BEGN 122A Fall 2024

Quiz 2

Tuesday, November 19, 2024

Name *(Last, First):* ______________________________

- This quiz is open book, open notes, and online, but web search is prohibited. You may follow electronic links from Canvas or the class web pages, but not any further. **No collaboration or communication in any form is allowed**, except for questions to the instructor and TAs.
- The quiz is due November 21, 2024 at 11:59pm, over Canvas. It should approximately take 2 hours to complete, but there is no time limit other than the submission deadline. Do not discuss any class-related topics among yourselves before or after you have completed your quiz, and until the submission deadline has passed.
- There are 3 problems. Points for each problem are given in **[brackets]**. There are 100 points total.

1. **[50 pts]** Consider the following linear time-invariant (LTI) biosystem:

$$H(s) = \frac{10s^2 + 101s + 10}{s + 1}.$$

(a) [10 pts] Sketch the Bode plot.

(b) [10 pts] First consider proportional control with 20 dB gain ($K_p = 10$), without measurement error. Find the DC error of the closed-loop system. What can you say about the phase margin, and the closed-loop bandwidth?

(c) [10 pts] Now consider error in the measurement of the biosystem, with the measurement system given by:

$$G(s) = \frac{1}{10s + 1}.$$

Find the value of the proportional gain K_p to give no more than 0.1% DC error. Does this affect the phase margin? Explain.

(d) [10 pts] Consider the biosystem again with the same measurement error and proportional control gain K_p, but now including integral control. Find the maximum value for the integral gain K_i to maintain stability. Find the DC error of the closed-loop system.

(e) [10 pts] Would additional derivative control help to improve system performance in any way? Explain.

2. **[30 pts]** Here we consider the dynamics of a coupled set of two ordinary differential equations in state variables $u(t)$ and $v(t)$ describing the biomechanics of a biosystem driven by a force input $f(t)$:

$$
\begin{aligned}
\frac{du}{dt} &= v(t) \\
m \frac{dv}{dt} &= -\gamma\, v(t) + f(t)
\end{aligned}
$$

with positive mass m, but with negative damping $\gamma < 0$.

(a) [10 pts] Find the transfer function $H(s) = u(s) \,/\, f(s)$ of the biosystem, and find the poles and zeros. Is the biosystem stable? Explain.

(b) [10 pts] Consider derivative control $F(s) = K_d\, s$ acting on the biosystem without measurement error. For what range of values of K_d is the closed-loop system stable? Explain.

(c) [10 pts] Now consider the effect of measurement error acting on the biosystem with additional gain

$$G(s) = \frac{1}{1 + \tau s}$$

in the open-loop transfer function. Find the value of the derivative control gain K_d for which the closed-loop system is stable and critically damped.

3. **[20 pts]** Short answer questions– let your imagination and creativity go loose!

   (a) [10 pts] Give an example of a biosystem where integral control substantially improves the closed-loop DC error. Explain.

   (b) [10 pts] Give an example of a biosystem where derivative control substantially improves the high-frequency response. Explain.