac{V(s)}{E_u(s)} = \frac{0.0025}{s^2 + 0.0216s + 0.0251} \tag{8}$$

E. Sensitivity Analysis

We performed a sensitivity analysis on the system, which provided these results detailed in the graphs below on the physical response value **u** and physiological response value **v** when altering physical stimulation input $\mathbf{E_u}$ and physiological stimulation $\mathbf{E_v}$ respectively, by small perturbation values of (0.01, 0.1, 0.5). This analysis provides an idea of how small changes in the inputs of the system affect the reaction of the system.

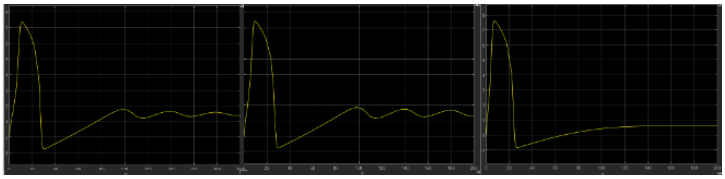

Figure 5. Sensitivity analysis of '$\mathbf{E_u}$*' on '****u****'*

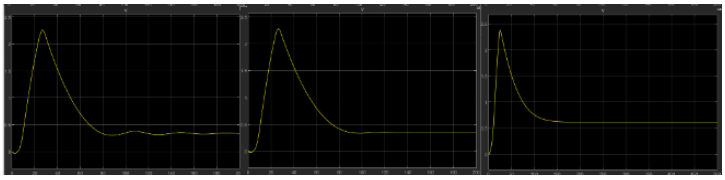

Figure 6. Sensitivity analysis of '$\mathbf{E_v}$*' on '****v****'*

As the value of $\mathbf{E_u}$ increases, the peak of the curve is reached after a shorter duration of time. The oscillating behavior observed in the tail of the function is also reduced, indicating a quicker return to steady state as small perturbations increase. A similar effect is observed in the sensitivity analysis of $\mathbf{E_v}$, as the peak of the curve is reached sooner, but also demonstrates its decay at a faster rate with less oscillations as perturbation increases. This alludes to the observation that as levels of psychological and physiological input increase, the climax of the system is reached more quickly, as well as the consequential return to steady state following the climax.

G. Clinical Syndrome

A clinical syndrome that would cause a modified version of our system would be anorgasma. An individual may have a desire for sex or an orgasm, and yet physiologically their body cannot orgasm, or not easily. This means that while the psychological system is working, the physiological system is incapable of breaching the level necessary for climax. Mathematically this would be represented as the physiological arousal reaching a horizontal asymptote.

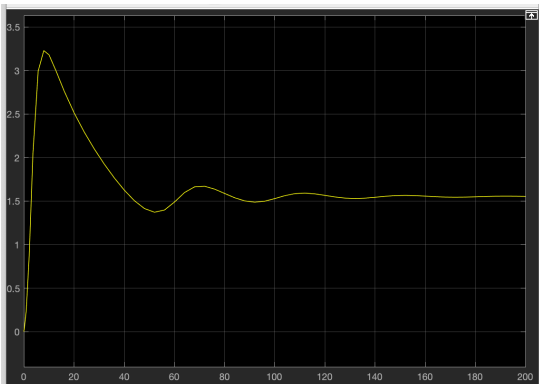

*Figure 7. Physiological Arousal for Anorgasma (**u**)*

Compared to the standard response, the physiological arousal displayed in Figure 7 doesn't reach the magnitude needed for climax and thus doesn't display the resolution behavior associated with it. The equation used to display this response is:

$$f(u) = (0.69*\sin(u))*\exp((-0.1*u)) \quad (9)$$

H. Simulation as an Alternative

Some advantages of this model would be, in a perfect system it would model the Masters–Johnson cycle fairly well. This model could be used to reproduce the arousal cycle without using a patient in a potentially embarrassing way. It can also be used to test the effects of various sexual dysfunctions without needing to seek out patients with potentially rare disorders.

Despite its advantages this model cannot be representative of individuals who have atypical psychological arousal behavior. Psychological arousal can depend on a number of factors outside of physiological arousal. There are assumptions made about the coefficients within the system, and therefore a model will never be as accurate as a physiological experiment.

V. CONCLUSION

A. Future Steps

As this model is based on the mechanisms of the male sexual response cycle – it may be beneficial to further develop this model for characterizing the female response cycle. The female response cycle has received comparatively little attention in the literature due to the historic academic landscape. However, it is notably more complex than that of male's due to a greatly diminished refractory period, allowing for multiple climaxes in one response cycle. Similarly to the male response, the female response is a mixture of physiological and psychological stimulations combined to complete the response cycle.

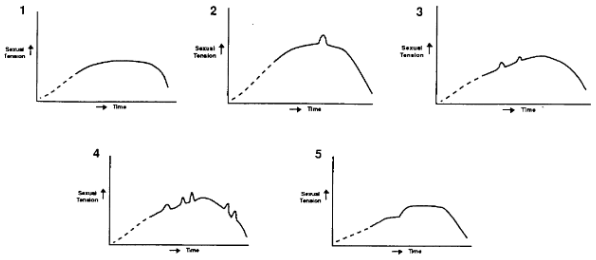

Figure 8. Female sexual response

In future versions of this problem, the next steps would be to develop a model based on this complicated system of female sexual response. Adapting the equations may be challenging but it may be beneficial for gynecological research.

B. Errors

There are a few steady state errors. The system increases proportionally with gain. This is visible in the system since the steady state value of **u** and **v** do not settle at 0 despite it being the baseline value. Upon further analysis of the stability of the system, we have determined that the system is unstable by nature and therefore is not appropriate for a stability analysis. Additionally, the paper our model is based upon by Blyuss & Kyrychko does not provide adequate reasoning behind the selection of the values for the parameters of the equations, so we do not have justification for the selection of most of the equation values.

The system is a general model that does not account for variances between individuals and may have some errors for clinical use.

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