BEGN 122A Fall 2021

Quiz 2

Tuesday, November 16, 2021

Name *(Last, First):* ____

- This quiz is open book, open notes, and online, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due November 17, 2021 at 11:59pm, over Canvas. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 3 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

1. **[50 pts]** Consider the following linear time-invariant (LTI) biosystem:

$$H(s) = \frac{10s}{(s+0.1)(s+1)(s+10)}.$$

(a) [10 pts] Sketch the Bode plot.

(b) [10 pts] First consider purely proportional control. Find the largest value of proportional gain K_p for which the closed-loop system is stable with at least 45 degree phase margin. Find the closed-loop DC error.

(c) [10 pts] Now add integral control. Find the values of integral gain K_i and proportional gain K_p that minimize the closed-loop DC error, while maintaining stable closed-loop response with at least 45 degree phase margin. Find the closed-loop DC error.

(d) [10 pts] Now add derivative control, keeping proportional and integral gains at the same levels. Find the value of derivative gain K_d to maintain a phase margin of at least 90°. Find the closed-loop DC error.

(e) [10 pts] Find the closed-loop transfer function for these values of proportional, integral, and derivative gain, and sketch the Bode plot. Validate that the closed-loop DC error and high-frequency dynamics are consistent with those predicted by the above open-loop analysis.

2. **[30 pts]** Here we consider the dynamics of a coupled set of two ordinary differential equations in state variables $u(t)$ and $v(t)$ describing a mechanical biosystem with mass m and stiffness k driven by a force input $f(t)$:

$$\begin{aligned} \frac{du}{dt} &= v(t) \\ m\,\frac{dv}{dt} &= -k\,u(t) + f(t). \end{aligned}$$

(a) [10 pts] Show a block diagram of a closed-loop system with a proportional-integral controller driving the velocity $v(t)$ of the biosystem towards a target $v_{\text{target}}(t)$ with proportional gain K and integral gain K'. You may assume that there is no measurement error in the output $v(t)$.

(b) [10 pts] Now consider a closed-loop system with a proportional-derivative controller driving the position $u(t)$ of the biosystem towards a target $u_{\text{target}}(t)$ with proportional gain K' and derivative gain K, again without measurement error. Show that the two closed-loop systems are equivalent, generating the same state variables $u(t)$ and $v(t)$, under the condition $u_{\text{target}}(t) = \int v_{\text{target}}(t)dt$.

(c) [10 pts] For mass $m = 2$ kg and stiffness $k = 1$ kg s^{-2}, find the control gain parameters K and K' to produce a critically damped closed-loop response with time constant $\tau = 1$ s (two coinciding poles at -1 s^{-1}). What is the closed-loop DC error, and how could you improve on it?

3. **[20 pts]** Short answer questions– let your imagination and creativity go loose!

   (a) [10 pts] Give an example of an unstable biosystem that is stabilized by proportional feedback control.

   (b) [10 pts] Give an example of a stable biosystem that becomes unstable under the effect of measurement delay in proportional feedback control.