BEGN 122A Fall 2022

Quiz 2

Tuesday, November 22, 2022

***Name** (Last, First):* ____

- This quiz is open book, open notes, and online, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due November 23, 2022 at 11:59pm, over Canvas. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 3 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

1. **[50 pts]** Consider the following linear time-invariant (LTI) biosystem:

$$H(s) = \frac{s+1}{s^2+10s}.$$

(a) [10 pts] Sketch the Bode plot.

(b) [10 pts] First consider proportional control with 60 dB gain ($K_p = 1{,}000$), without measurement error. Find the phase margin of the open-loop system, and find the DC error and the -3 dB bandwidth of the closed-loop system.

(c) [10 pts] Now consider error in the measurement of the biosystem, with the measurement system given by:

$$G(s) = \frac{1}{1 + 10s}.$$

Find the maximum value of the proportional gain K_p maintaining stable closed-loop response with at least 45° phase margin. How does this affect the DC error and the -3 dB bandwidth of the closed-loop system? Explain.

(d) [10 pts] Consider the biosystem again with measurement error, but now with proportional-derivative control with 60 dB proportional gain ($K_p = 1{,}000$). Find the minimum value for the derivative gain K_d to obtain a phase margin of at least 90°. Show that this design achieves the same DC error and -3 dB bandwidth of the closed-loop system without measurement error in (b).

(e) [10 pts] Would additional integral control help to improve system performance in any way? Explain.

2. **[30 pts]** Here we consider the dynamics of a coupled set of two ordinary differential equations in state variables $u(t)$ and $v(t)$ describing the interaction between two bacterial populations competing for growth, where the output population $u(t)$ is driven by a nutrient supply input $f(t)$:

$$\begin{aligned} \frac{du}{dt} &= -v(t) \\ \frac{dv}{dt} &= -u(t) + f(t). \end{aligned}$$

   (a) [10 pts] Find the transfer function $H(s) = u(s) \,/\, f(s)$ of the biosystem, and find the poles and zeros. Is the biosystem stable? Explain.

(b) [10 pts] Now consider proportional-derivative control $F(s) = K(1 - s)$ acting on the biosystem without measurement error, with $K > 1$. Find the phase margin and the closed-loop DC error. Explain what you observe.

(c) [10 pts] Now consider integral-derivative control $F(s) = K\left(\frac{1}{s} - s\right)$ acting on the biosystem without measurement error, with $K > 0$. Find the phase margin and the closed-loop DC error. Compare with (b), and explain.

3. **[20 pts]** Short answer questions– let your imagination and creativity go loose!

(a) [10 pts] Give an example where derivative control is essential to improve on closed-loop feedback control of a biosystem subject to measurement error at high frequencies.

(b) [10 pts] Give an example where integral control is essential to improve on closed-loop feedback control of a biosystem lacking gain at low frequencies.