

# BENG 122A Fall 2021

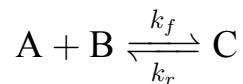
## Quiz 1

Tuesday, October 26, 2021

*Name (Last, First):* \_\_\_\_\_

- This quiz is open book, open notes, and online, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due October 27, 2021 at 11:59pm, over Canvas. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 3 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

1. [35 pts] Consider the following biochemical reaction taking place in an organ in the body:



where compounds A and B combine to generate compound C at rate  $k_f$ , and C decomposes to regenerate A and B at rate  $k_r$ . Compounds B and C are present at much higher concentrations than A, so we assume that their concentrations remain approximately constant,  $[B] \approx [B]_0$  and  $[C] \approx [C]_0$ . Compound A exits the volume  $V$  of the organ at a flow rate  $Q$ , while B and C recirculate in the organ without decay.

(a) [10 pts] Under these approximating assumptions, write the ODE in the concentration  $[A]$  that describes both the reaction kinetics and the flow.

(b) [5 pts] Find the equilibrium (*i.e.*, the steady-state) concentrations.

(c) [15 pts] Use Laplace transforms to find the concentration  $[A]$  as a function of time, starting from initial conditions  $[A](0) = [A]_0$ .

(d) [5 pts] Is the above approximating assumption that  $[B] \approx [B]_0$  and  $[C] \approx [C]_0$  reasonable under these conditions? Explain (in words) to what extent you expect your answers in (b) and (c) to change accounting for the reaction kinetics in compounds B and C.

2. [45 pts] Consider the following set of ODEs describing the dynamics of a biomechanical system with mass  $m$ , stiffness  $k$ , and damping  $\gamma$ , with force  $f(t)$  driving the input, and with position  $u(t)$  at the output:

$$\begin{aligned}\frac{du}{dt} &= v(t) \\ m \frac{dv}{dt} &= -\gamma v(t) - k u(t) + f(t).\end{aligned}$$

(a) [5 pts] Find the Laplace transform of position  $u(s)$  as a function of the Laplace transform of the force  $f(s)$ , for zero initial conditions in position  $u(0) = 0$  and velocity  $v(0) = 0$ .

(b) [15 pts] Find the step response of the system in two extreme cases: *i*) zero damping  $\gamma = 0$ ; and *ii*) zero stiffness  $k = 0$ . The step response is the position  $u(t)$  for a unit step in the force,  $f(t) = 1$  for  $t > 0$ , from zero initial conditions. Explain the differences you observe in the two cases.

(c) [10 pts] Find the transfer function  $H(s) = u(s)/f(s)$  of the system, and find the poles and zeros in the two extreme cases *i*) zero damping  $\gamma = 0$  and *ii*) zero stiffness  $k = 0$ . Compare the stability of the system in the two cases.

(d) [10 pts] Now consider closed-loop feedback, in which the force  $f(t)$  is given by

$$f(t) = f_{ext}(t) - K u(t)$$

where  $f_{ext}(t)$  is the externally applied force, and  $K$  is the feedback gain. Draw the closed-loop system block diagram, and find the closed-loop transfer function  $F(s) = u(s)/f_{ext}(s)$ . Show that the feedback gain  $K$  in the closed-loop system is equivalent to extra stiffness  $k$  in the open-loop system.

(e) [5 pts] Find the value of the feedback gain  $K$  for which the closed-loop system is critically damped. What does this mean in terms of the settling time?

3. [20 pts] Linear time invariant and conservative biosystems:

(a) [5 pts] In a series connection of multiple blocks of linear time invariant systems, does it matter in which order the blocks are connected? Explain.

(b) [5 pts] Show that the step response of a linear time invariant system is the time integral of its impulse response. *Hint:* consider the setting of Problem 3 (a) where one of the blocks is an integrator with transfer function  $\frac{1}{s}$ .

(c) [5 pts] To what extent does pressure in a blood vessel depend on altitude?  
Explain.

(d) [5 pts] To what extent does pressure increase or decrease with increasing diameter of the blood vessel? Explain.