BEGN 122A Fall 2020

Quiz 1

Tuesday, November 3, 2020

***Name** (Last, First):* ______________________________

- This quiz is open book, open notes, and online, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due November 4, 2020 at 11:59pm, over Canvas. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 3 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

1. **[35 pts]** Consider the following biochemical reaction taking place in an organ in the body:

$$\mathrm{A} + \mathrm{B} \underset{k_r}{\overset{k_f}{\rightleftharpoons}} \mathrm{C} + \mathrm{D}$$

where compounds A and B combine to generate compounds C and D at rate k_f, and C and D recombine to regenerate A and B at rate k_r.

(a) [5 pts] Show that if compounds B and C are present at much higher concentrations than either A or D, then you may assume that their concentrations are approximately constant, $[\mathrm{B}] \approx [\mathrm{B}]_0$ and $[\mathrm{C}] \approx [\mathrm{C}]_0$.

(b) [10 pts] Under those assumptions, and further assuming that both A and D exit the volume V of the organ at a flow rate Q, while B and C recirculate in the organ without decay, write the ODEs in the concentrations [A] and [D] that describe both the kinetics and the flow.

(c) [5 pts] Find the equilibrium (*i.e.*, the steady-state) concentrations.

(d) [15 pts] Use Laplace transforms to find the concentration $[\mathrm{A}]$ as a function of time, starting from initial conditions $[\mathrm{A}](0) = [\mathrm{A}]_0$ and $[\mathrm{D}](0) = 0$.

2. **[40 pts]** Consider the following set of ODEs describing the dynamics of a biomechanical system with mass m and damping γ, with force $f(t)$ driving the input, and with position $u(t)$ at the output:

$$\begin{aligned} \frac{du}{dt} &= v(t) \\ m\,\frac{dv}{dt} &= -\gamma\, v(t) + f(t)\,. \end{aligned}$$

(a) [5 pts] Find the Laplace transform of position $u(s)$ as a function of the Laplace transform of the force $f(s)$, and initial conditions in position $u(0) = u_0$ and velocity $v(0) = v_0$.

(b) [10 pts] For zero force $f(t) \equiv 0$, and for given initial conditions $u(0) = u_0$ and $v(0) = v_0$, find the position $u(t)$ as a function of time.

(c) [5 pts] Find the transfer function $H(s) = u(s)/f(s)$ of the system, and find the poles and zeros.

(d) [10 pts] Now consider closed-loop feedback, in which the force $f(t)$ is given by

$$f(t) = f_{ext}(t) - K\,u(t)$$

where $f_{ext}(t)$ is the externally applied force, and K is the feedback gain. Draw the closed-loop system block diagram, and find the closed-loop transfer function $F(s) = u(s)/f_{ext}(s)$.

(e) [10 pts] Find the value of the feedback gain K that minimizes the settling time of the closed-loop system. *Hint*: you may make approximate arguments. Think of settling time in terms of the poles of the system.

3. **[25 pts]** Linear time invariant biosystems:

   (a) [5 pts] Give two possible explanations of biophysical phenomena that cause non-zero pressure across a vessel.

(b) [10 pts] What is the time integral of the Delta-Dirac impulse? And what is the time integral of the impulse response of a linear time-invariant system? Explain the connection between the two.

(c) [10 pts] Show that the linear time-invariant system given by the ODE

$$\frac{du}{dt} = A\,u(t) + f(t) \tag{1}$$

with initial condition $u(0) = u_0$ is equivalent, for all strictly positive times $t > 0$, to a system with the same ODE and zero initial conditions $u(0) = 0$ but with modified force $f(t) + u_0\,\delta(t)$, *i.e.,* by absorbing the initial condition as an initial "kick" in the force. *Hint*: consider left and right limits of $u(t)$ at time $t = 0$.