BENG 122A Fall 2021

Quiz 1

Tuesday, October 26, 2021

***Name** (Last, First):* ____

- This quiz is open book, open notes, and online, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due October 27, 2021 at 11:59pm, over Canvas. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 3 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

1. **[35 pts]** Consider the following biochemical reaction taking place in an organ in the body:

$$\mathrm{A} + \mathrm{B} \underset{k_r}{\overset{k_f}{\rightleftharpoons}} \mathrm{C}$$

where compounds A and B combine to generate compound C at rate k_f, and C decomposes to regenerate A and B at rate k_r. Compounds B and C are present at much higher concentrations than A, so we assume that their concentrations remain approximately constant, $[\mathrm{B}] \approx [\mathrm{B}]_0$ and $[\mathrm{C}] \approx [\mathrm{C}]_0$. Compound A exits the volume V of the organ at a flow rate Q, while B and C recirculate in the organ without decay.

   (a) [10 pts] Under these approximating assumptions, write the ODE in the concentration $[\mathrm{A}]$ that describes both the reaction kinetics and the flow.

(b) [5 pts] Find the equilibrium (*i.e.*, the steady-state) concentrations.

(c) [15 pts] Use Laplace transforms to find the concentration $[\mathrm{A}]$ as a function of time, starting from initial conditions $[\mathrm{A}](0) = [\mathrm{A}]_0$.

(d) [5 pts] Is the above approximating assumption that $[B] \approx [B]_0$ and $[C] \approx [C]_0$ reasonable under these conditions? Explain (in words) to what extent you expect your answers in (b) and (c) to change accounting for the reaction kinetics in compounds B and C.

2. **[45 pts]** Consider the following set of ODEs describing the dynamics of a biomechanical system with mass m, stiffness k, and damping γ, with force $f(t)$ driving the input, and with position $u(t)$ at the output:

$$\begin{aligned} \frac{du}{dt} &= v(t) \\ m\,\frac{dv}{dt} &= -\gamma\, v(t) - k\, u(t) + f(t)\,. \end{aligned}$$

(a) [5 pts] Find the Laplace transform of position $u(s)$ as a function of the Laplace transform of the force $f(s)$, for zero initial conditions in position $u(0) = 0$ and velocity $v(0) = 0$.

(b) [15 pts] Find the step response of the system in two extreme cases: *i*) zero damping $\gamma = 0$; and *ii*) zero stiffness $k = 0$. The step response is the position $u(t)$ for a unit step in the force, $f(t) = 1$ for $t > 0$, from zero initial conditions. Explain the differences you observe in the two cases.

(c) [10 pts] Find the transfer function $H(s) = u(s)/f(s)$ of the system, and find the poles and zeros in the two extreme cases *i*) zero damping $\gamma = 0$ and *ii*) zero stiffness $k = 0$. Compare the stability of the system in the two cases.

(d) [10 pts] Now consider closed-loop feedback, in which the force $f(t)$ is given by

$$f(t) = f_{ext}(t) - K\,u(t)$$

where $f_{ext}(t)$ is the externally applied force, and K is the feedback gain. Draw the closed-loop system block diagram, and find the closed-loop transfer function $F(s) = u(s)/f_{ext}(s)$. Show that the feedback gain K in the closed-loop system is equivalent to extra stiffness k in the open-loop system.

(e) [5 pts] Find the value of the feedback gain K for which the closed-loop system is critically damped. What does this mean in terms of the settling time?

3. **[20 pts]** Linear time invariant and conservative biosystems:

(a) [5 pts] In a series connection of multiple blocks of linear time invariant systems, does it matter in which order the blocks are connected? Explain.

(b) [5 pts] Show that the step response of a linear time invariant system is the time integral of its impulse response. *Hint*: consider the setting of Problem 3 (a) where one of the blocks is an integrator with transfer function $\frac{1}{s}$.

(c) [5 pts] To what extent does pressure in a blood vessel depend on altitude? Explain.

(d) [5 pts] To what extent does pressure increase or decrease with increasing diameter of the blood vessel? Explain.