BENG 122A Fall 2024

Quiz 1

Tuesday, October 29, 2024

***Name** (Last, First):* ______________________________

- This quiz is open book, open notes, and online, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due October 31, 2024 at 11:59pm, over Canvas. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 3 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

1. **[40 pts]** Consider the following biochemical reaction taking place in an organ in the body:

$$\mathrm{A} + \mathrm{B} \underset{k_r}{\overset{k_f}{\rightleftharpoons}} \mathrm{C}$$

where compounds A and B combine to generate compound C at rate k_f, and C decomposes to regenerate A and B at rate k_r. Compound B is present at much higher concentration than A and C, so we assume that its concentration remains approximately constant, $[\mathrm{B}] \approx [\mathrm{B}]_0$. Compounds A and C exit the volume V of the organ at a flow rate Q, while B recirculates in the organ without decay.

   (a) [10 pts] Under these approximating assumptions, write the ODEs in the concentrations $[\mathrm{A}]$ and $[\mathrm{C}]$ that describe both the reaction kinetics and the flow.

(b) [5 pts] Find the equilibrium (*i.e.*, the steady-state) concentrations.

(c) [25 pts] Use Laplace transforms to find the concentrations $[\mathrm{A}]$ and $[\mathrm{C}]$ as a function of time, starting from initial conditions $[\mathrm{A}](0) = [\mathrm{A}]_0$ and $[\mathrm{C}](0) = [\mathrm{C}]_0$.

2. **[40 pts]** Consider the following set of ODEs describing the dynamics of a biomechanical system with mass m and damping γ, with force $f(t)$ driving the input, and with velocity $v(t)$ at the output:

$$\begin{aligned} \frac{du}{dt} &= v(t) \\ m\,\frac{dv}{dt} &= -\gamma\, v(t) + f(t)\,. \end{aligned}$$

(a) [10 pts] Find the Laplace transform of velocity $v(s)$ as a function of the Laplace transform of the force $f(s)$, and initial value of velocity $v(0) = v_0$. Does it depend on the initial value of position $u(0) = u_0$, and why?

(b) [10 pts] For zero force $f(t) \equiv 0$, and for given initial conditions $u(0) = u_0$ and $v(0) = v_0$, find the velocity $v(t)$ as a function of time. Explain what you find.

(c) [5 pts] Find the transfer function $H(s) = v(s)/f(s)$ of the system, and find the poles and zeros.

(d) [10 pts] Now consider closed-loop feedback, in which the force $f(t)$ is given by

$$f(t) = f_{ext}(t) - K\, v(t)$$

where $f_{ext}(t)$ is the externally applied force, and K is the feedback gain. Draw the closed-loop system block diagram, and find the closed-loop transfer function $F(s) = v(s)/f_{ext}(s)$.

(e) [5 pts] Find the range of values for the feedback gain K in the closed-loop system to stabilize the biomechanical system with negative damping, $\gamma < 0$.

3. **[20 pts]** Linear time invariant and conservative biosystems:

   (a) [5 pts] List various factors that cause blood pressure to vary across the body. Explain.

   (b) [5 pts] What determines the time constant of decay in the concentration of a compound in the blood stream? Explain.

(c) [5 pts] How are the impulse response and the step response of a linear time invariant system related? Explain.

(d) [5 pts] Explain the equivalence between initial conditions in the output of a linear time invariant system with zero input, and impulse activation ("all at once") of its input at time zero with zero initial conditions.