BENG 122A Fall 2025

Quiz 1

Tuesday, October 28, 2025

Name *(Last, First):* ______________________________

- This quiz is open book, open notes, and online, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due October 30, 2025 at 11:59pm, over Canvas. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 3 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

1. **[35 pts]** Consider the following biochemical reaction taking place in an organ in the body:

$$\mathrm{A} \underset{k_r}{\overset{k_f}{\rightleftharpoons}} \mathrm{B} + \mathrm{C}$$

where compound A decomposes to generate compounds B and C at rate k_f, and B and C recombine to regenerate A at rate k_r. Compounds A and B is present at much higher concentration than C, so we assume that their concentrations remain approximately constant, $[\mathrm{A}] \approx [\mathrm{A}]_0$ and $[\mathrm{B}] \approx [\mathrm{B}]_0$. Compounds A and B recirculates in the organ without decay, whereas C exits the volume V of the organ at a flow rate Q.

   (a) [10 pts] Under these approximating assumptions, write the ODE in the concentration $[\mathrm{C}]$ that describes both the reaction kinetics and the flow.

(b) [5 pts] Find the equilibrium (*i.e.*, the steady-state) concentrations.

(c) [15 pts] Use Laplace transforms to find the concentration $[\mathrm{C}]$ as a function of time, starting from initial conditions $[\mathrm{C}](0) = [\mathrm{C}]_0$.

(d) [5 pts] Is the above approximating assumption that $[A] \approx [A]_0$ and $[B] \approx [B]_0$ reasonable under these conditions? Explain (in words) to what extent you expect your answers in (b) and (c) to change accounting for the reaction kinetics in compounds A and B.

2. **[45 pts]** Consider the following set of ODEs describing the dynamics of a biomechanical system with positive mass m and positive stiffness k, with force $f(t)$ driving the input:

$$\begin{aligned} \frac{du}{dt} &= v(t) \\ m\,\frac{dv}{dt} &= -k\,u(t) + f(t)\,. \end{aligned}$$

(a) [10 pts] Find the Laplace transform of position $u(s)$ as a function of the Laplace transform of the force $f(s)$, and initial value of position $u(0) = u_0$ and velocity $v(0) = v_0$.

(b) [10 pts] For zero force $f(t) \equiv 0$, and for initial conditions $u(0) = u_0 \neq 0$ and $v(0) = 0$, find the position $u(t)$ as a function of time. Explain what you find.

(c) [10 pts] Find the transfer function $H(s) = u(s)/f(s)$ of the system, and find the poles and zeros. Is the system stable? Explain.

(d) [10 pts] Now consider closed-loop feedback, in which the force $f(t)$ is given by

$$f(t) = f_{ext}(t) - K\, u(t)$$

where $f_{ext}(t)$ is the externally applied force, and K is the feedback gain. Draw the closed-loop system block diagram, and find the closed-loop transfer function $F(s) = u(s)/f_{ext}(s)$.

(e) [5 pts] Now consider that the biomechanical system has negative stiffness, $k < 0$. Find the range of values for the feedback gain K in the closed-loop system to stabilize the biomechanical system. How stable can you make it? Explain.

3. **[20 pts]** Linear time invariant and conservative biosystems:

   (a) [5 pts] Give an example of a biosystem that is linear but not time-invariant.

   (b) [5 pts] Describe two ways in which blood pressure is affected by vessel diameter.

(c) [5 pts] How are the Laplace and Fourier transfer functions of a linear time-invariant system related? Explain.

(d) [5 pts] Show that an integrator, with transfer function $\frac{1}{s}$, is marginally stable.