BENG 122A Fall 2022

Quiz 1

Tuesday, October 25, 2022

Name *(Last, First):* ____

- This quiz is open book, open notes, and online, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due Thursday October 27, 2022 at 11:59pm, over Canvas. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 3 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

1. **[40 pts]** Consider the following biochemical reaction taking place in an organ in the body:

$$\mathrm{A} \underset{k_r}{\overset{k_f}{\rightleftharpoons}} \mathrm{B}$$

where compound A converts to compound B at a forward reaction rate k_f, and B converts back to A at a reverse reaction rate k_r. Compound A enters the volume V of the organ at an input rate $r_A(t)$, and both compounds A and B exit the volume at a flow rate Q.

   (a) [10 pts] Write the ODEs in the concentrations $[\mathrm{A}](t)$ and $[\mathrm{B}](t)$ that describe both the reaction kinetics in the volume and the flow through the volume. Is the system linear time-invariant? Explain why.

(b) [5 pts] Show that the steady-state concentrations $\overline{[\mathrm{A}]}$ and $\overline{[\mathrm{B}]}$, at zero steady-state input rate $\overline{r_A} = 0$, are zero.

(c) [10 pts] Here and further below, consider that the reverse reaction is much slower than the forward reaction, so that $k_r \approx 0$. We are interested in the dynamics of the concentration of compound B in response to variations in the input rate of compound A. Find the Laplace transfer function $H(s) = \tilde{[\mathrm{B}]}(s) \, / \, \tilde{r_A}(s)$, and find the poles. Show that the system is strictly stable at a positive flow rate $Q > 0$, and critically stable at zero flow $Q = 0$.

(d) [15 pts] Find the concentration $[\mathrm{B}]$ as a function of time, starting from zero initial conditions $[\mathrm{A}](0) = [\mathrm{B}](0) = 0$, where a molar quantity M_A of compound A is released in the volume, all at once, at time zero. Show that the concentration decays towards zero as time goes to infinity for a positive flow rate, but settles to a non-zero value at zero flow. What is that concentration? Explain.

2. **[40 pts]** Consider the following set of ODEs describing the dynamics in the position $u(t)$ and velocity $v(t)$ of a biomechanical system with mass m and damping γ driven by a force $f(t)$:

$$\begin{aligned} \frac{du}{dt} &= v(t) \\ m\,\frac{dv}{dt} &= -\gamma\, v(t) + f(t)\,. \end{aligned}$$

(a) [10 pts] Find the Laplace transform of output position $u(s)$ as a function of the Laplace transform of the input force $f(s)$, and the initial conditions in position $u(0) = u_0$ and velocity $v(0) = v_0$. Find the transfer function, and the poles and zeros.

(b) [10 pts] Here and further below, consider that the damping is negative, $\gamma < 0$. Show that the system is unstable. Could you think of a physical setting of a biosystem with negative damping giving rise to unstable dynamics?

(c) [10 pts] Now consider closed-loop feedback, in which the force $f(t)$ is given by

$$f(t) = f_{ext}(t) - K\,u(t)$$

where $f_{ext}(t)$ is the externally applied force, and K is the feedback gain. Draw the closed-loop system block diagram, and find the closed-loop transfer function $H_u(s) = u(s)/f_{ext}(s)$. What is the effect of the feedback gain K on the stability of the closed-loop system? Explain.

(d) [10 pts] Again consider closed-loop feedback, but now with velocity $v(t)$ as the output of the closed-loop system, with a force $f(t)$ given by

$$f(t) = f_{ext}(t) - C\,v(t)$$

where $f_{ext}(t)$ is the externally applied force, and C is the feedback gain. Draw the closed-loop system block diagram, and find the closed-loop transfer function $H_v(s) = v(s)/f_{ext}(s)$. Find the condition on the feedback gain C to ensure strict stability of the closed-loop system.

3. **[20 pts]** Linear time invariant and conservative biosystems:

(a) [5 pts] Give some examples of aspects of biosystems that are not linear, or that are not time invariant, and the implications of modeling them as linear time invariant systems.

(b) [5 pts] Show that $1/s$ represents the Laplace transform of a step function, as well as the Laplace transfer function of an integrator.

(c) [5 pts] How do blood pressure and blood volume vary under dilation of the blood vessels? Explain.

(d) [5 pts] Explain why blood velocity in the arterioles deep in the brain is comparable to that in the arteries entering the brain, despite their diameter being over a hundred times smaller.