BENG 122A Fall 2023

Quiz 1

Tuesday, October 31, 2023

***Name** (Last, First):* ______________________________

- This quiz is open book, open notes, and online, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due November 1, 2023 at 11:59pm, over Canvas. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 3 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

1. **[40 pts]** Consider the following biochemical reaction taking place in an organ in the body:

$$\mathrm{A} + \mathrm{B} \underset{k_r}{\overset{k_f}{\rightleftharpoons}} \mathrm{C} + \mathrm{D}$$

where compounds A and B combine to generate compounds C and D at rate k_f, and C and D recombine to regenerate A and B at rate k_r. Compound B enters the volume V of the organ at an input rate $r_B(t)$, and compounds B and D exit the organ at a flow rate Q, while A and C are maintained at constant concentrations $[\mathrm{A}]_0$ and $[\mathrm{C}]_0$ inside the organ.

   (a) [10 pts] Under these assumptions, write the ODEs in the concentrations $[\mathrm{B}](t)$ and $[\mathrm{D}](t)$ that describe both the reaction kinetics in the volume and the flow through the volume. Is the system linear time-invariant? Explain why.

(b) [5 pts] Find the steady-state (*i.e.*, the equilibrium) concentrations $\overline{[\mathrm{B}]}$ and $\overline{[\mathrm{D}]}$, at zero steady-state input rate $\overline{r_B} = 0$.

(c) [20 pts] A molar quantity M_B of compound B is released in the volume, all at once, at time zero, *i.e.*, $r_B(t) = M_B\delta(t)$. Use Laplace transforms to find the concentration $[\mathrm{D}](t)$ as a function of time, starting from steady-state initial conditions. Is the dynamics in [D] underdamped, critically damped, or overdamped? Explain.

(d) [5 pts] Now no longer consider that A and C are maintained at constant concentrations, but instead that they recirculate in the organ without decay, from initial concentrations $[\mathrm{A}](0) = [\mathrm{A}]_0$ and $[\mathrm{C}](0) = [\mathrm{C}]_0$. Is the above approximating assumption that $[\mathrm{A}](t) \approx [\mathrm{A}]_0$ and $[\mathrm{C}](t) \approx [\mathrm{C}]_0$ reasonable under these conditions? Explain (in words) to what extent you expect your answers in (a) through (c) to change accounting for the reaction kinetics in compounds A and C.

2. **[40 pts]** Consider the following set of ODEs describing the dynamics in the position $u(t)$ and velocity $v(t)$ of a biomechanical system with mass m, stiffness k, and damping γ driven by a force $f(t)$:

$$\begin{aligned} \frac{du}{dt} &= v(t) \\ m\,\frac{dv}{dt} &= -\gamma\, v(t) - k\, u(t) + f(t)\,. \end{aligned}$$

(a) [10 pts] Find the Laplace transform of output velocity $v(s)$ as a function of the Laplace transform of the input force $f(s)$, and the initial conditions in position $u(0) = u_0$ and velocity $v(0) = v_0$. Find the corresponding transfer function, and the poles and zeros.

(b) [10 pts] Here and further below, consider that the stiffness is negative, $k < 0$. Show that the system is unstable. Could you think of a physical setting of a biosystem with negative stiffness giving rise to unstable dynamics?

(c) [10 pts] Now consider closed-loop feedback, in which the force $f(t)$ is given by

$$f(t) = f_{ext}(t) - K\, v(t)$$

where $f_{ext}(t)$ is the externally applied force, and K is the feedback gain. Draw the closed-loop system block diagram, and find the closed-loop transfer function $H_v(s) = v(s)/f_{ext}(s)$. What is the effect of the feedback gain K on the stability of the closed-loop system? Explain.

(d) [10 pts] Again consider closed-loop feedback, but now with position $u(t)$ as the output of the closed-loop system, with a force $f(t)$ given by

$$f(t) = f_{ext}(t) - C\,u(t)$$

where $f_{ext}(t)$ is the externally applied force, and C is the feedback gain. Draw the closed-loop system block diagram, and find the closed-loop transfer function $H_u(s) = u(s)/f_{ext}(s)$. Find the condition on the feedback gain C to ensure stability of the closed-loop system with critical damping.

3. **[20 pts]** Linear time invariant and conservative biosystems:

   (a) [5 pts] What is the relationship between the Laplace and Fourier transfer functions of a linear time invariant system? Explain.

   (b) [5 pts] Do the imaginary components of the poles of a linear time invariant system affect its stability? Explain.

(c) [5 pts] What determines compliance of an organ? Explain.

(d) [5 pts] Is resistance of blood flow through a vessel inversely proportional to the vessel cross section area? Explain.