BENG 122A Fall 2026

Quiz 1

Tuesday, October 27, 2025

***Name** (Last, First):* _____

- This quiz is open book, open notes, and online, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due October 29, 2026 at 11:59pm, over Canvas. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 3 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

1. **[45 pts]** Consider the following biochemical reaction taking place in an organ in the body:

$$\mathrm{A} \underset{k_r}{\overset{k_f}{\rightleftharpoons}} \mathrm{B} + \mathrm{C}$$

where compound A decomposes to generate compounds B and C at rate k_f, and B and C recombine to regenerate A at rate k_r. Compound B is present at much higher concentration than A and C, so we assume that its concentration remains approximately constant, $[\mathrm{B}] \approx [\mathrm{B}]_0$. Compounds A and B recirculate in the organ without decay, whereas C exits the volume V of the organ at a non-zero flow rate Q.

   (a) [10 pts] Under these approximating assumptions, write the ODEs in the concentrations $[\mathrm{A}]$ and $[\mathrm{C}]$ that describe both the reaction kinetics and the flow.

(b) [5 pts] Find the equilibrium (*i.e.*, the steady-state) concentrations.

(c) [20 pts] Use Laplace transforms to find the concentrations $[\mathrm{A}]$ and $[\mathrm{C}]$ as a function of time, starting from initial conditions $[\mathrm{A}](0) = [\mathrm{A}]_0$ and $[\mathrm{C}](0) = [\mathrm{C}]_0$.

(d) [10 pts] Is the above approximating assumption that $[B] \approx [B]_0$ reasonable under these conditions? Quantify the change in the concentration $[B]$ from its initial value $[B]_0$ to its final steady state value, and how it affects all steady state values in (b).

2. **[35 pts]** Consider the following ODE describing the dynamics of a simplified model of the cardiovascular system with the heart delivering a pressure $p_{heart}(t)$ which through the vasculature produces a pressure $p_{organ}(t)$ in a single distant organ:

$$C\,\frac{d}{dt}p_{organ}(t) = \frac{1}{R}\,(p_{heart}(t) - p_{organ}(t)).$$

   (a) [10 pts] Sketch a circuit diagram of the simplified model of the cardiovascular system described by this ODE, and identify and interpret the parameters R and C in terms of its physical properties.

(b) [5 pts] Find the transfer function $H(s) = p_{organ}(s)/p_{heart}(s)$ of the system, and find the poles and zeros. Is the system stable? Explain.

(c) [10 pts] Find the step response of the system, as the organ pressure $p_{organ}(t)$ for a unit step in heart pressure at time zero, $p_{heart}(t) = 1$. What is the time constant τ? Express the 90% rise time and $\pm 10\%$ settling time in terms of the time constant.

(d) [10 pts] What would be the effect of closed-loop feedback, by establishing a parallel path of vasculature bypassing flow across the organ with effective shunt resistance R_{shunt}? Explain.

3. **[20 pts]** Linear time invariant and conservative biosystems:

   (a) [5 pts] Give an example of non-stationary stiffness in a biophysical system, and explain its effect on the dynamics.

   (b) [5 pts] Give an example of nonlinear damping in a biophysical system, and explain its effect on the dynamics.

(c) [5 pts] Give an example of a function $f(t)$ for which its Fourier transform $F(j\omega)$ and Laplace transform $F(s)$ are identical under the transformation $s = j\omega$. Is this true for any function? Explain.

(d) [5 pts] What can you say about stability of an undamped second-order system? Consider both positive or negative coefficients in the transfer function.