BENG 122A Fall 2020

Quiz 1

Tuesday, November 3, 2020

Name (Last, First): ________________________________

- This quiz is open book, open notes, and online, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.

- The quiz is due November 4, 2020 at 11:59pm, over Canvas. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.

- There are 3 problems. Points for each problem are given in [brackets]. There are 100 points total.
1. **[35 pts]** Consider the following biochemical reaction taking place in an organ in the body:

\[
A + B \xrightleftharpoons[k_f]{k_r} C + D
\]

where compounds A and B combine to generate compounds C and D at rate \( k_f \), and C and D recombine to regenerate A and B at rate \( k_r \).

(a) **[5 pts]** Show that if compounds B and C are present at much higher concentrations than either A or D, then you may assume that their concentrations are approximately constant, \( [B] \approx [B]_0 \) and \( [C] \approx [C]_0 \).
(b) [10 pts] Under those assumptions, and further assuming that both A and D exit the volume $V$ of the organ at a flow rate $Q$, while B and C recirculate in the organ without decay, write the ODEs in the concentrations $[A]$ and $[D]$ that describe both the kinetics and the flow.
(c) [5 pts] Find the equilibrium \((i.e.,\) the steady-state) concentrations.
(d) [15 pts] Use Laplace transforms to find the concentration $[A]$ as a function of time, starting from initial conditions $[A](0) = [A]_0$ and $[D](0) = 0$. 
2. **[40 pts]** Consider the following set of ODEs describing the dynamics of a biomechanical system with mass $m$ and damping $\gamma$, with force $f(t)$ driving the input, and with position $u(t)$ at the output:

\[
\frac{du}{dt} = v(t)
\]

\[
m \frac{dv}{dt} = -\gamma v(t) + f(t).
\]

(a) **[5 pts]** Find the Laplace transform of position $u(s)$ as a function of the Laplace transform of the force $f(s)$, and initial conditions in position $u(0) = u_0$ and velocity $v(0) = v_0$. 
(b) [10 pts] For zero force $f(t) = 0$, and for given initial conditions $u(0) = u_0$ and $v(0) = v_0$, find the position $u(t)$ as a function of time.
(c) [5 pts] Find the transfer function \( H(s) = \frac{u(s)}{f(s)} \) of the system, and find the poles and zeros.
(d) [10 pts] Now consider closed-loop feedback, in which the force \( f(t) \) is given by

\[
f(t) = f_{ext}(t) - K u(t)
\]

where \( f_{ext}(t) \) is the externally applied force, and \( K \) is the feedback gain. Draw the closed-loop system block diagram, and find the closed-loop transfer function \( F(s) = u(s)/f_{ext}(s) \).
(e) [10 pts] Find the value of the feedback gain $K$ that minimizes the settling time of the closed-loop system. *Hint:* you may make approximate arguments. Think of settling time in terms of the poles of the system.
3. **[25 pts]** Linear time invariant biosystems:

   (a) **[5 pts]** Give two possible explanations of biophysical phenomena that cause non-zero pressure across a vessel.
(b) [10 pts] What is the time integral of the Delta-Dirac impulse? And what is the time integral of the impulse response of a linear time-invariant system? Explain the connection between the two.
(c) [10 pts] Show that the linear time-invariant system given by the ODE

\[ \frac{du}{dt} = A u(t) + f(t) \]  

(1)

with initial condition \( u(0) = u_0 \) is equivalent, for all strictly positive times \( t > 0 \), to a system with the same ODE and zero initial conditions \( u(0) = 0 \) but with modified force \( f(t) + u_0 \delta(t) \), i.e., by absorbing the initial condition as an initial “kick” in the force. \textit{Hint:} consider left and right limits of \( u(t) \) at time \( t = 0 \).