BENG 122A Fall 2023

Quiz 2

Monday, November 20, 2023

***Name** (Last, First):* ______________________________

- This quiz is open book, open notes, and online, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due November 22, 2023 at 11:59pm, over Canvas. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 3 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

1. **[50 pts]** Consider the following linear time-invariant (LTI) biosystem:

$$H(s) = \frac{s + 10}{s^2 + 101s + 100}.$$

(a) [10 pts] Sketch the Bode plot.

(b) [10 pts] First consider proportional control with 60 dB gain ($K_p = 1{,}000$), without measurement error. Find the phase margin of the open-loop system, and find the DC error and the -3 dB bandwidth of the closed-loop system.

(c) [10 pts] Now consider error in the measurement of the biosystem, with the measurement system given by:

$$G(s) = \frac{1}{10s + 1}.$$

Find the maximum value of the proportional gain K_p maintaining stable closed-loop response with at least 45° phase margin. How does this affect the DC error and the -3 dB bandwidth of the closed-loop system? Explain.

(d) [10 pts] Consider the biosystem again with measurement error, but now with proportional-integral control. Find the maximal values for the proportional gain K_p and integral gain K_i maintaining a phase margin of at least 45°. Find the DC error and -3 dB bandwidth of the closed-loop system.

(e) [10 pts] Would additional derivative control help to improve system performance in any way? Explain.

2. **[30 pts]** Here we consider the dynamics of a coupled set of two ordinary differential equations in state variables $u(t)$ and $v(t)$ describing the biomechanics of a biosystem driven by a force input $f(t)$:

$$\begin{aligned} \frac{du}{dt} &= v(t) \\ m\,\frac{dv}{dt} &= -k\,u(t) + f(t) \end{aligned}$$

with positive mass m, but with negative stiffness $k < 0$.

(a) [10 pts] Find the transfer function $H(s) = u(s)\,/\,f(s)$ of the biosystem, and find the poles and zeros. Is the biosystem stable? Explain.

(b) [10 pts] Consider proportional control $F(s) = K_p$ acting on the biosystem without measurement error. For what range of values of K_p is the closed-loop system stable? Explain what you observe.

(c) [10 pts] Now consider proportional-integral (PI) control $F(s) = K_p + K_i \frac{1}{s}$ acting on the biosystem without measurement error. Setting the zero of the PI controller to cancel one of the poles of the biosystem in the open-loop transfer function, find the values of K_p and K_i for which the closed-loop system is stable and critically damped.

3. **[20 pts]** Short answer questions– let your imagination and creativity go loose!

   (a) [10 pts] Give an example where proportional feedback is inadequate to control a biosystem.

   (b) [10 pts] Give an example where derivative control stabilizes an unstable biosystem.